

Robustness issues on regulatory risk measures

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Robust Techniques in Quantitative Finance
Oxford University, UK September 3-7, 2018

Content

Based on some joint work with



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- ▶ **Embrechts-Wang-W.**, **Aggregation-robustness and model uncertainty of regulatory risk measures.** Finance and Stochastics, 2015
- ▶ **Embrechts-Liu-W.**, **Quantile-based risk sharing.** Operations Research, 2018
- ▶ **Embrechts-Schied-W.**, **Robustness in the optimization of risk measures.** Working paper, 2018

Agenda

- 1 Background
- 2 Classic statistical robustness
- 3 Some other perspectives of robustness
- 4 Robustness in optimization
- 5 Conclusion

Risk Measures

A **risk measure** $\rho : \mathcal{X} \rightarrow \overline{\mathbb{R}} = (-\infty, \infty]$

- ▶ **Risks** are modelled by random losses in a **specified period**
 - e.g. 10d in Basel III & IV market risk
- ▶ \mathcal{X} is a convex cone of rvs in some probability space $(\Omega, \mathcal{F}, \mathbb{P})$

Roles of risk measures

- ▶ regulatory capital calculation ← **our main interpretation**
- ▶ management, optimization and decision making
- ▶ performance analysis and capital allocation
- ▶ pricing

General Question

Question

What is a “good” risk measure for regulatory capital calculation?

- ▶ **Regulator's** and **firm manager's** perspectives can be different or even conflicting
 - **well-being of the society** versus **interest of the shareholders**
 - **systemic risk in an economy** versus **risk of a single firm**

Value-at-Risk and Expected Shortfall

Value-at-Risk (VaR) at level $p \in (0, 1)$

$\text{VaR}_p : L^0 \rightarrow \mathbb{R}$,

$$\text{VaR}_p(X) = F_X^{-1}(p) = \inf\{x \in \mathbb{R} : \mathbb{P}(X \leq x) \geq p\}.$$

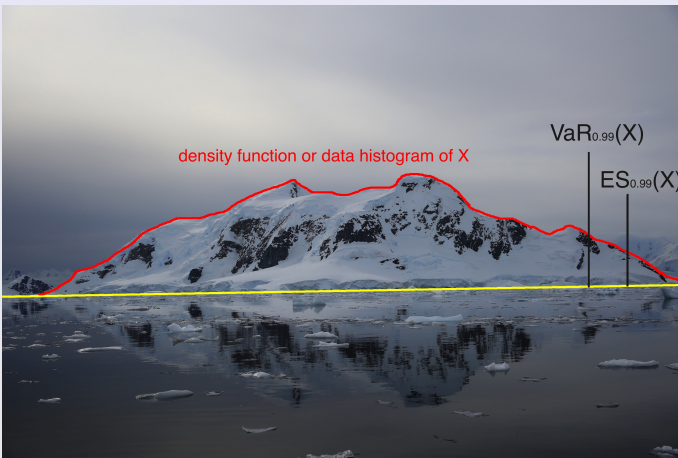
Expected Shortfall (ES/TVaR/CVaR/AVaR) at level $p \in (0, 1)$

$\text{ES}_p : L^0 \rightarrow \overline{\mathbb{R}}$,

$$\text{ES}_p(X) = \frac{1}{1-p} \int_p^1 \text{VaR}_q(X) dq \underset{(F_X \text{ cont.})}{=} \mathbb{E}[X | X > \text{VaR}_p(X)].$$

F_X above is the distribution function of X .

Value-at-Risk and Expected Shortfall



Value-at-Risk and Expected Shortfall

The ongoing **co-existence** of VaR and ES:

- ▶ Basel IV - **both**
- ▶ Solvency II - **VaR**
- ▶ Swiss Solvency Test - **ES**

Academic Inputs

- ▶ ES is generally **advocated by academia** for desirable properties in the past two decades; in particular,
 - **subadditivity** or **coherence** (**Artzner-Delbaen-Eber-Heath'99**)
 - **convex optimization** properties (**Rockafellar-Uryasev'00**)
- ▶ Some other examples of impact from academic research
 - **Gneiting'11**: **backtesting** ES is **unclear**, whereas backtesting VaR is **straightforward**
 - **Cont-Deguest-Scandolo'10**: **ES is not robust**, whereas **VaR is**

VaR versus ES

BCBS Consultative Document, May 2012, Page 41, *Question 8*:

“What are the likely constraints with moving from VaR to ES, including any challenges in delivering robust backtesting, and how might these be best overcome?”

VaR versus ES

Features/Risk measure	VaR	Tail-VaR
Frequency captured?	Yes	Yes
Severity captured?	No	Yes
Sub-additive?	Not always	Always
Diversification captured?	Issues	Yes
Back-testing?	Straight-forward	Issues
Estimation?	Feasible	Issues with data limitation
Model uncertainty?	Sensitive to aggregation	Sensitive to tail modelling
Robustness I (with respect to "Lévy metric" ³³)?	Almost, only minor issues	No
Robustness II (with respect to "Wasserstein metric" ³⁴)?	Yes	Yes

Table copied from [IAIS Consultation Document Dec 2014](#), page 42

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Model Uncertainty

VaR and ES are **law-based** (thus **statistical risk functionals**):

$\rho(X) = \rho(Y)$ if $X \stackrel{d}{=}_{\mathbb{P}} Y$ (equal in distribution under \mathbb{P})

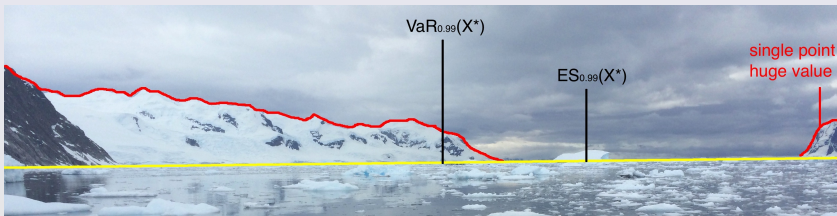
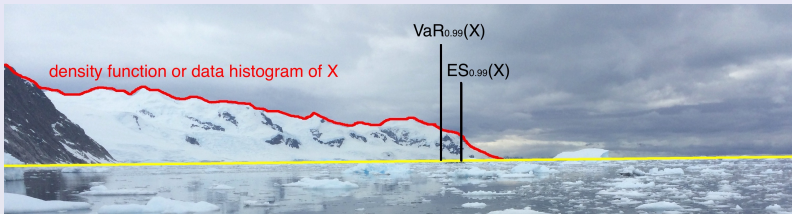
- ▶ The calculation requires **knowledge** of the distribution of a risk
- ▶ This may never be the exact case: **model uncertainty**
 - **statistical** error
 - **computational** error
 - **modeling** error
 - **conceptual** error
- ▶ Models are **at most** “**approximately correct**” \Rightarrow **robustness!**

Robust Statistics

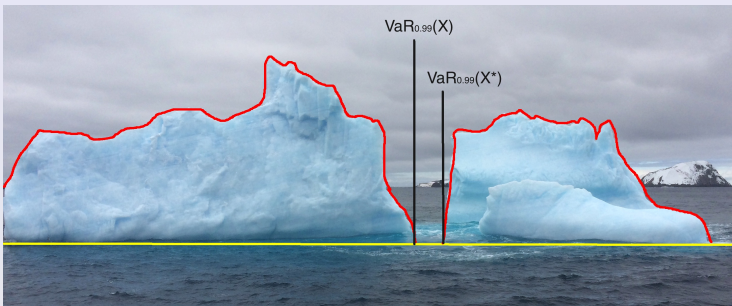
Statistical robustness addresses the question of “**what if the data is compromised with small error?**” (e.g. outlier)

- ▶ Originally **robustness** is defined on **estimators** (estimation procedures)
- ▶ Would the estimation be **ruined** if the underlying model is **compromised**?
 - e.g. an **outlier** is added to the sample

VaR and ES Robustness



VaR and ES Robustness



- ▶ Non-robustness of VaR_p only happens if the quantile has a gap at p
- ▶ Is this situation relevant for risk management practice?
 - one must be **very unlucky** to hit precisely where it has a gap ...

Robust Statistics

Classic qualitative robustness:

- ▶ **Hampel'71**: the robustness of a consistent estimator of T is equivalent to the **continuity of T** with respect to underlying distributions (both with respect to the same metric)
- ▶ When we talk about the **robustness** of a statistical functional, (**Huber-Hampel's**) robustness typically refers to **continuity** with respect to **some metric**.
- ▶ (Pseudo-)metrics: $\pi^q = L^q$ ($q \geq 1$), $\pi^\infty = L^\infty$, $\pi^W = \text{Lévy}$,
...

Robustness of Risk Measures

Consider the continuity of $\rho : \mathcal{X} \rightarrow \mathbb{R}$.

- ▶ A strong sense of continuity is w.r.t. **weak convergence**.
 - $X_n \rightarrow X$ in distribution $\Rightarrow \rho(X_n) \rightarrow \rho(X)$.
- ▶ Quite **restrictive**
- ▶ Practitioners like weak convergence (e.g. **estimation**, **simulation**)

Robustness of Risk Measures

- ▶ With respect to weak convergence $p \in (0, 1)$:
 - VaR_p is continuous at **distributions whose quantile is continuous at p** . VaR_p is argued as being **almost robust**.
 - ES_p is **not continuous** for any $\mathcal{X} \supset L^\infty$
- ▶ ES_p is continuous w.r.t. some other (stronger) metric, e.g. π^q (or the **Wasserstein- L^q metric**)

Range-Value-at-Risk (RVaR)

A two-parameter family of risk measures, for $\alpha, \beta > 0$, $\alpha + \beta < 1$,

$$\text{RVaR}_{\alpha,\beta}(X) = \frac{1}{\beta} \int_{\alpha}^{\alpha+\beta} \text{VaR}_{\gamma}(X) d\gamma, \quad X \in \mathcal{X}.$$

- ▶ RVaR **bridges the gap** between VaR and ES (limiting cases).
- ▶ RVaR is **continuous w.r.t. weak convergence**
- ▶ RVaR is **not convex or coherent**; it is finite on L^0
- ▶ Practically:

$$\text{RVaR}_{\alpha,\beta}(X) \underset{(F_X \text{ cont.})}{=} \mathbb{E}[X | \text{VaR}_{\alpha}(X) < X \leq \text{VaR}_{\alpha+\beta}(X)].$$

First proposed by [Cont-Deguest-Scandolo'10](#); name in [W.-Bignozzi-Tsanakas'15](#) 

Classic Robustness

The general perception of robustness, from worst to best:

$$ES \prec VaR \prec R VaR$$

Distortion Risk Measures

A **distortion risk measure** is defined as, for $X \in \mathcal{X}$,

$$\rho(X) = \int_0^\infty h(\mathbb{P}(X > x))dx + \int_{-\infty}^0 (h(\mathbb{P}(X > x)) - 1)dx,$$

where h is an increasing function on $[0, 1]$ with $h(0) = 0$ and $h(1) = 1$. h is called a **distortion function**. If h is **continuous**,

$$\rho(X) = \int_0^1 \text{VaR}_p(X)dg(p), \quad X \in \mathcal{X},$$

where $g(t) = 1 - h(1 - t)$, $t \in [0, 1]$.

- ▶ ES and VaR are special cases of distortion risk measures

Distortion Risk Measures

Some summary.

- ▶ A **distortion risk measure** is continuous (wrt π^W) on $L^\infty \Leftrightarrow$ its distortion function has a derivative which vanishes at neighbourhoods of 0 and 1 (classic property of **L-statistics**).
- ▶ From weak to strong:
 - Continuity w.r.t. π^∞ : all **monetary risk measures**
 - Continuity w.r.t. π^q , $q \geq 1$: finite **convex risk measures** on L^q , e.g. **ES_p**
 - Continuity w.r.t. **weak/a.s./P** convergence: e.g. **RVaR_{α,β}**, **VaR_p** (almost); **no convex risk measure satisfies this**

Some results: **Bäuerle-Müller'06**, **Cont-Deguest-Scandolo'10**, **Kou-Peng-Heyde'13**;
 general references: **Rüschendorf'13**, **Föllmer-Schied'16**

Robustness of Risk Measures

Is robustness w.r.t. weak convergence **necessarily a good thing?**

▶ Toy example.

- Let $X_n = n^2 I_{\{U \leq 1/n\}}$ for some $U[0,1]$ random variable U (e.g. a credit default risk). Clearly $X_n \rightarrow 0$ a.s. but X_n is getting more “dangerous” in many senses. **If ρ preserves weak convergence, then**

$$\rho(X_n) \rightarrow \rho(0) \quad (= 0 \text{ typically}).$$

- $\text{VaR}_{0.999}(X_{10000}) = 0$
 - $\text{ES}_{0.999}(X_{10000}) = 10^7$
- ▶ May be reasonable for **internal management**; not so much for **regulation**.

One-in-ten-thousand Event

On the other hand,

- ▶ the 1/10,000-event-type risks are very difficult to capture statistically (accuracy is impossible)

UK House of Lords/House of Commons, June 12, 2013, Output of a “stress test” exercise, from HBOS:

*“We actually got an external advisor [to assess how frequently a particular event might happen] and they came out with **one in 100,000 years** and **we said “no”**, and I think we submitted **one in 10,000 years**. But that was **a year and a half** before it happened. It doesn't mean to say it was wrong: **it was just unfortunate that the 10,000th year was so near.**”*

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Uncertainty in Risk Aggregation

General setup

- ▶ To calculate $\rho(S)$ where $S = \Lambda(X_1, \dots, X_n)$ for risk factors $X_1, \dots, X_n \in \mathcal{X}$ and aggregation function $\Lambda : \mathbb{R}^n \rightarrow \mathbb{R}$
- ▶ Two levels of model uncertainty:
 - the marginal distributions F_i of X_i , $i = 1, \dots, n$
 - the dependence structure (copula) of (X_1, \dots, X_n)
- ▶ Both $\text{VaR}_p(S)$ and $\text{ES}_p(S)$ depend on both levels
 - The second level of uncertainty is arguably more challenging due to data, computation and modeling limitations
 - In the Basel IV market risk formulas, the value of $\text{ES}_p(S)$ requires a calculation under the worst-case dependence

Some references on risk aggregation under dependence uncertainty:

[Embrechts-Puccetti-Rüschendorf'13](#), [Bernard-Jiang-W.'14](#), [Cai-Liu-W.'18](#)

Uncertainty in Risk Aggregation

- ▶ **Uncertainty at the second level** (with first level fixed):
 - **Robustness**: is $\rho \circ \Lambda$ continuous with respect to the modeling in dependence (π^W)? \Rightarrow **robustness in risk aggregation**
 - **Uncertainty spread**: how large is the spread of $\rho \circ \Lambda$ if we do not know about the dependence?
- ▶ We focus on the natural aggregation function
$$\Lambda(x_1 + \cdots + x_n) = \sum_{i=1}^n x_i.$$
- ▶ $\mathcal{X} = L^1, L^\infty, \dots$

Robustness in Risk Aggregation

Some results. In the problem of **risk aggregation**,

- ▶ A distortion risk measure is robust on $L^\infty \Leftrightarrow$ its distortion function is **continuous** on $[0, 1]$.
- ▶ ES_p is **robust** on L^1 ;
- ▶ VaR_p is **not robust** on L^∞ (but almost)
- ▶ $RVaR_{\alpha,\beta}$ is **robust** on L^0
- ▶ The **uncertainty spread** of VaR_p is generally **bigger** than that of ES_q for $q \leq p$
 - In Basel III & IV market risk calculation, $VaR_{0.99}$ is replaced by $ES_{0.975}$

Robustness in Risk Aggregation

On robustness in risk aggregation:

$$\text{VaR} \prec \text{ES} \prec \text{RVaR}$$

Remark.

- ▶ The robustness of ES_p is due to uniform integrability in risk aggregation.

Robustness in Risk Sharing

Simplistic setup

- ▶ n agents sharing a total risk (or asset) $X \in \mathcal{X}$ (set of rvs)
- ▶ ρ_1, \dots, ρ_n : underlying risk measures (objectives to minimize)
 - The risk measures are chosen as VaR, ES and R VaR.
- ▶ **Optimality**: aggregate risk \Leftrightarrow collaborative \Leftarrow competitive
- ▶ **Robustness**: small model misspecification does not lead to very different aggregate risk value

Robustness in Risk Sharing

Some results.

- ▶ There exists a π^1 -robust optimal allocation of $X \Leftrightarrow$ no VaR is involved
- ▶ If X is bounded, then there exists a π^∞ -robust optimal allocation of $X \Leftrightarrow$ no VaR is involved
- ▶ There exists a π^W -robust optimal allocation of $X \Leftrightarrow$ no VaR is involved and at least one R VaR.

On robustness in risk sharing:

$$\text{VaR} \prec\prec \text{ES} \prec \text{R VaR}$$

The Optimization Problem

General setup

- ▶ $\mathcal{G}_n = \{\text{measurable functions from } \mathbb{R}^n \text{ to } \mathbb{R}\}$
- ▶ $X \in (L^0)^n$ is an **economic vector**, representing all random sources
- ▶ $\mathcal{G} \subset \mathcal{G}_n$ is an **admissible set** (decision set)
- ▶ $g(X)$ for $g \in \mathcal{G}$ represents a **risky position** of an investor
- ▶ ρ is an **objective functional** mapping $\{g(X) : g \in \mathcal{G}\}$ to $\overline{\mathbb{R}}$

“The optimization problem”:
to minimize $\rho(g(X))$ over $g \in \mathcal{G}$

(e.g. think about a classic hedging/optimal investment problem)

The Optimization Problem

Denote

$$\rho(X; \mathcal{G}) = \inf\{\rho(g(X)) : g \in \mathcal{G}\},$$

and (possibly empty)

$$\mathcal{G}^*(X, \rho) = \{g \in \mathcal{G} : \rho(g(X)) = \rho(X; \mathcal{G})\},$$

We call

- ▶ $g^* \in \mathcal{G}^*(X, \rho)$ an **optimizing function**
- ▶ $g^*(X)$ an **optimized position**

Uncertainty in Optimization

- ▶ The **optimization problem** is often subject to **severe model uncertainty** resulting from the assumptions made for X .
- ▶ Let \mathcal{Z} be a set of **possible economic vectors** including X ; \mathcal{Z} may be interpreted as the set of alternative models.
 - E.g. a parametric family of models (**parameter uncertainty**)
- ▶ The **real** economic vector $Z \in \mathcal{Z}$ is likely different from the **perceived** economic vector X .
 - X : **best-of-knowledge** model
 - Z : **real** model (**unknowable**)

Uncertainty in Optimization

- ▶ We choose $g \in \mathcal{G}^*(X, \rho)$ to optimize our objective ρ (**best-of-knowledge decision**).
 - **real** position $g(Z)$
 - **perceived** position $g(X)$
- ▶ If the modeling is good, Z and X are **close to each other** according to some metric π
- ▶ $\rho(g(Z))$ should be **close to** $\rho(g(X))$ to make sense of the optimizing function g
- ▶ We desire some **continuity** of the mapping $Z \mapsto \rho(g(Z))$ at $Z = X$

Robustness in Optimization

- ▶ We call $(\mathcal{G}, \mathcal{Z}, \pi_{\mathcal{Z}})$ an **uncertainty triplet** if $\mathcal{G} \subset \mathcal{G}_n$ and $(\mathcal{Z}, \pi_{\mathcal{Z}})$ is a pseudo-metric space of n -random vectors.
- ▶ ρ is **compatible** if it maps $\mathcal{G}(\mathcal{Z})$ to $\overline{\mathbb{R}}$, and $\rho(g(Y)) = \rho(g(Z))$ for all $g \in \mathcal{G}$ and $Y, Z \in \mathcal{Z}$ with $\pi_{\mathcal{Z}}(Y, Z) = 0$.

Definition 1

A compatible objective functional ρ is **robust at $X \in \mathcal{Z}$ relative to the uncertainty triplet $(\mathcal{G}, \mathcal{Z}, \pi_{\mathcal{Z}})$** if there exists $g \in \mathcal{G}^*(X, \rho)$ such that the function $Z \mapsto \rho(g(Z))$ is $\pi_{\mathcal{Z}}$ -continuous at $Z = X$.

Robustness in Optimization

Remarks.

- ▶ Robustness is a **joint property** of the tuple $(\rho, X, \mathcal{G}, \mathcal{Z}, \pi_{\mathcal{Z}})$
- ▶ Only a **$\pi_{\mathcal{Z}}$ -neighbourhood** of X in \mathcal{Z} matters
- ▶ If ρ is robust at X relative to $(\mathcal{G}, \mathcal{Z}, \pi_{\mathcal{Z}})$, then ρ is also
 - robust at X relative to $(\mathcal{G}, \mathcal{Y}, \pi_{\mathcal{Z}})$ if $X \in \mathcal{Y} \subset \mathcal{Z}$;
 - robust at X relative to $(\mathcal{G}, \mathcal{Z}, \hat{\pi}_{\mathcal{Z}})$ if $\hat{\pi}_{\mathcal{Z}}$ is **stronger** than $\pi_{\mathcal{Z}}$
- ▶ If $\mathcal{G}^*(X, \rho) = \emptyset$, then ρ is not robust at X
- ▶
 - One can use topologies instead of metrics
 - One can use uncertainty on \mathbb{P} instead of on X
 - **Conceptually different** from the field of **robust optimization** or **optimizing robust preferences**

Representative Optimization Problems

Representative optimization problems.

- ▶ $n = 1$ and X is a **random loss**
- ▶ The **pricing density** $\gamma = \gamma(X)$ is a measurable function of X
 - $\gamma > 0$, $\mathbb{E}[\gamma] = 1$ and $\mathbb{E}[\gamma X] < \infty$
- ▶ The **budget constraint** is $\mathbb{E}[\gamma g(X)] \geq x_0$
- ▶ Problems: to minimize $\rho(g(X))$ over $g \in \mathcal{G}$ for some $\mathcal{G} \subset \mathcal{G}_n$ in three settings $\mathcal{G} = \mathcal{G}_{\text{cm}}, \mathcal{G}_{\text{ns}}, \mathcal{G}_{\text{bd}}$

Representative Optimization Problems

(a) Complete market:

$$\mathcal{G}_{\text{cm}} = \{g \in \mathcal{G}_1 : \mathbb{E}[\gamma g(X)] \geq x_0\}.$$

(b) No short-selling or over-hedging constraint:

$$\mathcal{G}_{\text{ns}} = \{g \in \mathcal{G}_1 : \mathbb{E}[\gamma g(X)] \geq x_0, 0 \leq g(X) \leq X\}.$$

Assume $0 \leq x_0 < \mathbb{E}[\gamma X]$ to avoid triviality.

(c) Bounded constraint: for some $m > 0$,

$$\mathcal{G}_{\text{bd}} = \{g \in \mathcal{G}_1 : \mathbb{E}[\gamma g(X)] \geq x_0, 0 \leq g(X) \leq m\}.$$

Assume $0 \leq x_0 < m$ to avoid triviality.

Representative Optimization Problems

Remark.

- ▶ Problem (c) is not a special case of Problem (b) as X in (b) is both the **constraint** and the **source of randomness**

For (a)-(c), assume

- ▶ $X \geq 0$ and the distribution function of X is continuous and strictly increasing on $(\text{ess-inf}X, \text{ess-sup}X)$.
- ▶ $X \in \mathcal{Z}$, and $(\mathcal{Z}, \pi_{\mathcal{Z}})$ is one of the classic choices (L^q, π^q) for $q \in [1, \infty]$ and (L^0, π^W) .

Robustness in the Optimization of VaR

Let

$$q = \text{VaR}_p(X; \mathcal{G}_{\text{ns}}) = \inf \{ \text{VaR}_p(g(X)) : g \in \mathcal{G}_{\text{ns}} \},$$

$$q' = \text{VaR}_p(X; \mathcal{G}_{\text{bd}}) = \inf \{ \text{VaR}_p(g(X)) : g \in \mathcal{G}_{\text{bd}} \}.$$

Assumption 1

$q > 0$ and $\mathbb{P}(\gamma(X - q) \leq \text{VaR}_p(\gamma(X - q))) = p$.

Assumption 2

$q' > 0$ and $\mathbb{P}(\gamma \leq \text{VaR}_p(\gamma)) = p$.

- ▶ $q, q' > 0$ means the optimization does not result in zero risk
- ▶ Assumptions 1-2 are very weak

Robustness in the Optimization of VaR

Theorem 2

For $p \in (0, 1)$ and $X \in \mathcal{Z}$,

- (i) VaR_p is *not robust* relative to $(\mathcal{G}_{\text{cm}}, \mathcal{Z}, \pi_{\mathcal{Z}})$;
- (ii) under Assumption 1, VaR_p is *not robust* at X relative to $(\mathcal{G}_{\text{ns}}, \mathcal{Z}, \pi_{\mathcal{Z}})$;
- (iii) under Assumption 2, VaR_p is *not robust* at X relative to $(\mathcal{G}_{\text{bd}}, \mathcal{Z}, \pi_{\mathcal{Z}})$.

- ▶ Robustness of VaR in optimization is *very bad*

Robustness in the Optimization of ES

Assumption 3

$$\text{ess-sup} \gamma \leq \frac{1}{1-p}.$$

- ▶ Assumption 3 may be interpreted as a **no-arbitrage** condition for a market with ES participants

Assumption 4

Either γ is a constant, or γ is a continuous function and $\gamma(X)$ is continuously distributed.

- ▶ Assumption 4 is commonly satisfied

Robustness in the Optimization of ES

Theorem 3

For $p \in (0, 1)$ and $X \in \mathcal{Z}$,

- (i) under Assumption 3, ES_p is *robust* at X relative to $(\mathcal{G}_{\text{cm}}, \mathcal{Z}, \pi_{\mathcal{Z}})$;
- (ii) under Assumption 4, ES_p is *robust* at X relative to $(\mathcal{G}_{\text{ns}}, \mathcal{Z}, \pi_{\mathcal{Z}})$, where $(\mathcal{Z}, \pi_{\mathcal{Z}}) = (L^q, \pi^q)$ for $q \in [1, \infty]$;
- (iii) under Assumption 4, ES_p is *robust* at X relative to $(\mathcal{G}_{\text{bd}}, \mathcal{Z}, \pi_{\mathcal{Z}})$, where $(\mathcal{Z}, \pi_{\mathcal{Z}}) = (L^q, \pi^q)$ for $q \in [1, \infty]$.

- ▶ Robustness of ES in optimization is *quite good*

Robustness in Optimization for VaR and ES

On robustness in optimization:

VaR \Leftarrow **ES** (RVaR/ES not easy to compare)

Observations.

- ▶ The discontinuity in $Z \mapsto g^*(Z)$ comes from the fact that optimizing VaR is “too greedy”: always ignores tail risk, and hoping the probability of the tail risk is correctly modelled.
- ▶ None of the two values

$$\text{VaR}_p(g^*(X)) \quad \text{and} \quad \text{VaR}_p(g^*(Z))$$

is a rational measure of the “optimized” risk.

Robustness in Optimization for VaR and ES

Is risk positions of type g^* realistic?

*“Starting in 2006, the CDO group at UBS noticed that their risk-management systems treated AAA securities as essentially **riskless** even though they yielded a premium (the proverbial **free lunch**). So they decided to **hold onto them** rather than sell them.”*

- ▶ From Feb 06 to Sep 07, UBS increased investment in AAA-rated CDOs by **more than 10 times**; many large banks did the same.
 - Take a risk of **big loss** with **small probability**, $X_i = XI_{A_i}$
 - Treat it as free money - **profit**
 - **Model uncertainty?**

quoted from **Acharya-Cooley-Richardson-Walter'10**

Other Questions

Other questions

- ▶ other risk measures
- ▶ other optimization problems
- ▶ utility maximization problems
- ▶ risk measures as constraints instead of objectives
- ▶ robust preferences

Conclusion

Some conclusions on robustness

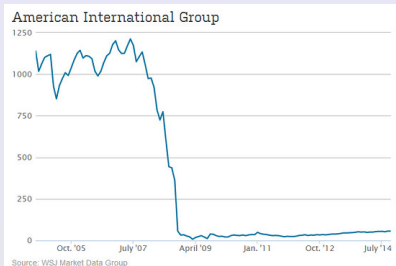
▶ Classic notion

- $ES \prec VaR \prec R VaR$
- However this robustness may not be desirable

▶ Novel perspectives

- $VaR \prec ES \prec R VaR$ in risk aggregation
- $VaR \prec\prec ES \prec R VaR$ in risk sharing
- $VaR \prec\prec ES$ in optimization
- The rationality of optimizing VaR under model uncertainty is **questionable**

AIG



CEO of AIG Financial Products, August 2007:

*"It is **hard** for us, without being flippant, to even see **a scenario within any kind of realm of reason** that would see us **losing one dollar** in any of those transactions."*

- ▶ AIGFP sold protection on super-senior tranches of CDOs
- ▶ \$180 billion bailout from the federal government in September 2008

Thank You



↑
VaR

↑
Real danger

Industry Perspectives

From the **International Association of Insurance Supervisors**:

- ▶ Document (version June 2015)
[Compiled Responses to ICS Consultation 17 Dec 2014 - 16 Feb 2015](#)

In summary

- ▶ Responses from insurance organizations and companies in the world.
- ▶ 49 responses are public
- ▶ 34 commented on Q42: VaR versus ES (TVaR)

Industry Perspectives

- ▶ 5 responses are supportive about ES:
 - Canadian Institute of Actuaries, CA
 - Liberty Mutual Insurance Group, US
 - National Association of Insurance Commissioners, US
 - Nematrian Limited, UK
 - Swiss Reinsurance Company, CH
- ▶ Some are indecisive; most favour VaR.

Regulator and firms may have **different views**

Discussion

Major reasons to favour VaR from the insurance industry (**IAIS report June 2015**)

- ▶ Implementation of ES is expensive (staff, software, capital)
- ▶ ES does not exist for certain heavy-tailed risks
- ▶ ES is more costly on distributional information, data and simulation
- ▶ ES has trouble with a change of currency