The geometry of our world

Jason Lotay explains how mathematicians studying special geometries are collaborating with physicists to explore M-theory, an 11-dimensional description of the world that unifies the various string theories

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I am a mathematician at University College London with a key interest in seven-dimensional geometry. This sounds pretty far away from the real world of physics, so why am I writing an article in *Physics World*? I hope to convince you that this type of geometry not only is an exciting area of research being pursued by some of the world's top mathematicians, but may also play a crucial role in enabling us to formulate a unified theory of physics.

I have been fascinated by both maths and physics ever since my school days – a combined interest that has fundamentally shaped my academic career. Quantum theory and gravity in particular captured my imagination and I wanted to learn more. During my maths degree, I did very little geometry and instead took every physics option I could. It was only when I wanted to study Albert Einstein's general theory of relativity that I realized it would be useful for me to know some more sophisticated aspects of geometry. Once I took my first geometry class, I was hooked, and when I found out that the connections between geometry and physics went much further than I thought, I knew that it was the research topic for me.

Luckily, I managed to do a PhD in this area and ended up studying it for a living. Along the way, I have found that it is possible to interpret many ideas in physics using geometry and, conversely, to use physics as a motivation in geometry to spectacular effect. The links between geometry and physics go back a long way and perhaps the most prominent player in the interaction is gravity.

Dimensions of gravity

When observing the everyday effects of gravity such as objects falling to the ground, one might intuitively think of gravity as a force of attraction between objects. This was the point of view Isaac Newton took, and although it was certainly very useful, it led to an incomplete theory.

Geometry is an invaluable (and arguably indispensable) tool in understanding gravity

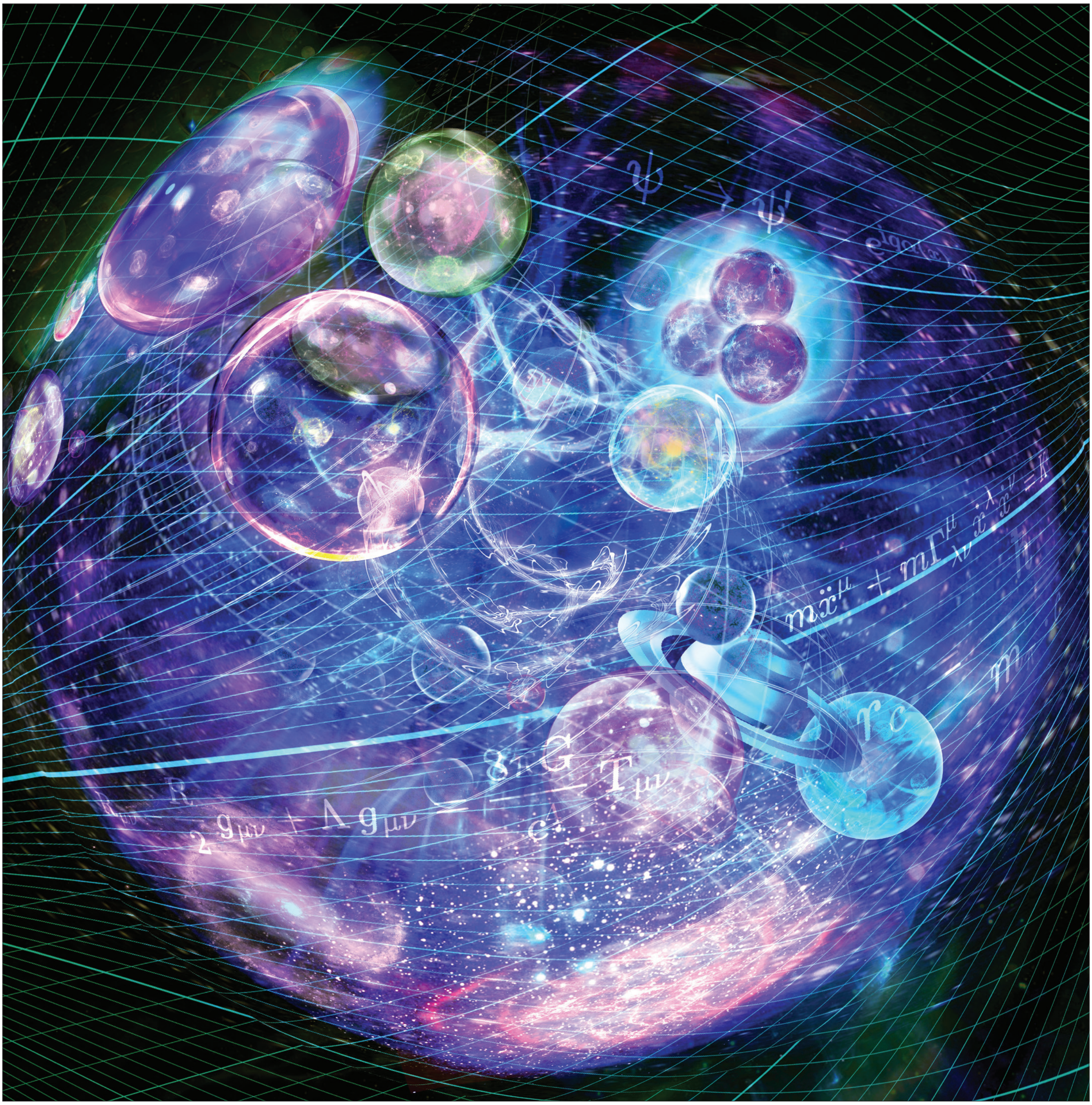
Einstein's fundamental idea in general relativity is to replace the notion of gravity as a force with, instead, gravity as an effect of the curvature of the universe. Describing gravity in this way shows that general relativity is inherently geometric. Einstein's theory has concrete physical implications that have stood up to all experimental data so far, even those at extremely high precision: it enables us to use GPS; it correctly describes the behaviour of Mercury's orbit; and it predicted gravitational waves, which were detected in 2015 by the Laser Interferometer Gravitational-Wave Observatory in one of the most exciting recent developments in physics.

Since Einstein's pioneering work, it has been clear that geometry is an invaluable (and arguably indispensable) tool in understanding gravity. Additionally, we must consider geometry in more than our usual three dimensions, since general relativity is formulated in terms of space–time: a four-dimensional view of our universe where the three dimensions of space and one dimension of time can interact.

Most of us perceive the world in the familiar three spatial dimensions, so it can be challenging to conceptualize additional dimensions, which are not necessarily spatial. Let me illustrate with an example. Suppose you want to buy a piece of furniture, say a wardrobe. The size of a wardrobe is obviously important, since it has to fit in your home, so you need to know its height, width and depth – the three dimensions we know very well. However, there are many other factors involved in choosing the wardrobe, including its weight (if you have to get it up some stairs) and its cost. These are properties of the wardrobe that we can measure on a scale, just like height. Colour is another characteristic that can be assigned a number on a scale, using wavelength. In fact we can measure all sorts of aspects of the wardrobe, all of which can be thought of as dimensions.

In this way, we can have as many dimensions as we like, and we should not be worried by the idea of adding in more dimensions; they are just ways of talking about additional properties of an object. Of course, it becomes hard to picture what higher dimensions look like. This is where I and other mathematicians come in, since we have the tools to deal with geometry in any dimensions, not just describing it but solving problems there too.

In relativity, the idea of adding a fourth dimension, time, should now not be a concern. In fact, having time as another dimension is a very old idea in physics, but the key observation of Einstein is that the



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universe should *curve* in the time dimension; this is why clocks run more slowly when gravity is stronger.

For mathematicians, even though adding another dimension is easy, the geometries in three and four dimensions are very different, so the number of dimensions in the theory is very important. In fact, some of the most celebrated works in mathematics involve these geometries. In three dimensions we have the famous Poincaré conjecture, a Millennium Prize Problem worth \$1m, which was solved by Grigori Perelman (though he refused both the prize and the Fields Medal – the top award in mathematics). On the other hand, Simon Donaldson was awarded the Fields Medal primarily for his work using inspiration from physics (specifically Yang–Mills theory, which forms the basis for the Standard Model of particle physics) to understand 4D geometry.

Despite the successes of Newton and Einstein's theories, our understanding of gravity is still incomplete. The most well-known shortcoming is that we have no theory that unifies gravity with quantum theory, which explains the behaviour of elementary particles. The struggle for this unified theory plagued Einstein and still remains an open problem.

There are also three gravitational phenomena that general relativity struggles to explain. The first is the "missing mass" known as dark matter, which various clues point to, including a mismatch between the speeds at which stars are predicted to move around their galactic centres, and those observed. The second is that, using the cosmic microwave background, the universe appears to "look the same" in all directions, which is most easily explained using the idea of inflation: that the universe underwent a period of rapid expansion after the Big Bang. The mechanism of inflation, however, is not readily compatible with general relativity. Finally, there is the problem that not only is the universe expanding, but that the rate of expansion is increasing; this is typically explained by so-called dark energy. General relativity can account for the rate of expansion, but only by introducing a cosmological constant, as Einstein himself did. However, the observed value of the constant does not match with any currently consistent theoretical prediction.

Taking a jump

In an attempt to unify gravity with quantum theory, physicists introduced string theory. The key idea of this theory is that rather than modelling particles by points or little round balls, as one does in quantum theory or from an intuitive perspective, we should instead view particles as being little "strings": one-dimensional objects that can either be closed loops or open pieces with free ends. These strings can vibrate just like the strings on a guitar or in a piano, and understanding these vibrations then allows us (or at least string theorists) to describe and understand the particles.

This relatively simple idea has important physical consequences in that it can potentially provide a unified theory. It also adds geometry, and in particular curvature, into the game in a fundamental way. Unlike a point or round ball, a string can be curved and how it is curved can be influenced by the world around it. For example, if we lay a string flat on the table in a straight line it is not curved at all, but if we push it flush against a sphere, say a globe, then it will be curved since the sphere is curved.

Although string theory seems like a pretty simple idea, it has a complicated consequence. In order for the theory to make sense one needs to take a major jump: we have to add extra dimensions to the universe beyond the four we know. For a mathematician this is easy, but for a typical physicist this is quite tricky and hard to swallow (though hopefully my wardrobe analogy has made it a little bit easier). The theory does not say what these extra dimensions are: they are not something as concrete as space or time that we can add on. However, they do behave a bit like our usual spatial dimensions, and they are curved too in a special way, inspired by relativity.

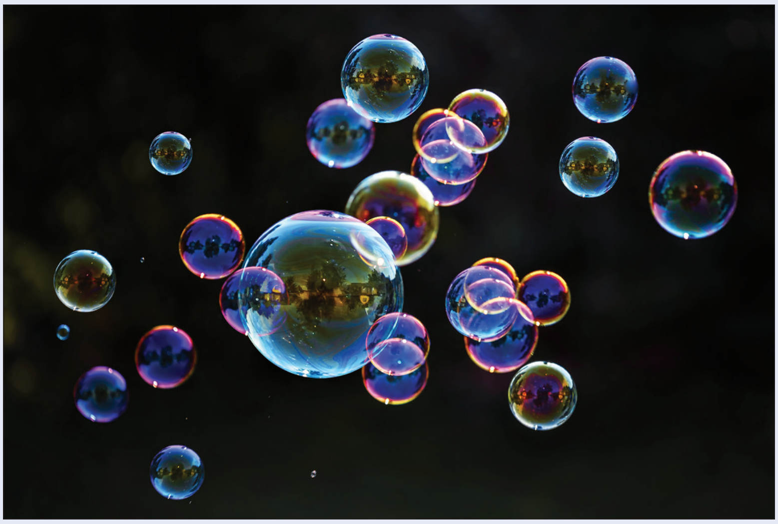
So how many extra dimensions do we need to describe the geometry of our world? Well, it varies, but most string theories use 10 dimensions, so six more than the usual four. This seems like a lot, but in the past string theorists have considered using as many as 26, and for mathematicians the number six is still pretty small.

Actually, when I said there is a theory called string theory in 10 dimensions, this is not quite true. There are actually several different string theories in 10 dimensions. This is quite embarrassing, because if we are looking for a unified theory then there should be just one. This problem caused consternation in the field until theoretical physicist and Fields Medal winner Edward Witten proposed a new theory of physics called M-theory.

M-theory happens in 11 dimensions, so seven more than our usual space and time. It has the great property that it shows that all of the string theories in 10 dimensions, which as I said all look different, are actually all special cases of this single 11D theory. So M-theory seems to be the unification of the string theories that the community was looking for. This means adding another dimension, but since we were already at 10, going to 11 does not seem like much of a stretch.

There is some debate as to what the M actually stands for. Some say it is for master theory or mother-

Inspired by bubbles



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We can write down equations for the seven-dimensional objects we are interested in, but it is hard to solve them. This should come as no surprise, as Albert Einstein's equations describing only four dimensions gave rise to the same problem. Solving such equations is the key reason why studying gravity, and the analogous geometric problems that arise, is particularly challenging. However, I have been looking into a new approach to solving these equations that takes inspiration from a much more mundane topic in physics: bubbles.

When you blow a soap bubble, it starts off as some weird blob, but gradually becomes a sphere, as long as it does not pop. The reason is simple physics: the bubble will reach equilibrium when the pressure on the inside and outside match, which means the bubble will become a sphere. The bubble's route to equilibrium can also be phrased in terms of its surface tension, which mathematically can be expressed as the fact that the bubble wants to minimize its surface area, given the volume of air it contains. The bubble does this automatically, and since we can model its evolution using an equation similar to one of the simplest evolution equations in physics, which describes how heat dissipates, we can solve it, or at least analyse it quite effectively.

Now, it turns out that the 7D objects we are looking for minimize a kind of area or energy like a bubble. This is not so surprising since these objects are supposed to come from physics, and we know that physical objects try to reach the state of least energy if they can. So, when starting from some (not quite random) 7D blob, we can write down a kind of heat equation, as devised by mathematician Robert Bryant of Duke University, North Carolina, US. When we solve this, it should hopefully lead us to the G_2 geometry we are looking for (see main article), just like a soap bubble blob eventually becomes a sphere.

As I warned you before, the soap bubble can pop, and this is a real problem for our 7D equation, where the blob may well burst before we can reach the answer we want. However, I have been able to show that sometimes it does work and finds the 7D spaces we want.


Bright Club

Clarifying M-theory Jason Lotay performs science stand-up explaining the concept of multiple dimensions using a toy lightsaber.

of-all theory (since there is also an F-theory that might be father-of-all), or perhaps membrane theory. The last one makes sense because M-theory is not a string theory, as the fundamental objects are no longer 1D, but are instead higher-dimensional surfaces or membranes.

The seven extra dimensions can be studied completely separately from the four dimensions we are familiar with from space–time, before being later combined in the full 11D M-theory. Although we cannot say what the seven extra dimensions are, they are not completely arbitrary. In fact, they are very special, satisfying equations similar to those appearing in general relativity, which makes sense because they are supposed to help us describe gravity.

What is really fascinating is that the simplest case of these equations also appears in geometry and is a key equation that mathematicians have long been studying and continue to explore. Some of the best mathematicians study this geometry, including three Fields Medallists: Michael Atiyah, as well as Simon Donaldson and Witten. This is something I have also been working on, taking inspiration from physics (see box opposite).

The key to why 7D geometry is interesting in mathematics and physics is symmetry

The key to why 7D geometry is interesting in mathematics and physics is symmetry. We know that objects like cubes and spheres have lots of symmetry, in that they look the same from many (and sometimes all) angles, whereas other shapes such as oblongs and rugby balls have less symmetry. A crucial mathematical fact is that the types of symmetries that can occur for various geometric objects depends very much on how many dimensions we are working in. Even more important is that there is a special type of symmetry that can occur only in seven dimensions. This symmetry leads to so-called G_2 geometry in seven dimensions, and it is this geometry that plays a major role both in modern mathematics and in M-theory.

Progress through collaboration

Theory is all well and good, but can we link any of this M-theory stuff to experiments? Well, yes we can. I have been discussing research with King's College London physicist Bobby Acharya, who has worked with Witten on studying fermions in M-theory and is currently focused on trying to link the theory to observations in cosmology as well as experiments at CERN's Large Hadron Collider.

One of the most exciting recent discoveries in particle physics has been the Higgs boson, but why does it have the mass we observed? This is a question that M-theorists hope to answer. As we get more information from space telescopes, and powerful ground-based telescopes too, we learn more about black holes, the acceleration of the universe and the rotation of galaxies. As a consequence, we get more observations that help us to understand dark matter and dark energy, and their effects. With these insights, it is hoped that one can use M-theory to give a satisfying explanation of these phenomena, which currently cannot be explained well by general relativity.

Again, in order to achieve this, we need to know a lot about the possible 7D geometries that can occur, and so I (and other mathematicians) have been talking with Acharya and other physicists such as Sergei Gukov at the California Institute of Technology, US, and James Sparks at the University of Oxford, UK, to see if we can make progress in both maths and physics through collaboration.

Although G_2 geometry plays a key role in M-theory, there is still much that we do not understand. On the mathematical side, we have a limited understanding of 7D geometry and so we need to work hard to find and analyse the kinds of objects that are needed to make M-theory work. On the physics side, we need to continue to strive to connect M-theory to concrete observations so it can be tested, and we need to pin down precisely the 7D geometry that forms the extra dimensions in M-theory. These are certainly difficult problems, but there has been a recent upsurge in activity in this area so it is an exciting time in the field, on both the maths and the physics side. I am hopeful that soon, by having mathematicians and physicists working together, we will have major breakthroughs that will shed light on 7D geometry and bring us a step closer to that elusive unified theory of physics. ■