MATH UN1101

CALCULUS I (SECTION 5) - SPRING 2019

PRACTICE MIDTERM 1

The exam is **75 minutes**. No additional material or calculators are allowed.

- Write your **name and UNI** clearly on your exam booklet.
- **Show your work** and reasoning, not just the final answer. Partial credit will be given for correct reasoning, even if the final answer is completely wrong.
- **Don't cheat**!
- Don't panic!

(1) (10 points) State whether the following are true/false. No explanations necessary.

(a) For every real number x, the identity $\sin(\sin^{-1}(x)) = x$ holds.

(b) If $f(x)$ is continuous at $x = a$, then $f(x)$ is differentiable at $x = a$.

(c) The function $f(x) = xe^{\sin(x)}$ is continuous on its domain.

(d) The limit

$$\lim_{x\to 0} \frac{x\sin(4/x^3)}{\sqrt{x}}$$

exists.

(e) If f is continuous at 2, then $\lim_{x\to 2} f(x+4)$ must exist.

(f) The equation $x^7 + x - 1 = 0$ has a solution in $(0, 1)$.

(g) If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$, then $\lim_{x\to a} f(x)/g(x)$ does not exist.

(h) $f(a) = g(a)$ for all real numbers a, where

$$f(x) = \frac{x^2-4}{x-2}, \qquad g(x) = x + 2.$$

(i) $\lim_{x\to a} f(x) = g(a)$ for all real numbers a, where

$$f(x) = \frac{x^2-4}{x-2}, \qquad g(x) = x + 2.$$

(j) The derivative of $f(x) = (2x)^3$ is $3 \cdot (2x)^2 = 12x^2$.

(2) Find the limit, if it exists. If it does not exist, explain why.

(a) (5 points)

$$\lim_{x\to 0} f(|x|),$$

where the function $f(x)$ is

$$f(x) = \begin{cases} \ln(-x) & x < 0 \\ \cos^{-1}(x) & x \geq 0 \end{cases}.$$

(b) (5 points)

$$\lim_{x\to\infty} \frac{e^{-x}}{6+\cos(x)}.$$

(3) Consider the function

$$f(x) = \frac{g(x)+1}{g(x)-1}.$$

(a) (5 points) If $g(x)$ is continuous everywhere except $x = 2$, where is $f(x)$ continuous?

(b) (5 points) State what it means for $f(x)$ to have a horizontal asymptote $y = L$. If $\lim_{x\to\pm\infty} g(x) = \infty$, what must L be?

(c) (5 points) Compute the derivative of $f(x)$. State important differentiation rules that you use.

(4) Let $f(x) = A/(3-x)$ for some constant A.

(a) (5 points) Write down the definition of the derivative $f'(x)$, and compute the derivative $f'(1)$ using the definition.

(b) (5 points) If the tangent line to $f(x)$ at $x = 1$ is $y = 2x + 2$, what is A?

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(5) (5 points) For the following graph, state where it fails to be continuous and where it fails to be differentiable. Where the derivative exists, sketch its derivative.