M3/4/5P12 PROGRESS TEST 2

PLEASE WRITE YOUR NAME AND CID NUMBER ON EVERY SCRIPT THAT YOU HAND IN. FAILURE TO DO THIS MAY RESULT IN YOU NOT RECEIVING MARKS FOR QUESTIONS THAT YOU ANSWER.

Note: all representations are assumed to be on finite dimensional complex vector spaces. Unless a question specifies otherwise, all results from the course may be assumed if they are clearly stated.

Question 1. Let G be a finite group. Let V be a representation of G, with character χ_V.

(a) What is the definition of the *dual representation* V^*?

(b) What is the character χ_{V^*} of the dual representation V^*, in terms of χ_V? Justify your answer.

(c) Let W be another representation of G, with character χ_W. What is the character $\chi_{V\otimes W}$ of the tensor product representation $V \otimes W$, in terms of χ_V and χ_W? *You just need to state the answer.*

(d) Let V_{triv} be the one-dimensional trivial representation of G, with character χ_{triv}. Show that

$$\langle \chi_{V\otimes W}, \chi_{triv}\rangle = \langle \chi_V, \chi_{W^*}\rangle.$$

(e) Suppose V and W are irreducible representations. If W^* is not isomorphic to V, how many copies of V_{triv} appear in the decomposition of $V \otimes W$ into irreducibles? How many copies of V_{triv} appear in the decomposition of $V \otimes V^*$ into irreducibles? Justify your answers.

Question 2. (a) Let G be a finite group. State the column orthogonality relations for the irreducible characters of G.

(b) Here is an incomplete character table for a group of order 24, with 7 conjugacy classes.

	$g_1 = e$	g_2	g_3	g_4	g_5	g_6	g_7
Size of conj. class	1	1	6	4	4	4	4
χ_1	1	1	1	1	1	1	1
χ_2	1	1	1	ω	ω^2	ω^2	ω
χ_4	2	-2	0	-1	-1	1	1

In the table, $\omega = e^{2\pi i/3}$.

(i) Find another irreducible character χ_3 of dimension 1 (i.e. with $\chi_3(e) = 1$).

(ii) Find two more distinct irreducible characters χ_5, χ_6 of dimension two.

(iii) Work out the complete character table for this group of order 24, justifying your answer.

Date: Tuesday March 15, 2016.