Theory and Applications of Computability

In cooperation with the association Computability in Europe

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S. Barry Cooper • Mariya I. Soskova
Editors

The Incomputable
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Madison, WI, USA

Mariya I. Soskova

28 March 2016
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