WEIGHT PART OF SERRE'S CONJECTURE AND ITS RELATION TO CRYSTALLINE LIFTS

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Disclaimer. *These are notes for a talk I gave in the Summer 2020 London study group about the weight part of Serre's conjecture (on Zoom). These notes might contain some typos and mistakes. I did not have time to go through everything carefully but have included references for those interested in more detail.*

1. Goals of this talk

Goal 1.1. *Prove the weight part of Serre's conjecture*

Goal 1.2. *Relate Serre's recipe for the weight to crystalline lifts of particular weights.*

In both parts we will see phenomena which we will encounter in later talks, such as weight filtrations, θ-cycles and crystalline lifts.

Note by a form of type (N, k, ε) we now mean a geometric mod p modular form of this type (in the sense of last week's talk).

The following two theorems will be very important for both of these goals:

Theorem 1.3 (Deligne). *Let f be a cuspidal Hecke eigenform of type (N, k, ε) with $2 \leq k \leq p+1$ with eigenvalues a_l. Suppose that $a_p \neq 0$ (the ordinary case). Then $\rho_{f,p}$ is reducible and*

$$\rho_{f,p} = \begin{pmatrix} \omega^{k-1}\lambda(\varepsilon(p)/a_p) & * \\ 0 & \lambda(a_p) \end{pmatrix}.$$

with ω the mod p cyclotomic character and $\lambda(a) : G_{\mathbb{Q}_p} \to \overline{\mathbb{F}}_p^{\times}$ the unramified character sending Frob_p to $a \in \overline{\mathbb{F}}_p^{\times}$.

Proof. See [7, Proposition 11.1]. □

Theorem 1.4 (Fontaine). *Let f be a cuspidal Hecke eigenform of type (N, k, ε) with $2 \leq k \leq p+1$ with eigenvalues a_l. Suppose that $a_p = 0$ (supersingular case). Then $\rho_{f,p}$ is irreducible and*

$$\rho_f|_I = \begin{pmatrix} \omega_2^{k-1} & 0 \\ 0 & \omega_2'^{k-1} \end{pmatrix}$$

with ω_2 and ω_2' the two fundamental characters of level 2.

Proof. See [5, Section 6.8]. □

2. The weight part of Serre's conjecture

In this section we discuss the proof of the weight part by Edixhoven ([5, Section 4]).

Theorem 2.1 (The weight part)**.** *Let $\rho : G_{\mathbb{Q}} \to \mathrm{GL}_2(\overline{\mathbb{F}}_p)$ be continous, irreducible and odd. Suppose we have a cuspidal eigenform g of type (N, k, ε) such that $\rho \cong \rho_g$. Then*

(1) (existence): there exists a cuspidal eigenform of type (N, k_ρ, ε) (Serre's recipe) with the same eigenvalues for all $l \neq p$ such that $\rho \cong \rho_f$
(2) (refinement): there exists an eigenform f of type $(N, k(\rho), \varepsilon)$ (Edixhoven's adjusted weight) with the same eigenvalues for all $l \neq p$ such that $\rho \cong \rho_f$.
(3) (minimality): there is no eigenform of level prime to p and of weight less than $k(\rho)$ whose associated Galois representation is isomorphic to ρ.

Remark 2.2. Proof below is for odd p. (For $p = 2$ we need to be more careful in some places.)

2.1. **Existence.** This consists of the following three steps

(1) Twist by the cyclotomic character to get a form f_1 of weight $k_1 \leq p + 1$
(2) Use results on ρ_{f_1, G_p} to determine k_1
(3) Use θ-cycles to obtain weight k_ρ

Step 1: Twist by the cyclotomic character to get weight $k_1 \leq p + 1$

Question 2.3. How does twisting affect the weight of f?

Recall the θ operator:

$$\theta : \sum a_n q^n \to \sum n a_n q^n$$

with $a_n \in \overline{\mathbb{F}}_p$.

If f is an eigenform of type (N, k, ε) with eigenvalues a_l, then θf is an eigenform of $(N, k + p + 1, \varepsilon)$ with eigenvalues $l a_l$. (This should follow from last week's talk) We can use this to translate twists of a modular form by the θ operator to twists of its representation by the cyclotomic character:

Fact 2.4.

$$\rho_{\theta f} = \rho_f \otimes \omega$$

To complete step 1 we will use the following, which says that any form of some weight k 'comes from' a form of a lower weight.

Theorem 2.5. *Let f be an eigenform of some types (N, k, ε), then there exist integers i and k' with $0 \leq i \leq p - 1, k' \leq p + 1$ and an eigenform g of type (N, k', ε) such that f and $\theta^i g$ have the same eigenvalues for all Hecke operators $T_l (l \neq p)$.*

Proof. [5, Section 7]. □

By this theorem and the fact above we obtain f_1 of weight $k_1 \leq p + 1$ and $i \in \mathbb{Z}/(p - 1)/\mathbb{Z}$ such that $\rho \otimes \omega^{-i}$ is isomorphic to ρ_{f_1} f_1 where f_1 is of type (N, k_1, ε) with $k_1 \leq p + 1$. This finishes step 1.

Step 2: study ρ_{f_1, G_p} to determine k_1

2.1.1. *The weight filtration.* Mod p modular forms of different weights can have the same q-expansion, but we do know the following:

Theorem 2.6. *If $f \in M(N,k)$ and $f' \in M(N,k')$ have the same q-expansion (at some cusp) then $k' \equiv k \mod p-1$.*

Proof. See [11, Theorem 2]. □

We define:

Definition 2.7 (The weight filtration)**.** The filtration $w(f)$ of $f \in M(N,k)$ is the smallest integer k' for which there exists a form in $M(N,k')$ which, at some cusp, has the same q-expansion as f. We can also define it as

$$w(f) = \min\{k - i(p-1) \,|\, f \in A^i M(N, k-i(p-1))\}.$$

with A the Hasse invariant as introduced last week.

Remark 2.8. We think of the Hasse invariant A as a modular form of weight $p-1$, and, moreover, $f \cong Af \mod p$.

2.1.2. *Results on $\rho_{f,p}$.* We will get congruence relations in our proof and we will sometimes need to decide between weights 1 and p, and weight 2 and $p+1$. We will have tools for this in terms of the ramification behaviour.

Tool I: Excluding weight 1

Proposition 2.9. *Let f be a cuspidal eigenform of type $(N,1,\varepsilon)$. Then ρ_f is unramified at p ($p>2$).*

Proof. See [5, Proposition 2.7]. □

Tool II: Deciding between weight 2 and $p+1$

Theorem 2.10 (Mazur)**.** *Let f be a cuspidal eigenform of type $(N,p+1,\varepsilon)$. Suppose that ρ_f is irreducible and that there exists no form g of type $(N,2)$ such that $f = Ag$. Then $a_p^2 = \varepsilon(p)$, and $\rho_{f,p}$ in Deligne's theorem is not finite at p.*

Proof. See [10, Section 6] and [5, Section 2.4]. □

Remark 2.11.

- Ordinary case ([5, Section 8]): if f is a cuspidal eigenform of type $(N,p+1,\varepsilon)$ then the notions 'finite at p' and 'peu ramifie' are equivalent. Moreover ρ_f is finite at p if f has weight 2.
- Supersingular case: we note that $p \nmid N$, so that $\varepsilon(p) \neq 0$, hence there must exist a form of weight 2.

Tool III: Criterion for $\rho_{f,p}$ for to be tamely ramified

Theorem 2.12 (Gross)**.** *Let f be as in Deligne's theorem. Then $\rho_{f,p}$ is tamely ramified (i.e., its restriction to the inertia group is diagonalisable) if and only if there exists a cuspidal eigenform f' of type (N,k',ε), with $k' = p+1-k$, such that $la'_l = l^{k'}a_l$ for all l.*

Proof. See [7, Theorem 13.10]. □

If such a form exists, it is called a *companion form*, explicitly, this gives:

$$\rho_f \otimes \omega \cong \rho_{f'} \otimes \omega^k$$

2.1.3. *Finishing Step 2.* Let ϕ, ϕ' be the two characters $I_t \to \overline{\mathbb{F}}_p^\times$ associated to ρ_p (as in [5, Section 2.4]).

The level 2 case

Then $\phi \neq \phi^p = \phi'$ is irreducible and we can write

$$\rho|_I = \begin{pmatrix} \omega_2^b \omega_2'^a & 0 \\ 0 & \omega_2^a \omega_2'^b \end{pmatrix}$$

with ω_2, ω_2' the two fundamental characters of level 2 and $0 \leq a < b \leq p-1$.

By Step 1 we have i such that $\rho \otimes \omega^{-i} \cong \rho_{f_1}$ with weight $k_1 \leq p+1$. We may suppose $k_1 = w(f_1)$. We deduce:

- Comparing Fontaine and Deligne's theorems, we find $a_p = 0$.
- f_1 has eigenvalues $l^{-i} a_l (l \neq p)$, with a_l the eigenvalues of g.
- By Tool I we find $k_1 \geq 2$
- By Tool II we find $k_1 \leq p$.

To find possible values for (i, k_1) such that $\rho \cong \omega^i \otimes \rho_{f_1}$, we write

$$\begin{pmatrix} \omega_2^b \omega_2'^a & 0 \\ 0 & \omega_2^a \omega_2'^b \end{pmatrix} = \omega^i \otimes \begin{pmatrix} \omega_2^{b-a} & 0 \\ 0 & \omega_2'^{b-a} \end{pmatrix}.$$

Suppose we need to solve

$$\omega^a \otimes \begin{pmatrix} \omega_2^{b-a} & 0 \\ 0 & \omega_2'^{b-a} \end{pmatrix} = \omega^i \otimes \begin{pmatrix} \omega_2^{k_1-1} & 0 \\ 0 & \omega_2'^{k_1-1} \end{pmatrix}.$$

one clear possibility is $i = a$, and $k = b - a + 1$. We can also write

$$\begin{pmatrix} \omega_2^b \omega_2'^a & 0 \\ 0 & \omega_2^a \omega_2'^b \end{pmatrix} = \omega^{b-1} \otimes \begin{pmatrix} \omega_2'^{a-b+p+1} & 0 \\ 0 & \omega_2^{a-b+p+1} \end{pmatrix},$$

so that we also get $i = b - 1$ and $k = p + 2 + a - b$. If $b - a = 1$, then the only option is $i = a$ and $k = 2$, so

$$(i, k_1) = \begin{cases} (a, 1 + b - a), (b - 1, p + 2 + a - b), & \text{if } b - a \neq 1, \\ (a, 2), & \text{if } b - a = 1. \end{cases}$$

The level 1 case

Then we have

$$\rho_p = \begin{pmatrix} \omega^\beta \varepsilon_1 & * \\ 0 & \omega^\alpha \varepsilon_2 \end{pmatrix},$$

for some α, β and with $\varepsilon_1, \varepsilon_2$ two unramified characters $G_p \to \overline{\mathbb{F}}_p^\times$

By Step 1, we have i such that $\rho \otimes \omega^{-i} \cong \rho_{f_1}$ with weight $k_1 \leq p+1$. We may suppose $k_1 = w(f_1)$.

We have to split this in two cases: $* \neq 0$ and $* = 0$, we will just treat the first case here.

Suppose $* \neq 0$. Normalise α, β as follows

$$0 \leq \alpha \leq p-2, 1 \leq \beta \leq p-1$$

We get one solution for (i, k_1) with $i = \alpha$ and $k_1 \geq 2$ (by Tool I) with this gives us

$$k_1 - 1 \equiv \beta - \alpha \mod p - 1$$

If $\alpha < \beta$, we have $1 \leq \beta - \alpha \leq p - 1$, this determines $k_1 = 1 + \beta - \alpha$ unless $\beta - \alpha = 1$, then we have to decide between weight 2 and $p + 1$, for which we use Tool II.

When $\beta - \alpha = 1$ we find $k_1 = 1 + \beta - \alpha$ unless ρ is tres ramifie, then $k_1 = 1 + \beta - \alpha + p - 1$.

Step 3: Use θ-cycles to obtain weight k_ρ

Question 2.13. We know how θ affects the weight, but how does it affect $w(f)$?

We have the following result.

Lemma 2.14. *If f has filtration k and $p \nmid k$, then θf has filtration $k + p + 1$. If $p \mid k$, then $w(\theta f) < w(f) + p + 1$.*

Proof. See e.g. [11, Corollaire 3]. □

To give a more precise answer, we study θ-cycles. We motivate the terminology by noting that in the supersingular case we have $w(\theta^{p-1} f) = w(f)$ and we always have $w(\theta^p f) = w(\theta f)$.

Definition 2.15. Let f be such that $\theta f \neq 0$. Then the θ-cycle of f is the sequence of p integers

$$(w(f), (w(\theta f), \ldots, w(\theta^{p-1} f))$$

For more information about these, we refer to [8]. Edixhoven determines such cycles for low weights $1 \leq k \leq p + 1$ ([5, Proposition 3.3]):

Example. Let f be a cuspidal eigenform of type (N, k, ε) with $3 \leq k \leq p - 1$ with eigenvalues a_l and $w(f) = k$. Suppose $a_p = 0$, then the θ-cycle of f is

$$(k, k + p + 1, \ldots, k + (p - k)(p + 1), k_1, \ldots, k_1 + (k - 3)(p + 1), k)$$

where $k_1 = p + 3 - k$.

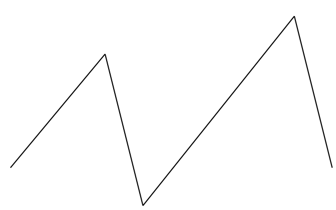

Figure 1: Schematic view of θ-cycles for f as above (where $k_1 < k$)

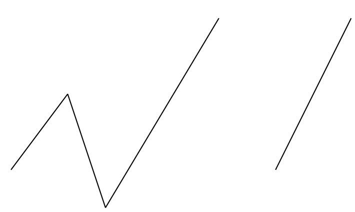

Figure 2: Schematic view of θ-cycles for ordinary f with $2 \leq w(f) \leq p - 1$ on the left and $w(f) = 1, p, p + 1$ on the right

In general the idea is that when the weight filtration becomes divisible by p, we first add $p + 1$ and then substract a multiple of $p - 1$.

Continued level 2 case Recall

$$(i,k_1) = \begin{cases} (a, 1+b-a), (b-1, p+2+a-b) & \text{if } b-a \neq 1 \\ (a,2) & \text{if } b-a = 1. \end{cases}$$

If f_1 is an eigenform of type $(N, 1+b-a, \varepsilon)$ with eigenvalues $l^{-a}a_l$, we can take $f = \theta^a f_1$. We want to compute $w(f)$ to see if this form gives the right weight. We do this using the θ-cycles for f_1, we want to see if the drop in θ-cycles happens before or after we have twisted by a. We can do this by looking at the following congruence

$$1+b-a+n(p+1) \equiv 0 \mod p$$

happens when $n = a-b-1+p \geq a$ (for $0 \leq n \leq p-1$). So this implies

$$w(f) = 1+b-a+(p+1)a = 1+pa+b = k_\rho.$$

Now if f_1 is an eigenform of type $(N, p+2+a-b, \varepsilon)$ with eigenvalues $l^{-(b-1)}a_l$, and we can take $f = \theta^{b-1} f_1$, again using θ-cycles we can show

$$w(f) = 1+pa+b = k_\rho$$

Continued level 1 case

For $* \neq 0$,

$$(i,k_1) = \begin{cases} (\alpha, 1+\beta-\alpha+p-1), & \text{if } \beta-\alpha = 1 \text{ and } \rho_p \text{ is tres ramifie,} \\ (\alpha, 1+\beta-\alpha), & \text{otherwise.} \end{cases}$$

Using Edixhoven's explicit description we find that twisting by θ^α is before any drop, so that

$$w(\theta^\alpha f_1) = k_1 + (p+1)\alpha = k_\rho.$$

We proceed similarly for $\alpha \geq \beta$ and for $* = 0$ (note that this final case uses Tool III). This finishes existence.

2.2. **Refinement.** Edixhoven uses Katz' notion of modular forms to refine $k(\rho)$, but they only differ in two cases, and only in the level one case.

(1) $\rho|_I$ is trivial, $a = 0 = b$, then $k(\rho) = 1$ and $k_\rho = p$
(2) $\rho|_I$ is not trivial, $p = 2, \alpha = 0, \beta = 1, \rho_p$ is not finite at p, then $k(\rho) = 3$ and $k_\rho = 4$

We can prove the first case using Tool III, this will give us a companion form of weight 1. This finishes refinement.

2.3. **Minimality.** Suppose the contrary, suppose g is an eigenform of some type (M, k, ε') with $p \nmid M$ and $\rho \cong \rho_g$. Then by the previous (existence and refinement) we obtain a form f of type $(M, k(\rho), \varepsilon')$ with $\rho \cong \rho_f$ such that f has the same eigenvalues a_l for all $l \neq p$. If $a_p(f) = 0$ and $a_p(g) = 0$, then the q-expansions of f and g at a fixed cusp differ by a constant factor and we have

$$k(\rho) = w(f) = w(g) \leq k$$

If $a_p(g) \neq 0$, then we have to use the following proposition:

Proposition 2.16. *(Gross) If $a_p \neq 0$, then f has filtration k with $2 \leq k \leq p+1$.*

Proof. [7, Proposition 4.12]. □

We use this to check minimality case by case. This finishes minimality and the proof of the weight part of Serre's conjecture.

3. Relation to crystalline lifts

First we note that p-adic representations have very different properties from l-adic ones. The p-adic analogue of an unramified l-adic representation is that of a *crystalline* representation. These are studied in p-adic Hodge theory. There are many introductions to the theory, see e.g. [1] or [4]. In this section we use these tools to prove Fontaine's and Deligne's theorems.

In this section we write χ for the p-adic cyclotomic character.

Definition 3.1 (Tate twists)**.** We denote $\mathbb{C}_p$ for the completion of $\overline{\mathbb{Q}}_p$ and we denote $\mathbb{C}_p(n) = \mathbb{C}_p(\chi^n)$ for the n-th Tate twist, which is the one-dimensional representation with basis e described by

$$g(\alpha e) := g(\alpha)\chi(g)^n e, g \in G_{\mathbb{Q}_p}, \alpha \in \mathbb{C}_p$$

Definition 3.2 (Hodge–Tate weights)**.** If V is a crystalline representation of $G_{\mathbb{Q}_p}$ over $\overline{\mathbb{Q}}_p$, then i is a Hodge–Tate weight of V if

$$(V \otimes_{\mathbb{Q}_p} \mathbb{C}_p(-i))^{G_{\mathbb{Q}_p}} \neq 0.$$

Remark 3.3. Note the representation does not have to be crystalline for the above definition to make sense.

Example. The p-adic cyclotomic character χ has Hodge–Tate weight one.

Fact 3.4. *Say we have f of type (N, k, ε), then we know $\rho_{f,p}$ is crystalline at p with Hodge–Tate weights $(0, k-1)$ (due to Faltings, see [6]).*

3.1. Reduction of crystalline representations.

Proposition 3.5. *Let V be a two-dimensional crystalline representation of G_p over $\overline{\mathbb{Q}}_p$ with Hodge–Tate weights $(0, k-1)$ for $1 < k < p$. Let T be a $\overline{\mathbb{Z}}_p$-lattice in V stable under G_p and let $\overline{T}$ be its reduction modulo $\mathfrak{m}_{\overline{\mathbb{Z}}_p}$.*

- *If V is reducible then*
$$\overline{T} \cong \begin{pmatrix} \omega^{k-1}\lambda(\mu_2) & * \\ 0 & \lambda(\mu_1) \end{pmatrix}$$
for $\mu_1, \mu_2 \in \overline{\mathbb{F}}_p^{\times}$, with $*$ peu ramifie if $k = 2$.
- *If V is irreducible then*
$$\overline{T} \cong (\mathrm{Ind}_{G_{p^2}}^{G_p} \omega_2^{k-1}) \otimes \lambda(\mu), \mu \in \overline{\mathbb{F}}_p^{\times}.$$

Proof. [3, Proposition 4.1.1]. □

Remark 3.6.
- Using Dieudonne modules, one can obtain a similar result in the reducible case for all $k > 1$ (with $*$ peu ramifie if $k = 2$), see [9].
- The above proposition is proved using Fontaine-Lafaille theory. In the irreducible case there is a proof using Wach modules instead and going up to $k = p + 1$, see [2].

Now we can give a 'proof' of the Theorems by Deligne and Fontaine.

Rough idea. *Suppose we have f of type (N, k, ε), then we know $\rho_{f,p}$ is crystalline at p with Hodge-Tate weights $(0, k-1)$. We know that the reduction of crystalline Galois representations with Hodge-Tate weights has the form giving the results of the theorems.*

3.2. **Crystalline lifts.** Suppose we have a representation $\rho : G_{\mathbb{Q}} \to \mathrm{GL}_2(\overline{\mathbb{F}}_p)$. In this section we show one can construct explicit crystalline lifts of the local representation.

3.2.1. *The ordinary case.* Suppose first for some $k \leq p$

$$\rho_{G_{\mathbb{Q}_p}} = \begin{pmatrix} \omega^{k-1}\lambda(\mu_1) & 0 \\ 0 & \lambda(\mu_2) \end{pmatrix}$$

then we obtain a lift with Hodge-Tate weights $(0, k-1)$.

$$\begin{pmatrix} \chi^{k-1}\lambda(\tilde{\mu}_1) & 0 \\ 0 & \lambda(\tilde{\mu}_2) \end{pmatrix}$$

with χ the p-adic cyclotomic character and $\sim$ Teichmuller lifts.

Note here that $* = 0$, i.e. we are dealing with a split extension, the case $* \neq 0$ is similar but a bit more involved.

3.2.2. *The supersingular case.* Suppose next that $\rho_{G_{\mathbb{Q}_p}}$ is irreducible, then there is a character $\xi : G_{\mathbb{Q}_p^2} \to \overline{\mathbb{F}}_p^{\times}$ such that

$$\rho_{G_{\mathbb{Q}_p}} \sim \mathrm{Ind}_{G_{\mathbb{Q}_p^2}}^{G_{\mathbb{Q}_p}} \xi$$

with $\xi|_I = \omega_2^{k-1}$ for some k.

Fact 3.7. *We can lift the character ξ to a crystalline character with Hodge-Tate weights $(0, k-1)$.*

Then the representation $\mathrm{Ind}\,\tilde{\xi}$ is a crystalline lift of $\rho_{G_{\mathbb{Q}_p}}$ with Hodge-Tate weights $(0, k-1)$.

3.2.3. *Minimal crystalline lifts.* We can also define a minimal crystalline lift as follows:

Definition 3.8 ($k_{\mathrm{cris}}(\rho)$)**.** For a representation $\rho : G_{\mathbb{Q}} \to \mathrm{GL}_2(\overline{\mathbb{F}}_p)$ we define $k_{\mathrm{cris}}(\rho)$ to be the least $k \geq 1$ such that ρ_p has a crystalline lift of weight $(0, k-1)$.

We can relate this to Serre's minimal weight as follows. Suppose ρ is modular of some weight k, then by Fact 3.4 we have a lift with weights $(0, k-1)$. Hence $k_{\mathrm{cris}}(\rho) \leq k$. In fact, we know $k(\rho) = k_{\mathrm{cris}}(\rho)$ (for $k \leq p+1$ this can be seen from the previous).

Remark 3.9. We will soon see more crystalline lifts in a more general context - it is closely related to modularity and Serre weight conjectures.

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