Baldassarri: "Radius of convergence of p-adic connections & the Berkovich ramification locus"

k is a p-adic field, alg closed, k° ring of integers, $\tilde{k}$ residue field

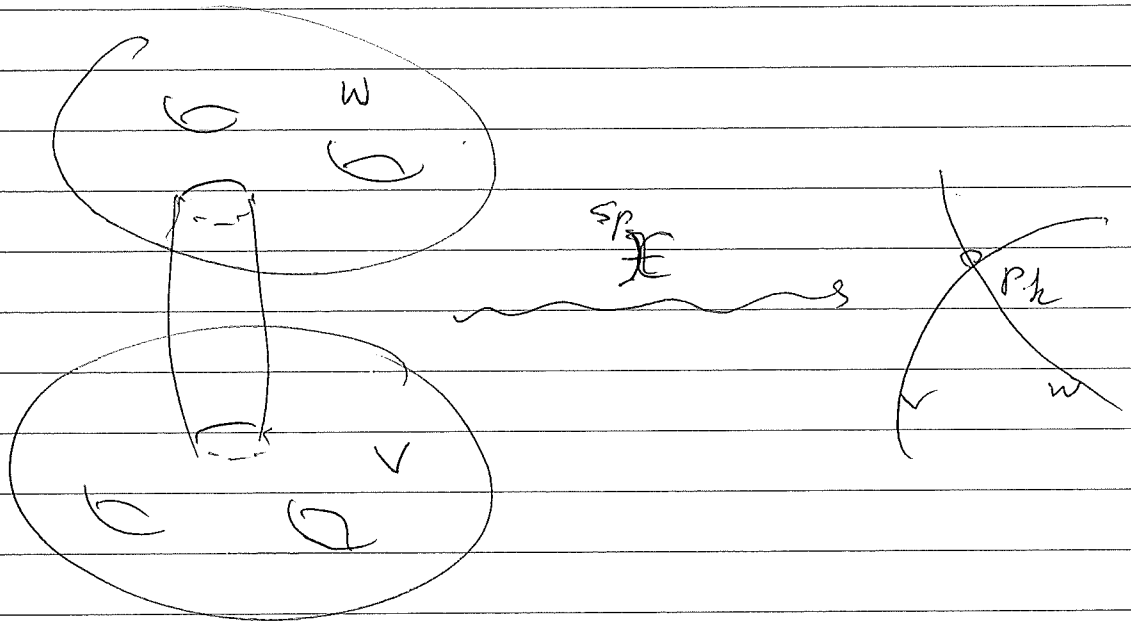
let X be a compact k-analytic curve (Berkovich sense) smooth (rigid analytic sense)

$\mathfrak{X}$ = strictly semistable k°-formal model

$(\mathcal{E}, \nabla) \in MIC(X/k)$ $\qquad sp_{\mathfrak{X}} : X \longrightarrow \mathfrak{X}_S \quad \forall x \in X(k)$

module with integrable connection

$D = D_{\mathfrak{X}}(x, 1^-) \ni x \qquad sp_{\mathfrak{X}}(D) = \{pt\}$



$\forall x \in X(k)$ $\mathfrak{X}$-optimal basis of solutions of $(\mathcal{E},\nabla)$ = a k-basis of $\mathcal{E}_x^\nabla$

$(v_1, \dots, v_\mu)$ $\exists$?? $\rho_1(x) \geq \rho_2(x) \geq \dots > \rho_\mu(x) \geq 0$ s.t.

v_i prolongs to a horizontal section of $(\mathcal{E},\nabla)$ on $\mathfrak{X}$, $D_{\mathfrak{X}}(x, p^{-\delta_i(x)})$ s.t.

$h_{\mathcal{E}}(u) = \sum \rho_i(x) \geq 0$ is <u>minimal</u>

the ρ_i are the slopes of the <u>convergence polygon</u> at $x \in X(k)$

$\mathcal{H}(x) \supset k$

$X \xrightarrow{\psi} X$

$x' \longmapsto x$

CANONICAL

$x \in X$ $\quad \mathcal{N}_{\mathfrak{X}}(x, (\mathcal{E},\nabla)) = \mathcal{N}_{\mathfrak{X} \hat{\otimes}_k \mathcal{H}(x)}(x', \psi^*(\mathcal{E},\nabla))$

$X \xrightarrow{\mathcal{N}}$ Polygons

In general $H^*_{dR}(X,(\mathcal{E},\nabla))$ not f.g.

can give conditions for finite dimensionality in terms of factorization

$\mathfrak{X}$ $\quad RAM(\mathcal{E},\nabla) = (\mathrm{rk}\,\mathcal{E})\chi_{dR}(X) - \chi_{dR}(X,(\mathcal{E},\nabla))$

$\mathcal{E} \curvearrowleft h_{\mathbb{C}}$

Q does $(\varepsilon, \vec{\nabla})$ admit a controlling graph

$$X \xrightarrow{\tau_y} T_y \longrightarrow T_{\mathfrak{X}} \qquad y \longrightarrow \mathfrak{X}$$

STRONG: Given $\mathfrak{X}$ $\exists\, y \to \mathfrak{X}$??

WEAK: Does there exist $\mathfrak{X}$ such that $N_{\mathfrak{X}}$ factors through $T_{\mathfrak{X}}$

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Curves $\varphi: Y \to X$ finite say étale

$\mathfrak{X}$ sstable mod of X

sections $s_1, \dots s_\ell$ of φ at $x \in X(k)$ in $\hat{Q}_{X,x}$

Radii of conv. ? w.r.t. $\mathfrak{X}$

(Dagger) R-H formula? (l-adic) Huber.

Xander Faber "dop and geom of berkovich ramification"

basic result: p-adic Rolle thm:

p-adic Rolle: $\varphi: D \longrightarrow \Delta^n$ open disk étale

$\forall$ open disk $D' \subset D$ rel radius $\leq p^{-\frac{1}{p-1}}$, φ is open embedding.

(board turned up again)

Thm $\varphi: Y \to X$ finite

$\varphi^{-1}(B) = Z \to B$ branch locus. $D \subset Y \setminus Z$

$T: D \xrightarrow{\sim} D(0,1^-)$

D' T-Radius $\leq p^{-1/p-1}$

$\varphi_{D'}$ open embedding

if φ is residually separable at the maximal point ξ of D in Y, then is an open embedding.

$Sect(\mathcal{H}/X) = Hom_{\mathcal{O}_X\text{-alg}}(\varphi_* \mathcal{O}_Y, \mathcal{O}_X)$ on X

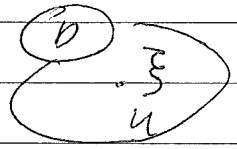
$\varphi_*(\mathcal{O}_Y, d_Y) = (\mathcal{F}, \nabla) \in MIC(X(!B)/k)$

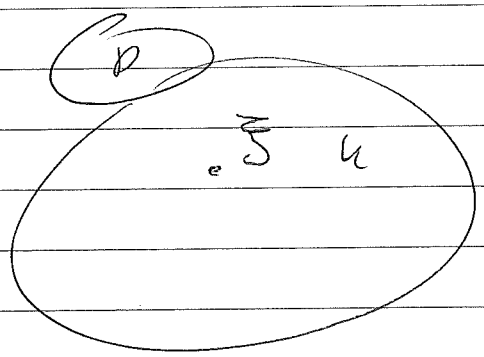
$Sect(\mathcal{F}, \nabla)$ $\quad x \in X \setminus B$

$\varphi^+(\mathcal{F}, \nabla) = (\mathcal{E}, \nabla) \in MIC(Y/k)$ $\quad \mathcal{E} = \varphi^* \varphi_* \mathcal{O}_Y$.

$D \subset Y$ T formal coord

$\mathcal{O}_Y(D) = \{\xi\}$





$\mathcal{H}_U(D)$ analytic elements completion of $\mathcal{O}_X(D) \cap \mathcal{O}_{U,\xi}$??

can't follow anymore ---

Coleman, LIII:

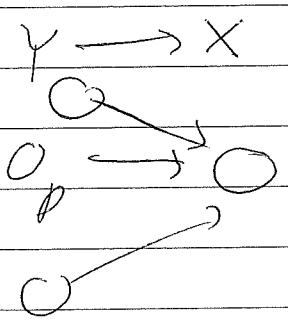
$\mathcal{Y}'$ s.st model of $\mathcal{Y}'_\eta = Y$

$\mathcal{X}'$ in $\mathcal{X}'_\eta = X$ $\quad \exists \; \mathcal{Y} \to \mathcal{Y}'$ $\quad \phi_\eta = \varphi$

ϕ finite.

$\mathcal{X} \dashrightarrow \mathcal{X}'$

Thm the connection $(\mathcal{E}, \nabla)$ prolongs to a log stratification on $\phi^* \phi_* \mathcal{O}_Y$



ξ = generic point of a component of $\mathcal{Y}$

again cannot follow, all over the blackboard.