M3/4/5P12 PROGRESS TEST 1

PLEASE WRITE YOUR NAME AND CID NUMBER ON EVERY SCRIPT THAT YOU HAND IN. FAILURE TO DO THIS MAY RESULT IN YOU NOT RECEIVING MARKS FOR QUESTIONS THAT YOU ANSWER.

Note: all representations are assumed to be on finite dimensional complex vector spaces.

Question 1. Let G be a finite group.

(a) What does it mean for a (non-zero) representation of G to be *irreducible*?

(b) Show that if V is an irreducible representation of G and

$$f : V \to V$$

is a G-linear map then f is equal to multiplication by a scalar $\lambda \in \mathbb{C}$. *You may assume the fact that every linear map from V to V has an eigenvalue.*

(c) Using part (b), show that a non-zero irreducible representation of a finite Abelian group is one-dimensional.

(d) Now let $G = C_3 = \{e, g, g^2\}$ the cyclic group of order 3. Consider the regular representation $\mathbb{C}G$. What are the irreducible non-zero subrepresentations of $\mathbb{C}G$?

Question 2. Consider the dihedral group of order 8,

$$D_8 = \langle s, t : s^4 = t^2 = e, tst = s^{-1} \rangle.$$

There is a three-dimensional representation of D_8 on $V = \mathbb{C}^3$ defined by

$$\rho_V(s) = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix} \qquad \rho_V(t) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

You don't need to check that this defines a representation.

(a) Find a one-dimensional subrepresentation U_1 of V. Can you find another one?

(b) Deduce that V is isomorphic as a representation of D_8 to a direct sum $U_1 \oplus U_2$ where U_2 is a two-dimensional irreducible representation of D_8. *You don't need to find U_2.*

(c) Show that D_8 has one isomorphism class of two-dimensional irreducible representations and four isomorphism classes of one-dimensional representations. You may assume any results you need from the course.

Date: Tuesday February 16, 2016.