MATH UN1101

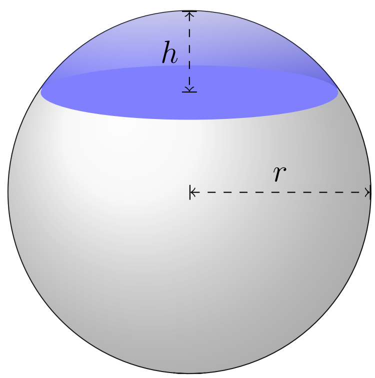
CALCULUS I (SECTION 5) - SPRING 2019

HOMEWORK 13 (DUE MAY 07)

Each part (labeled by letters) of every question is worth 2 points. There are 10 parts, for a total of 20 points. You are encouraged to discuss the homework with other students but you must write your solutions individually, in your own words.

(1) Find the area enclosed by the two curves. Roughly sketch the area.
   (a) $y = x^3$ and $y = x$.
   (b) $y = \cos x$ and $y = \sin x$ on $[0, \pi]$.

(2) Consider the cap of height h in a sphere with radius r.



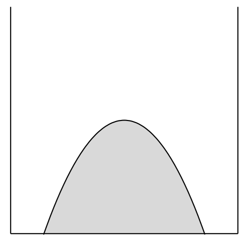
   (a) Write an integral which computes the volume of the cap. (Hint: rotate the situation 90° first.)
   (b) Explain in words what is calculated by the Riemann sum corresponding to the integral, and why it approximates the volume.
   (c) Evaluate the integral in (a) to get the volume of the cap.
   (d) Explain what answer you expect to get in (c) when $h = r$. Check that this is indeed the case.

(3) Let $f(x)$ be a continuous function on $[a, b]$. By analogy with volumes of solids of revolution, make a guess for what the following integral represents:

$$\int_a^b 2\pi f(x)\, dx.$$

Explain your guess. Pick an example for $f(x)$ to illustrate why your guess is correct.

(4) After a whole semester of throwing your homework into a hole, you discover the hole is not actually infinitely deep and has a bottom! All the homeworks you threw in have formed a nice little pile at the bottom.



The pile is the solid of revolution obtained by rotating $y = 1 - x^2$ on $[0, 1]$ around *the y-axis*. We want to find its volume.

(a) Sketch a 3d diagram of the solid, with x, y, z axes labeled.

(b) Write x as a function of y, so that we can do the usual thing with solids of revolution but around the y-axis.

(c) Using (b), write the volume as an integral of the form $\int_0^1 f(y)\,dy$, for some function $f(y)$. Evaluate the integral to find the volume.