

# STUDY GROUP ON THE WEIGHT PART OF SERRE'S CONJECTURE

JAMES NEWTON

The goal of the study group is eventually to discuss some recent work of Diamond, Kassaei and Sasaki on a geometric version (including irregular weights) of the weight part of Serre's conjecture for Hilbert modular forms [DK16, DS17, DKS20].

Along the way, we will have some introductory talks on mod  $p$  modular forms (elliptic and Hilbert) and other aspects of Serre's conjectures on the modularity of mod  $p$  Galois representations.

Preliminary schedule:

- April 22* Introduction (James Newton)
- April 29* An overview of Serre's conjectures for modular forms, particularly the weight aspect. [Ser87] (Miriam Norris)
- May 6* Preliminaries on modular curves and mod  $p$  modular forms. The Hasse invariant and theta operators.
- May 13* The weight part of Serre's conjecture for modular forms: theta cycles, companion forms ([Edi92]). Interpretation in terms of crystalline lifts.
- May 20* Hilbert modular forms and Hilbert modular varieties. Partial Hasse invariants, partial theta operators. [AG05]
- May 27* Minimal weights [DK16]
- June 3* Cohomological interlude: the BDJ conjectures [BDJ10] and more on crystalline lifts.
- June 10* The Diamond–Sasaki conjectures on geometric weights [DS17] and the relationship to BDJ.
- June 17* First talk on [DKS20]: geometric Jacquet–Langlands.
- June 24* Second talk on [DKS20]: applications to mod  $p$  Hilbert modular forms.

## REFERENCES

- [AG05] F. Andreatta and E. Z. Goren, *Hilbert modular forms: mod  $p$  and  $p$ -adic aspects*, Mem. Amer. Math. Soc. **173** (2005), no. 819, vi+100. MR 2110225
- [BDJ10] Kevin Buzzard, Fred Diamond, and Frazer Jarvis, *On Serre's conjecture for mod  $l$  Galois representations over totally real fields*, Duke Math. J. **155** (2010), no. 1, 105–161. MR 2730374
- [DK16] Fred Diamond and Payman Kassaei, *Minimal weights of hilbert modular forms in characteristic  $p$* , 2016, arXiv:1612.08725.
- [DKS20] Fred Diamond, Payman Kassaei, and Shu Sasaki, *A mod  $p$  jacquet-langlands relation and serre filtration via the geometry of hilbert modular varieties: Splicing and dicing*, 2020, arXiv:2001.00530.
- [DS17] Fred Diamond and Shu Sasaki, *A serre weight conjecture for geometric hilbert modular forms in characteristic  $p$* , 2017, arXiv:1712.03775.
- [Edi92] Bas Edixhoven, *The weight in Serre's conjectures on modular forms*, Invent. Math. **109** (1992), no. 3, 563–594. MR 1176206
- [Ser87] Jean-Pierre Serre, *Sur les représentations modulaires de degré 2 de  $\text{Gal}(\overline{\mathbf{Q}}/\mathbf{Q})$* , Duke Math. J. **54** (1987), no. 1, 179–230. MR 885783