Penrose's

Weyl Curvature Hypothesis

WCH ($\lesssim 1978$)

& his

Conformal Cyclic Cosmology

CCC ($\gtrsim 2005$)

WCH : RP in "General Relativity: an Einstein Centennial Survey" C.U.P. 1979

CCC : RP in "On Space and Time" C.U.P. 2008
or "Cycles of Time" Bodley Head 2010

Connected by conformal geometry

Plan of this talk:

- generalities on conformal geometry
- the Big Bang
  - how special it was, WCH
  - producing examples with WCH
- the remote future with $\Lambda > 0$
  - future infinity $\mathscr{I}$
- CCC : "an outrageous suggestion"
  - circles in the CMB

The idea of conformal geometry:

- $g_{ab} \longrightarrow \tilde{g}_{ab} = \Omega^2(x)\, g_{ab}$ rescaling

- $[g] = \{ \tilde{g} \mid \tilde{g} = \Omega^2 g \}$ conformal class

Some conformal generalities

Metric vs. Conformal metric

$g = g_{ab}$ | $[g] = \{\Omega^2(x)\, g_{ab}\}$

10 functions | 9 functions

length | angles

time | ratios L_1/L_2

clocks | light-cone

massive particles | massless particles

$e, p, q, \dots$ | $\gamma, \nu, \dots$

Curvature:

$$\overset{20}{\text{Riem}} = \overset{10}{\text{Weyl}} + \overset{10}{\text{Ricci}}$$

General Relativity

$$\text{Ricci} = \text{Matter}$$

if $\tilde{g}_{ab} = \Omega^2 g_{ab}$ then

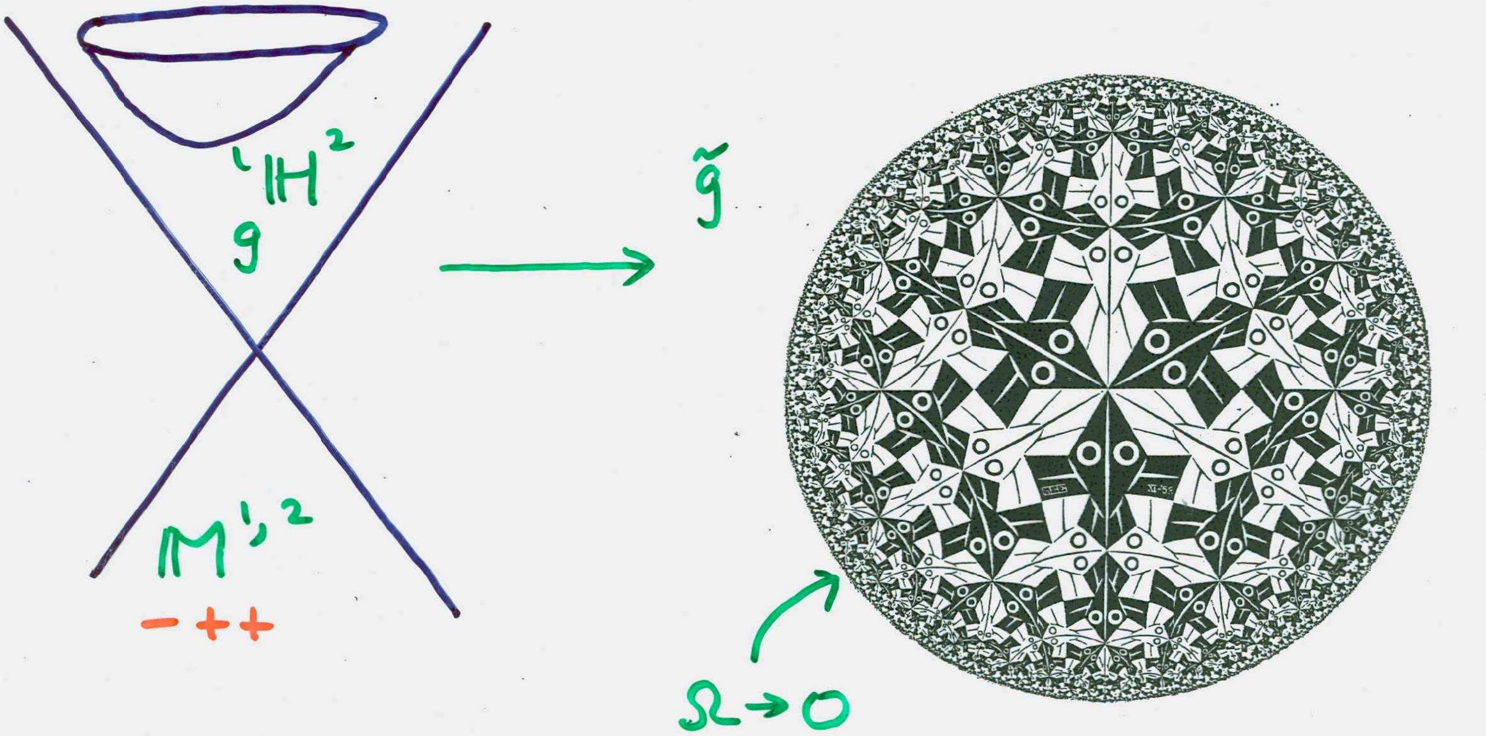
$$\widetilde{\text{Weyl}} = \text{Weyl}$$

$$\widetilde{\text{Ricci}} = \text{Ricci} + \frac{\nabla\nabla\Omega}{\Omega} + \frac{\nabla\Omega\nabla\Omega}{\Omega^2} + \cdots$$

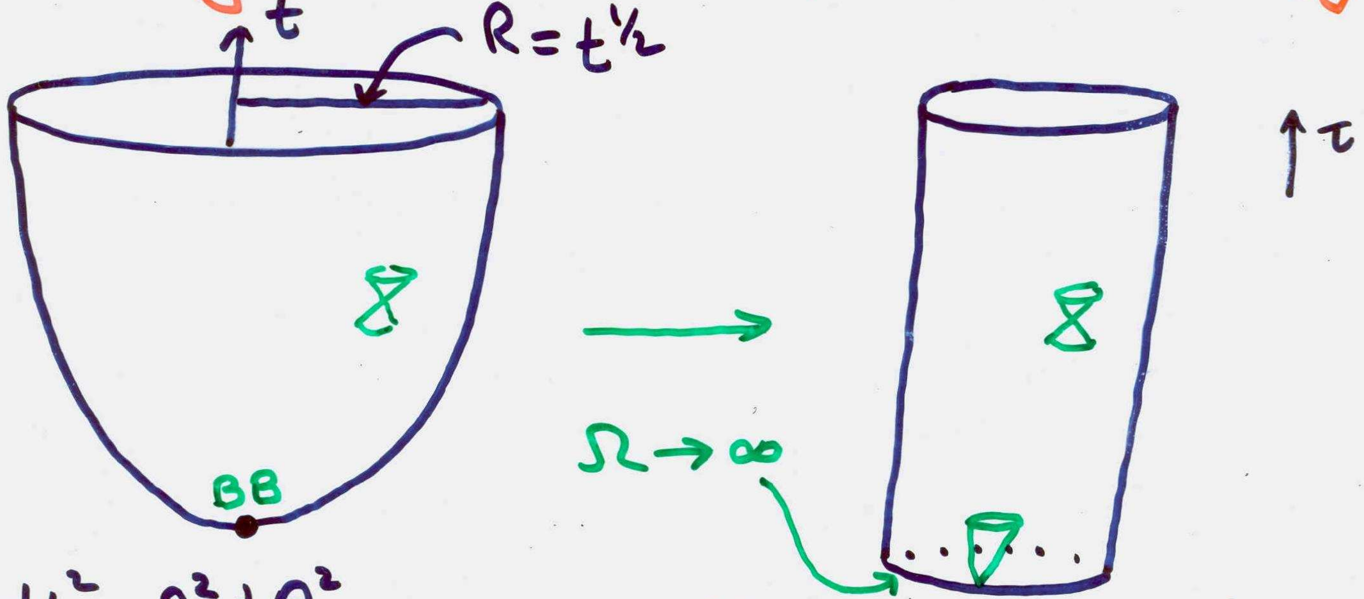
Two faces of conformal rescaling:

$$\tilde{g}_{ab} = \Omega^2 g_{ab}$$

- Escher's "Circle Limit"



- Homogeneous Isotropic Cosmology

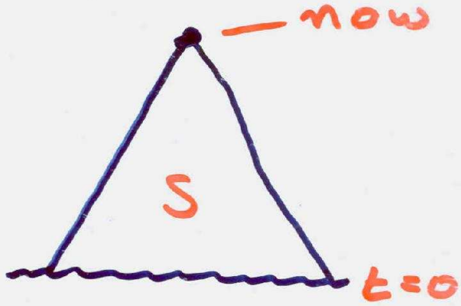


$$g = dt^2 - R^2 d\theta^2$$

$$\tilde{g} = d\tau^2 - d\theta^2 = R^{-2} g$$

$$d\tau = \frac{dt}{R} \quad : \text{conformal time}$$

The Big Bang



- was very special* RP

$S \simeq 10^{88} k - 10^{111} k$

observed

$S_{BH} \simeq 10^{123} k$

possible

$V = 10^{10^{88}} \ll \cdots < 10^{10^{123}}$

- "This restriction on the early geometry
... something like the Weyl curvature vanishes at any initial singularity" RP 79

- Riem = Weyl + Ricci   WCH

- was hot : $E \gg mc^2$

effectively massless

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* • ?2nd law of TD now but
• matter was in thermal eq^{m} ie high entropy then
• so gravity was in low entropy then.

Observation

"The right quantum gravity"

$\Downarrow$

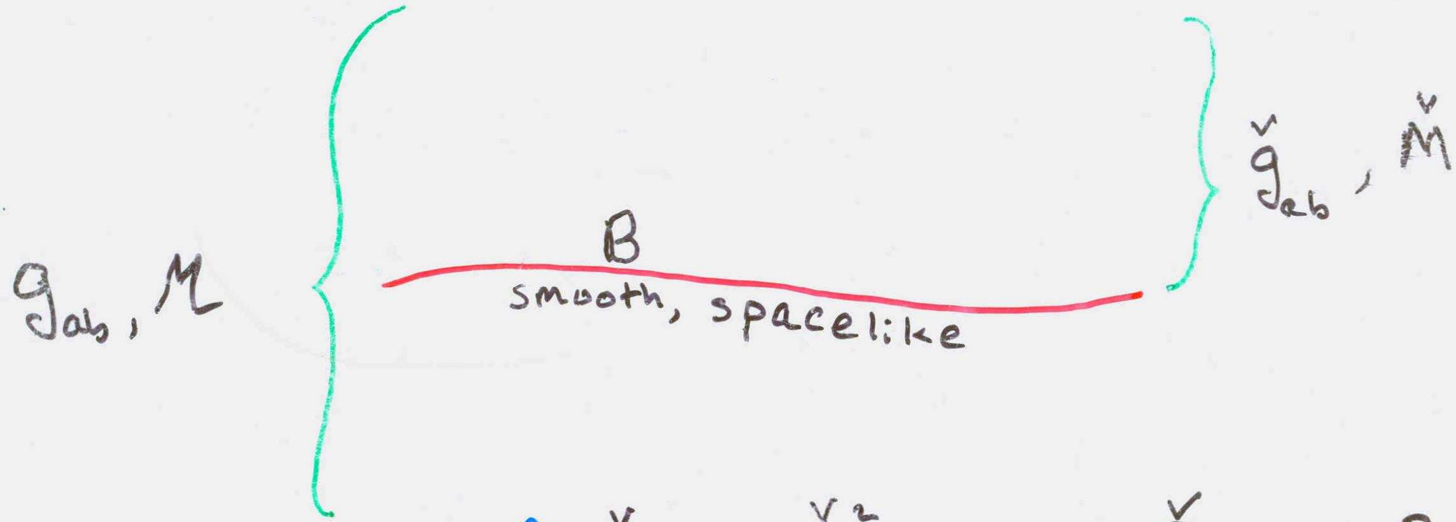
WCH

$\Downarrow$ as a selection principle

Examples

- how do you recognise <u>finite</u> $\boxed{\text{Weyl}}$ in the midst of <u>singular</u> $\boxed{\text{Riemann}}$?

One way:



If $\check{g}_{ab} = \check{\Omega}^2 g_{ab}$ with $\check{\Omega} = 0$ at B

then $\check{C}_{abc}{}^{d} = C_{abc}{}^{d}$ $\therefore$ finite at B

but $\check{R}_{ab}$ (Ricci) singular at B

IDEA: work with g_{ab}

Claim: there is a well-posed initial value problem for (g, m) with data at B making $(\check{g}, \check{m})$ a solution of the Einstein equations with matter for

- perfect fluid with $p = (\gamma - 1)\rho$, $1 < \gamma \leq 2$ and with Λ; the data are the unconstrained 3-metric of B

  (no separate matter data; Weyl(0) = 0 ⇒ Weyl(t) = 0)

  gr-qc 9903008 Anguige-Tod

- massless Einstein-Vlasov (massless, collisionless)

  $$T_{ab}(x) = \int P_a P_b f(x, p)\, d\omega_p \qquad \mathcal{L}_X f = 0$$

  with data $f(t=0, x^i, P_a)$ $\qquad \int P_i f_0 = 0$

  (no separate geometric data;

  Weyl(0) = 0 ⇏ Weyl(t) = 0) 9903009 Anguige

- a range of other matter models and spatial homogeneity. 0209071 Tod

  0704.2506

Conversely: when does "finite Weyl" imply the existence of a conformal extension?

An answer is known 0710.5552/.5723

Lübbe - Tod

We should ask: what could make the Big Bang so special?

We could ask: is the region of M before $\check{M}$ really there? What is before the Big Bang in M?

Augustine

Veneziano hep-th 9802057

Hoyle Ap. J. **196** (1975) 661-670

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  0704.2506

observed

Positive Λ & the remote future

- Recall de Sitter space: $\hat{R}_{ab} = 3H^2 \hat{g}_{ab}$

$$\hat{g} = dt^2 - \cosh^2(Ht)\, d\sigma_3^2$$

unit S^3

$$= \cosh^2(Ht) \left[d\tau^2 - d\sigma_3^2 \right]$$

$$d\tau = \frac{dt}{\cosh Ht}$$ conformal time

$t \to \infty \; ; \; \tau \to \tau_F = \frac{\pi}{H}$

Infinite proper time but finite conformal time.

- $\hat{g}_{ab} = \hat{\Omega}^2 g_{ab} \; ; \; \hat{\Omega} \to \infty ; t \to \infty ; \tau \to \tau_F$

This is generic with $\Lambda > 0$;

like Escher, rescale and add a

boundary $\mathscr{I}$ at infinity.

- Weyl $\to 0$ at $\mathscr{I}$
- RP: everything fades to radiation

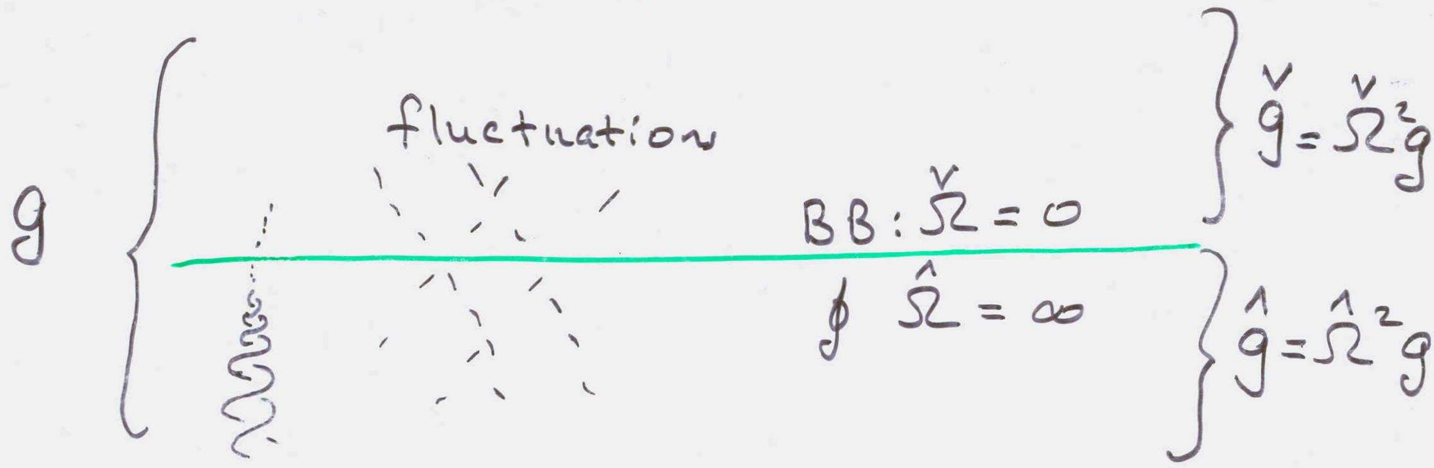
stars, galaxies, blackholes, protons ...

so there are no clocks measuring t

(VBE)

The Big Idea : match $\mathscr{I}$ to BB conformally

Conformal cyclic cosmology



Then:

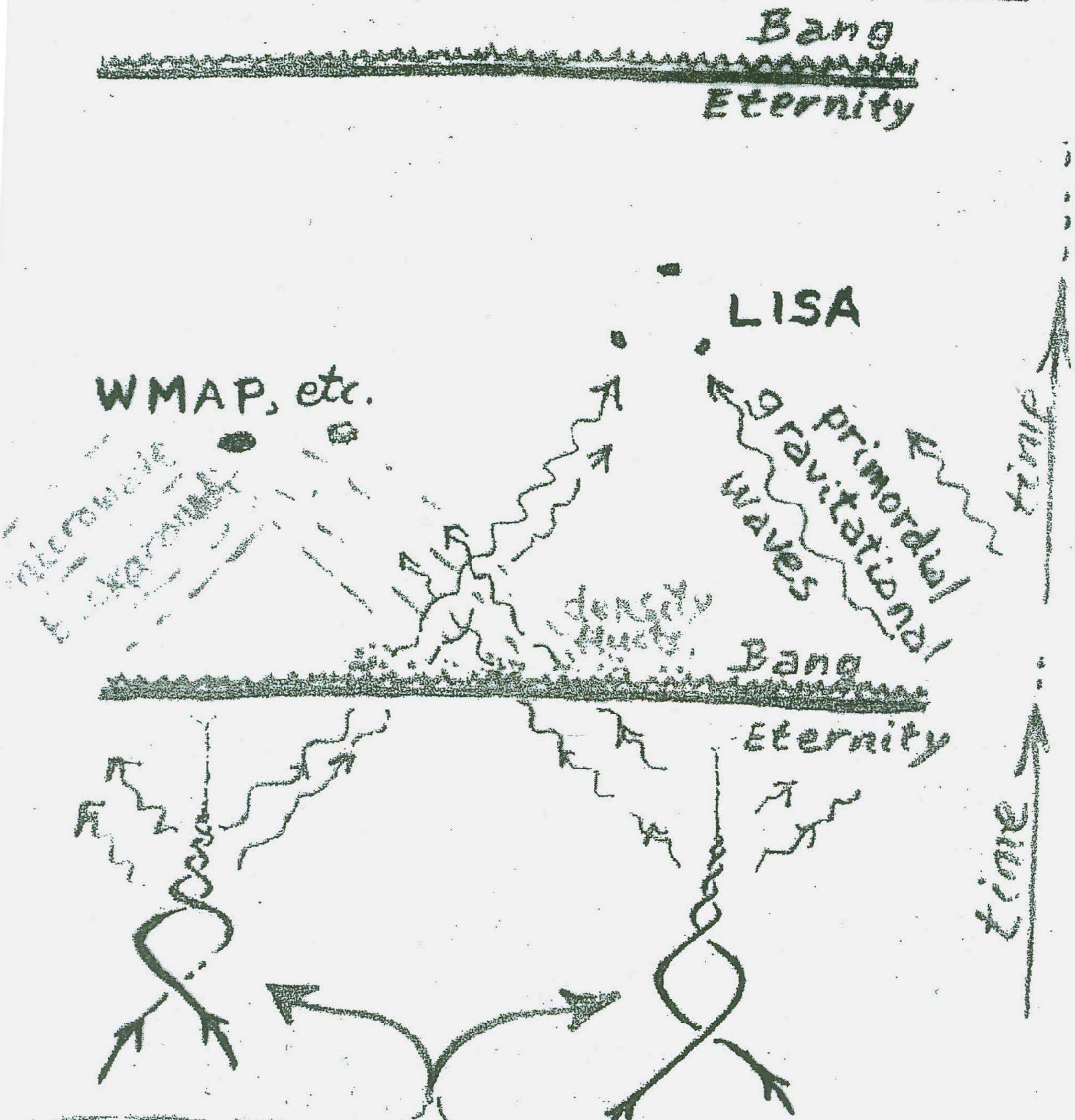
- initial Weyl is automatically 0
- primordial fluctuations come from the previous **aeon**
- inflation was before the bang
- $\check{\mathrm{Ricci}} = \hat{\mathrm{Ricci}} + \varphi\nabla\nabla\varphi + \nabla\varphi\nabla\varphi + \dots$

  where $\varphi \sim \log(\check{\Omega}\hat{\Omega}^{-1})$

  a natural scalar field to be dark matter.

Observational Implications

- Primordial gravitational waves
- Primordial density fluctuations



Inspiralling black holes, producing gravitational radiation. Gets through to next cycle, causing density fluctuations.

In the red corner:

1011.3706 Concentric circles in WMAP data may provide evidence of violent pre-Big-Bang activity

Authors: V.G.Gurzadyan, R.Penrose (Submitted on 16 Nov 2010)

1012.1486 More on the low variance circles in CMB sky

Authors: V.G.Gurzadyan, R.Penrose

1104.5675 CCC-predicted low-variance circles in CMB sky and LCDM

Authors: V. G. Gurzadyan, R. Penrose

In the blue corner:

1012.1268 A search for concentric circles in the 7-year WMAP temperature sky maps

Authors: I. K. Wehus, H. K. Eriksen

1012.1305 No evidence for anomalously low variance circles on the sky

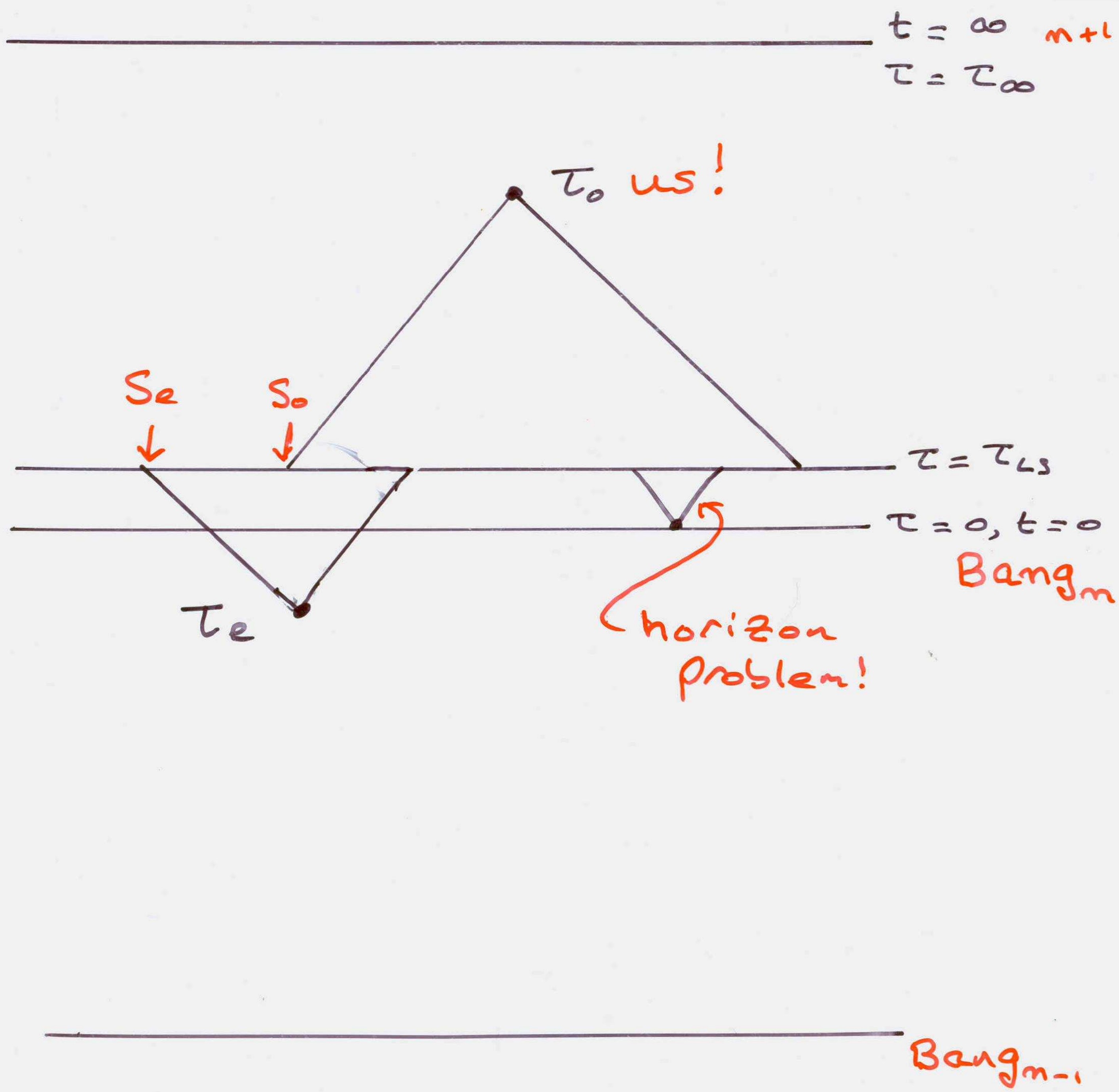
Authors: Adam Moss, Douglas Scott, James P. Zibin

1012.1656 Are There Echoes From The Pre-Big Bang Universe? A Search for Low Variance Circles in the CMB Sky

Authors: Amir Hajian

1105.1081 Comment on "CCC-predicted low-variance circles in CMB sky and LCDM"

Authors: H. K. Eriksen, I. K. Wehus

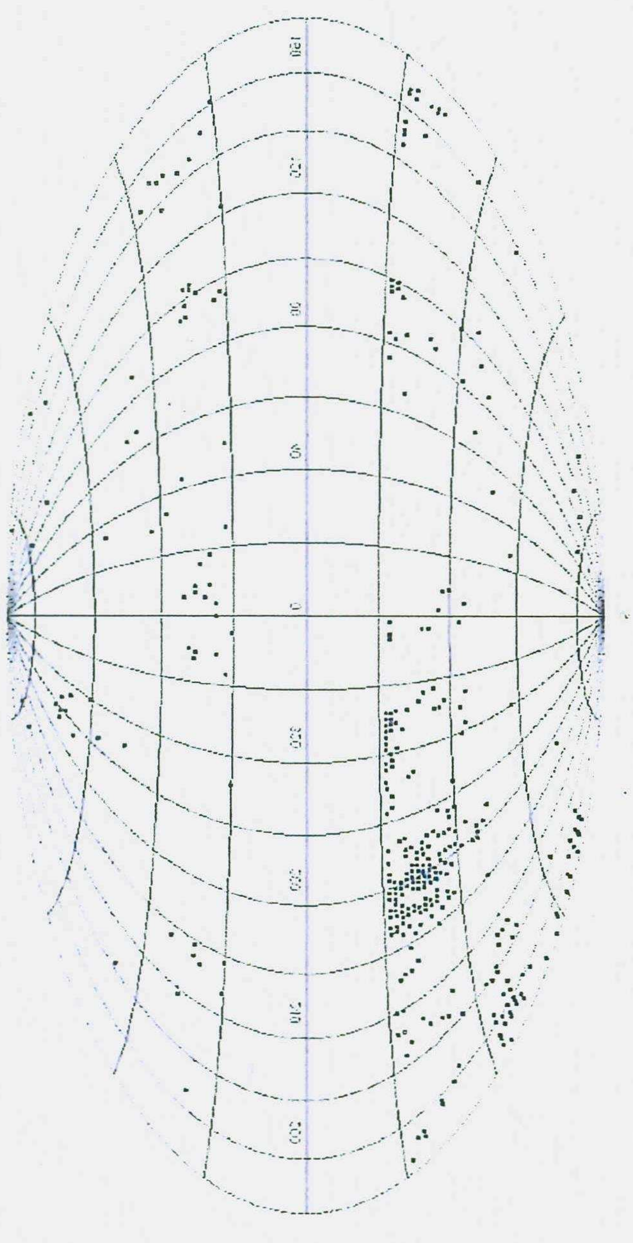


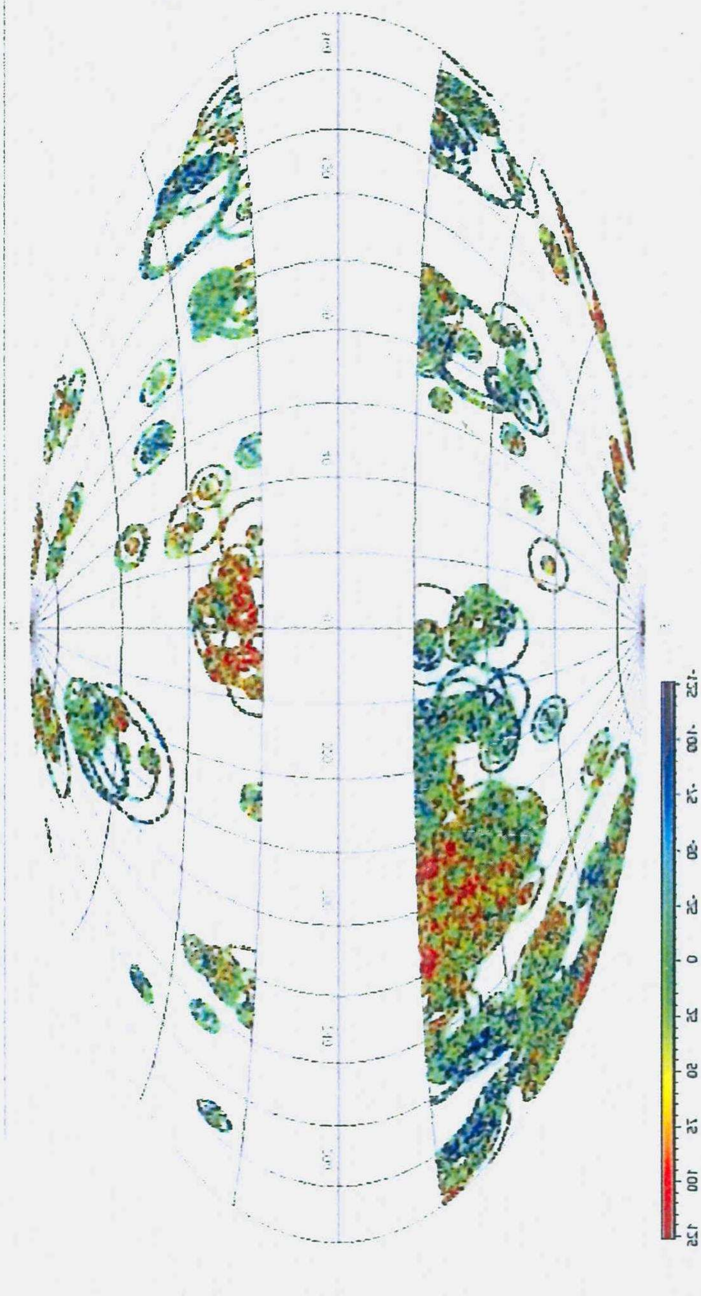
- calculate $\frac{\tau_0}{\tau_\infty} \simeq \frac{3}{4}$: no more V BE!
- $S_o \cap S_e$ is a circle on the sky.

Angular radius Θ say then suppose

  - $\Delta \tau_{n+1} \approx \Delta \tau_n$
  - $\tau_e \gtrsim \tau_0$

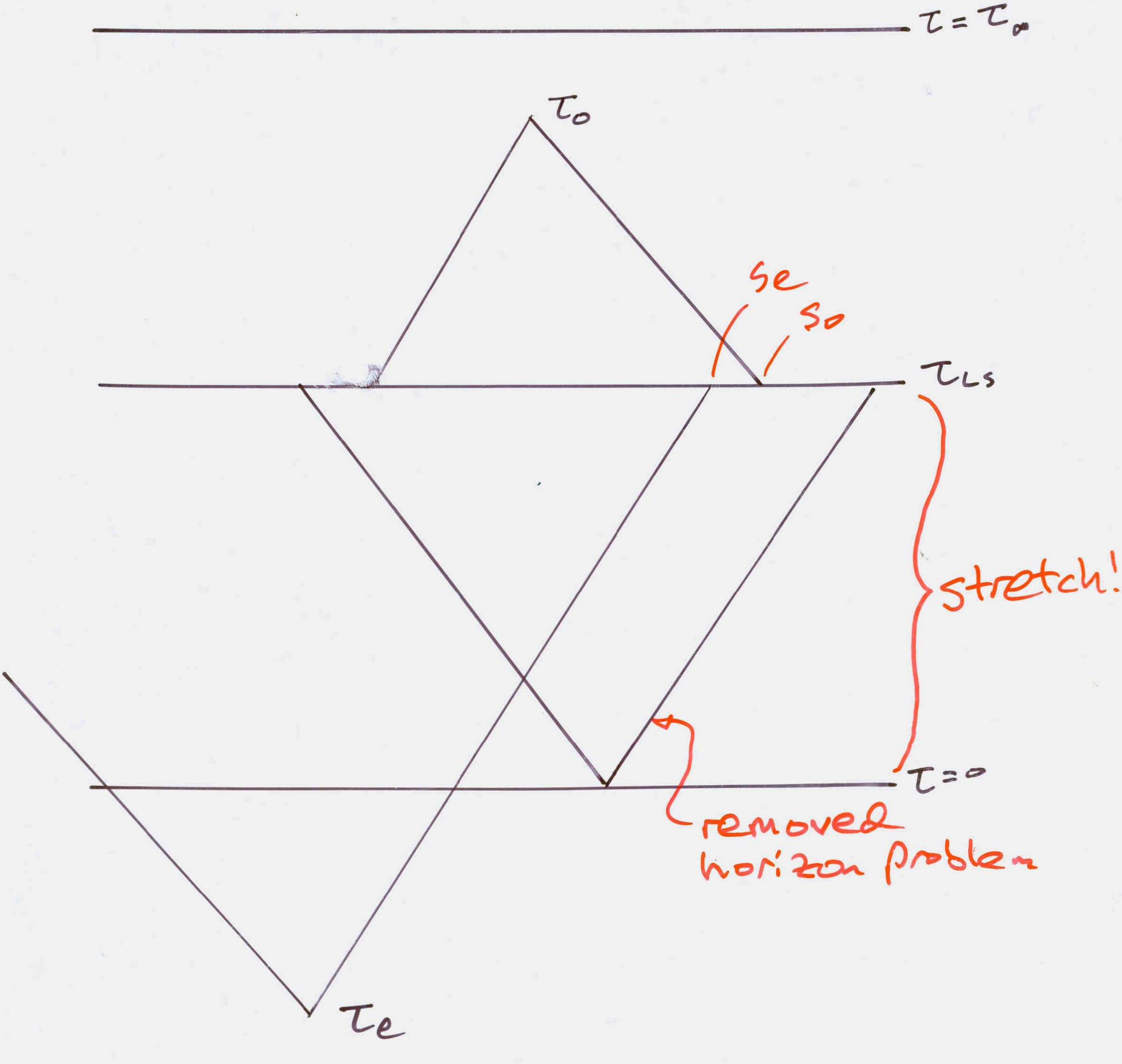
deduce $\Theta \lesssim 20^\circ$





from VG + RP
2010

With inflation:



Now S_e can be much bigger than S_0

& the circles can have any size.

T. 1107.1421