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ERRATUM: PERTURBATION OF PARTITIONED HERMITIAN DEFINITE GENERALIZED EIGENVALUE PROBLEMS*

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Abstract. The main purpose of this erratum is to correct mistakes in the proof of Theorem 2.4 of [R.-C. Li et al., *SIAM J. Matrix Anal. Appl.*, 32 (2011), pp. 642–663] and in the inequalities (2.23), (2.24), and (2.25) on p. 653 of the same paper.

Key words. quadratic eigenvalue perturbation bound, generalized eigenvalue problem, multiple eigenvalue

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For the proof of Theorem 2.4 of [1], line 6 from the bottom of p. 650 of the same paper applied the induction assumption incorrectly. The correct application would have used ν_i instead of $\widetilde{\lambda}_i$. Despite this mistake, the theorem itself remains true. We shall now present a new proof. To do so, we need the following lemma.

LEMMA A (see [2, Lemma 2.1]). *Suppose $H - \lambda(S + \Delta S)$ and $H - \mu_i \Delta S - \lambda S$ are Hermitian positive definite pencils. If μ_i is the ith[1] eigenvalue of the first pencil, then it is also the ith eigenvalue of the second pencil.*

THEOREM 2.4. *Under the conditions of Theorem 2.2, we have, for all $1 \le i \le N$,*

$$|\widetilde{\lambda}_i - \lambda_i| \le \frac{2\|E_{21} - \widetilde{\lambda}_i F_{21}\|_2^2}{\eta_i + \sqrt{\eta_i^2 + 4\|E_{21} - \widetilde{\lambda}_i F_{21}\|_2^2}} \le \frac{2\|E_{21} - \widetilde{\lambda}_i F_{21}\|_2^2}{\eta + \sqrt{\eta^2 + 4\|E_{21} - \widetilde{\lambda}_i F_{21}\|_2^2}}. \tag{2.15}$$

Proof. Notice that $\widetilde{\lambda}_i$ is the ith largest eigenvalue of the Hermitian positive definite pencil

$$\begin{aligned}\widetilde{A} - \lambda\widetilde{B} &= \begin{pmatrix} A_{11} & E_{21}^* \\ E_{21} & A_{22} \end{pmatrix} - \lambda \begin{pmatrix} I_m & F_{21}^* \\ F_{21} & I_n \end{pmatrix} \\ &= \begin{pmatrix} A_{11} & E_{21}^* \\ E_{21} & A_{22} \end{pmatrix} - \lambda \left[I_N + \begin{pmatrix} 0 & F_{21}^* \\ F_{21} & 0 \end{pmatrix} \right]\end{aligned}$$

whose ith largest eigenvalue $\widetilde{\lambda}_i$, by Lemma A, is the same as that of

$$\begin{pmatrix} A_{11} & E_{21}^* \\ E_{21} & A_{22} \end{pmatrix} - \widetilde{\lambda}_i \begin{pmatrix} 0 & F_{21}^* \\ F_{21} & 0 \end{pmatrix} - \lambda I_N.$$

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[1] In [2], the eigenvalues are ordered in ascending order. But the proof there works for, without any change, the case in which the eigenvalues are ordered in descending order.

That is to say that $\widetilde{\lambda}_i$ is the ith largest eigenvalue of the Hermitian matrix

$$\begin{pmatrix} A_{11} & E_{21}^* - \widetilde{\lambda}_i F_{21}^* \\ E_{21} - \widetilde{\lambda}_i F_{21} & A_{22} \end{pmatrix}.$$

Now apply Lemma 2.1(c) in [1] to conclude (2.15). ∎

The mistakes in the inequalities (2.23), (2.24), and (2.25) in [1] can be corrected by replacing all $\widetilde{\lambda}_j$ in their right-hand sides by $\lambda_j^{(\mathrm{b})}$:

$$\begin{aligned} |\lambda_j^{(\mathrm{b})} - \lambda_j^{(\mathrm{c})}| \le{}& \|\widehat{F}_{21}\|_2^2\,|\lambda_j^{(\mathrm{b})}| + \|\widehat{E}_{21}\widehat{F}_{21}^* + \widehat{F}_{21}\widehat{E}_{21}^* - \widehat{F}_{21}\widehat{A}_{11}\widehat{F}_{21}^*\|_2 \\ &+ \frac{2\|\widehat{E}_{21} - \widehat{F}_{21}\widehat{A}_{11}\|_2^2}{\eta_j + \sqrt{\eta_j^2 + 4\|\widehat{E}_{21} - \widehat{F}_{21}\widehat{A}_{11}\|_2^2}}, \end{aligned} \tag{2.23}$$

$$|\lambda_j^{(\mathrm{b})} - \lambda_j^{(\mathrm{c})}| \le \frac{2\|\widehat{E}_{21} - \lambda_j^{(\mathrm{b})}\widehat{F}_{21}\|_2^2}{\eta_j + \sqrt{\eta_j^2 + 4\|\widehat{E}_{21} - \lambda_j^{(\mathrm{b})}\widehat{F}_{21}\|_2^2}}, \tag{2.24}$$

$$|\lambda_j^{(\mathrm{b})} - \lambda_j^{(\mathrm{c})}| \le \|\widehat{F}_{21}\|_2\,|\lambda_j^{(\mathrm{b})}| + \frac{2\|\widehat{E}_{21}\|_2^2}{\eta_j + \sqrt{\eta_j^2 + 4\|\widehat{E}_{21}\|_2^2}}. \tag{2.25}$$

These changes affect Example 2.2 of [1], in which (2.23), (2.24), and (2.25) were used to generate the plots in Figure 2.1 there. Fortunately, these plots have no visible differences from the corresponding ones generated by the corrected (2.23), (2.24), and (2.25) above.

There are several typographical errors in [1]. At line 8 on p. 643, A and B should be $\widetilde{A}$ and $\widetilde{B}$. At line −9 on p. 643 and also line 20 on p. 654, $\|\cdot\|$F should be $\|\cdot\|_{\mathrm{F}}$. At line 14 on p. 645, insert "positive" before "definite." At line −5 on p. 645, (2.3) should be (2.4). At the end of line −10 on p. 651, (2.18) should be (2.15). At line 5 on p. 653, $Y^*\widetilde{A}Y$ and $Y^*\widetilde{B}Y$ should be $Y\widetilde{A}Y^*$ and $Y\widetilde{B}Y^*$.

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REFERENCES

[1] R.-C. Li, Y. Nakatsukasa, N. Truhar, and S. Xu, *Perturbation of partitioned Hermitian definite generalized eigenvalue problems*, SIAM J. Matrix Anal. Appl., 32 (2011), pp. 642–663.

[2] Y. Nakatsukasa, *Perturbation behavior of a multiple eigenvalue in generalized Hermitian eigenvalue problems*, BIT, 50 (2010), pp. 109–121.

[3] W.-W. Xu and H. Liu, *private communication*, 2011.