Alan Lauder

level weight
N, k, prime $p \nmid N$, "precision" m

Problem compute $P_p(t) = \det(1 - t U_p \mid M_k(Np))$

$$U_p : \sum a_n q^n \longmapsto a_{np} q^n$$

mod p^m for k very large

Motivation Gouvea Mazur conj:

$k \equiv k' \bmod (p-1)$ and p-adically close
$\Rightarrow$
low slopes $P_k(t)$ and $P_{k'}(t)$ should agree

Stein, Buzzard–Calegari.

Problem Compute $P_p(t) = \det(1 - tU_p \mid M_k(N))$ mod p^m

Buzzard, Loeffler, Kilford $N=1$, $p < 23$

Thm can do this in polynomial time in N, p, m and in linear time in $\log k$.

$N=1$ classical modular form of weight k $\longleftrightarrow$ $k/2$ fold differential on $X_0(1) \simeq \mathbb{P}^1$.

p prime overconvergent modular form $\hookrightarrow$ differential on $\mathbb{C}$ $X_0(1)$ with disks of radius < 1 around the "supersingular pts" removed.

$p \geq 5$

$$\begin{array}{ccc} \mathbb{Z}_p & \subset & B \\ \cap & & \cap \\ \mathbb{Q}_p & \subset & K \end{array} \qquad r \in B.$$

$$M_k(B,r) = \left\{ \sum_{j=0}^{\infty} r^j \frac{b_j}{E_{p-1}^j} \;\middle|\; b_j \in W_j(B) \text{ and } \lim_{j\to\infty} b_j = 0 \right\} \text{ where}$$

$W_0(B) := M_k(B) :=$ classical forms wt k with coefficients in B, and for $j \geq 1$ $W_j(B)$ is chosen so that:

$$M_{k+j(p-1)}(B) = W_j(B) \oplus E_{p-1}^j \cdot M_{k+(j-1)(p-1)}(B)$$

$$M_k(K,r) = M_k(B,r) \otimes_B K.$$

$0 < \mathrm{ord}_p(r) < \frac{p}{p+1} \Rightarrow U_p$ acts c.c on $M_k(K,r)$.

Coleman's trick

$k = k_0 + j(p-1), \quad 0 \leq k_0 < p-1$

$E_{p-1}^j : M_{k_0}(K,r) \to M_k(K,r)$

Consider $E_{p-1}^{-j} \circ U_p \circ E_{p-1}^j$ on $M_{k_0}(K,r)$

$U_p \circ g, \quad g := \frac{E_{p-1}(q)}{E_{p-1}(qt)}$

Lem
$g \in M_0(B,r)$ for $0 \leq \mathrm{ord}_p(r) < \frac{1}{p+1}$

g, f, g wts ℓ, r, ℓ, level N

Hida families Darmon Rotger.

$$\mathcal{L}_p(\mathcal{G}, F, \mathcal{G})(\ell, r, \ell) = * \times \text{p-adic integral} \in \mathbb{C}_p$$

$$\wr$$

$$\log_p P \qquad P \in E(\mathbb{Q})$$

$$\| \qquad \| \lim_{r \to \infty} U_p^{n!} \qquad \text{overconvergent}$$

$$c(g(p), \underbrace{e_{ord}(\underbrace{d^{-1}(f) \times g}_{\theta})}_{N_p})$$

levels $\overline{N_p}$

problem is to compute the ordinary projection operator

matrix U_p:

$$\text{ord}_p r \simeq \frac{1}{p+1}$$

$$\text{mod } p^m \quad \left(* \;\middle|\; 0 \right) \begin{pmatrix} \ulcorner \\ * \\ \lrcorner \end{pmatrix} \quad \text{if } \theta \text{ is } \frac{p}{p+1} \text{ overconvergent}$$

reduces to a finite matrix product.

(showed slides)