Not $\left(\frac{dy}{dx}\right)^2$

Today: try to solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0.$

$x \in \mathbb{R}$

y real function

Try $y = e^{-x}$ $\quad \frac{dy}{dx} = -e^{-x}$ $\quad \frac{d^2y}{dx^2} = (-1)^2 e^{-x}$

$$(-1)^2 e^{-x} + 3(-e^{-x}) + 2(e^{-x}) = 0$$

✓ For all x.

or even Ae^{-x} works because

$$(-1)^2 + 3(-1) + 2 = 0$$

Try $y = e^{mx}$ works if $\underbrace{m^2 + 3m + 2 = 0}_{\text{auxiliary equation.}}$

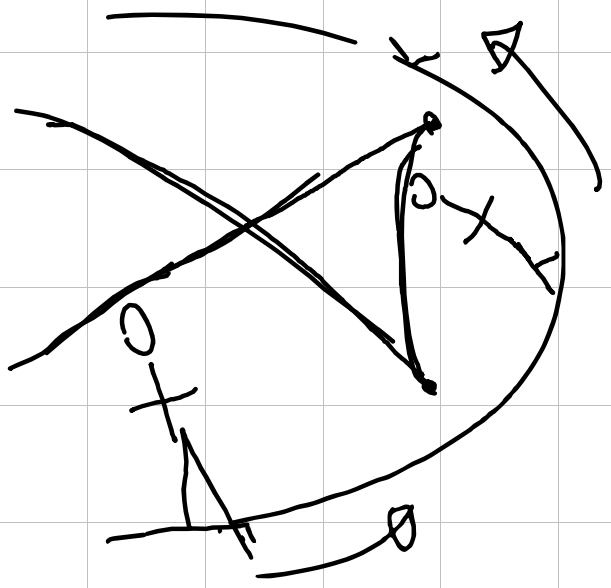
$y = Be^{mx}$

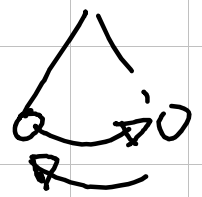
2D vector space

$A\underline{i} + B\underline{j}$

"Vectors" $\vec{i}$

General solution $y = Ae^{-x} + Be^{-2x}$





$$\frac{d^2y}{dx^2} + y = 0 \quad \text{with} \quad y(0) = 0, \quad \left.\frac{dy}{dx}\right|_{x=0} = 1$$

Try e^{mx}, solve $m^2 + 1 = 0$ auxiliary equation

$(m+i)(m-i) = 0$ $\quad m^2 - (i)^2$

1×1 matrix (3)

3

$$y = Ae^{ix} + Be^{-ix}$$

$$= A(\cos x + i \sin x) + B(\cos x - i \sin x)$$

$$= (A+B)\cos x + (Ai - Bi)\sin x$$

call this C — call this D

$C + D = 1$

$2C + 2D = 2$

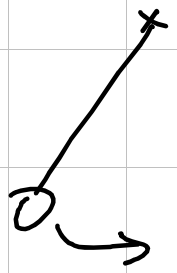
$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$= C\cos x + D\sin x$$

$[= R\cos(x - \alpha)]$

$y(0) = 0$ means $C = 0$

$\left.\frac{dy}{dx}\right|_{x=0} = 1$ means $\left.-C\sin x + D\cos x\right|_{x=0} = 0$ $\quad D = 1$



$$\frac{d^2y}{dx^2} + y = 0 \qquad y(0) = 0 \qquad y(\pi) = 1$$

Solve $y = C\cos x + D\sin x = R\cos(x-\alpha)$

↑ known period.

$0 = y(0) = C$

$1 = y(\pi) = C$

$\therefore$ oh dear.

TRIG CORNER

$R\cos(x-\alpha)$

$R(\cos x \cos\alpha + \sin x \sin\alpha)$

if that's

$C\cos x + D\sin x$

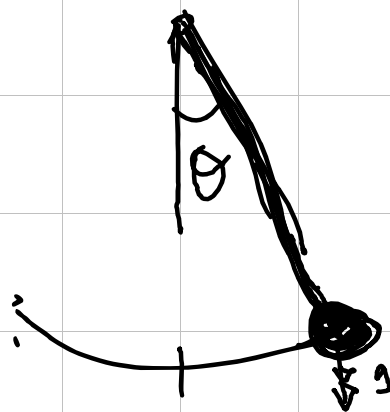
then

$R\cos\alpha = C$

$R\sin\alpha = D$

idea: $\tan\alpha = D/C$

$R = \sqrt{C^2+D^2}$ (±)

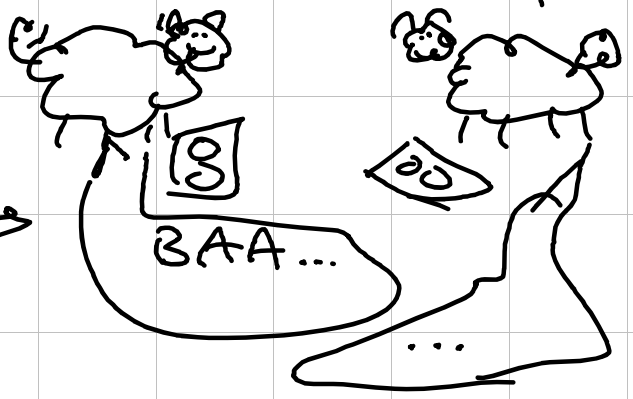


acceleration $\approx \frac{d^2\theta}{dt^2}$

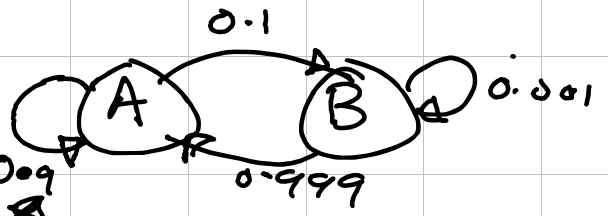
force of gravity $\approx -mg\sin\theta \approx -\theta$ if θ small.

Very quick intro to Markov chains:

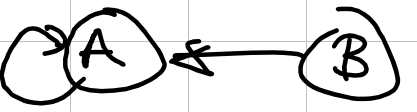
- Design predictive text for dog. A, B button.
- 50/50 As / Bs.
- 60/40 As / Bs
- Different depending previous letters.



BAAA BABAABAA.

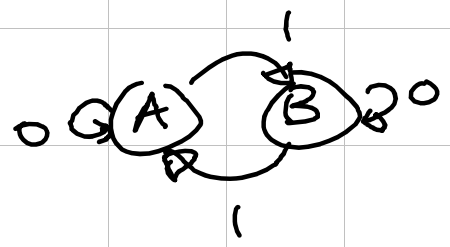


(invariant)
long-term probability of A / B?

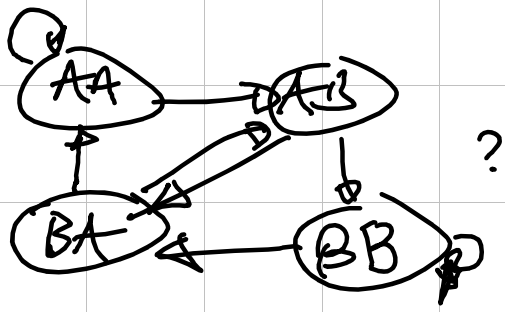


$$\begin{pmatrix} 0.9 & 0.1 \\ 0.999 & 0.001 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}$$ eigenvectors

BAAAAAAAA A...
AAAAAAAAAA....



$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \lambda^2 + 1 = 0$$



H / T TTH ✓

4 or 5 or 30 ↙   ☺

1 = 2	ϕ
3	1.839
4	1.92
30	≈ 2

Choose 3 numbers. Positive whole numbers.

Repeat the following:

- Add them.
- Delete the smallest.

Keep going until you get bored.

Then divide largest number you've got by the second-largest.

solve
$37m + 14n = 3$
m, n whole numbers

1.839... ish

Why? What's that?

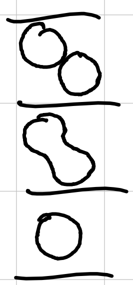
$$\frac{1 + \sqrt[3]{19 - 3\sqrt{33}} + \sqrt[3]{19 + 3\sqrt{33}}}{3}$$

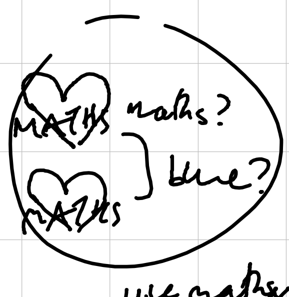
1.83928675521416(113255185?)

fool's gold ratio

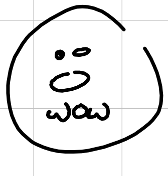
Mosi 1921

All comes together.





use maths heart

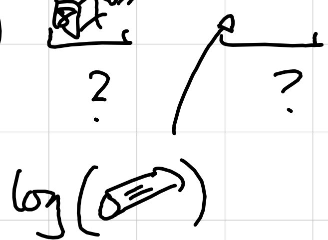


Risky Dice

?

Logarithms

?



Confused

?

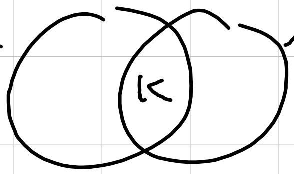
Tier 1 } Prime
2 ✗
3 ✗

oh no   ooer

∃ K emote
↑
Greek letter used in maths.

Volt: ½

Twitch emotes



Maths symbols

Pyrhz equation.

Solve $F_{n+3} - F_{n+2} - F_{n+1} - F_n = 0$

$F_0 = ?$
$F_1 = ?$
$F_2 = ?$

$F_n = 0$ works

F_n = polynomial in n?

~~$\frac{\sqrt{5}-1}{2}$~~ ?

Try $F_n = \lambda^n$ for some constant λ.

~~solves ratio~~

$$\lambda^{n+3} - \lambda^{n+2} - \lambda^{n+1} - \lambda^n = 0$$

Divide by λ^n

Auxiliary equation

$$\lambda^3 - \lambda^2 - \lambda - 1 = 0$$

cubic

plastic number ?!

solve: Real soln $\simeq (1.839\ldots) = \lambda_0$

2 complex solns. λ_1, λ_2

General solution is $A\ (1.839?..)^n + \overbrace{B\lambda_1^n + C\lambda_2^n}^{\text{small for large } n.}$ exp. decay complex part so oscillate

$A\ (1.839?..)^{n+1}$ wins. $+[\text{small}]$

10, 10, x

$s = \frac{20+x}{2} \therefore 10 + \frac{x}{2}$

$\frac{1}{2} \times 10 \times 10 \times \sin C$ ↑ large at 90°

Extreme values of $s(s-10)(s-10)(s-x)$

$\left(10+\frac{x}{2}\right)\left(\frac{x}{2}\right)\left(\frac{x}{2}\right)\left(10-\frac{x}{2}\right)$

$(20+x)(20-x)x^2$

$400x^2 - x^4$

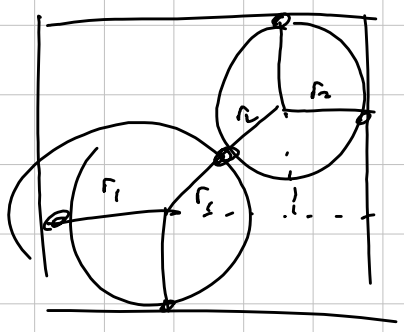
$\frac{d}{dx}$ $800x - 4x^3$

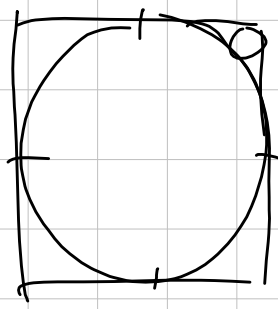
$x=0$ or $x^2 = 200$

$x = 10\sqrt{2}$

Pitch the closest?!

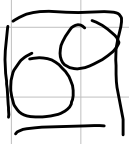
15





$r_1 + \frac{r_1+r_2}{\sqrt{2}} + r_2 = 1$

$(r_1+r_2)\left(1+\frac{1}{\sqrt{2}}\right) = 1$



Draw diagram not to scale

$e^{2i\theta} + e^{-2i\theta} = 2\cos 2\theta$

Prüfer code

$x^2 + 3 + x^{-2} \simeq \left(x+\frac{1}{x}\right)^2 + 1$

$ax^2+bx+c=0$ $\quad$ $bx^2+cx+a=0$

repeated roots

$b^2-4ac=0$ $\quad$ $c^2-4ab=0$

$b^2=4ac$ $\quad$ $c^2=4ab$

want $a^2-4bc \geq 0$?? $\quad$ $=0$ $\quad$ $\times$

$b^2c^2 = 16a^2bc$

$=-63a^2<0$ $\quad$ $bc = 4a^2$

TODAY

- $\frac{d^2y}{dx^2} + y = 0$ ✓
- Markov chains ✓
- $F_{n+3} = F_n + F_{n+1} + F_{n+2}$ ✓

  1.839...
- MATT ✓