# Skorokhod embeddings, Martingale Optimal Transport and their applications

14th – 16th March 2016

University of Oxford

held at the Oxford-Man Institute, Eagle House

## Organised by:

Alexander Cox (University of Bath)

Jan Obłój (University of Oxford)

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**AGENDA**

## Monday 14th March – All talks held in AHL Lecture Theatre

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|---|---|
| **10:15-11:00** | *Coffee and registration* |
| **11:00-11:45** | Welcome and Opening statements |
| **11:45-12:45** | **David Hobson**<br><br>"On the value of being American" |
| **12:45-13:45** | *Lunch - conservatory* |
| **13:45-14:45** | **Yan Dolinsky**<br><br>"Super-replication in Extremely Incomplete Markets" |
| **14:45-15:45** | **Zhaoxu Hou**<br><br>"On robust pricing-hedging duality in continuous time" |
| **15:45-16:15** | *Coffee - conservatory* |
| **16:15-17:15** | **Gaoyue Guo**<br><br>"An optimal embedding problem by Brownian motion" |
| **17:15-18:15** | **Tongseok Lim**<br><br>"On the structure of optimal Skorohod embedding between radically symmetric marginal" |
| **18:15** | *Wine reception – common room* |

## Tuesday 15th March – All talks held in AHL Lecture Theatre

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|---|---|
| **9:15-10:15** | **Paul Gassiat**<br><br>"On Root's solution to the Skorohod embedding problem" |
| **10:15-10:45** | *Coffee - conservatory* |
| **10:45-11:45** | **Nizar Touzi**<br><br>"The Root solution to the multi-marginal embedding problem: an optimal stopping and the time-reversal approach" |

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|---|---|
| **11:45-12:45** | **Tiziano De Angelis**<br><br>"Root's barriers and optimal stopping boundaries" |
| **12:45-13:45** | *Lunch - conservatory* |
| **13:45-14:45** | **Sigrid Källblad**<br><br>"Model-independent bounds for Asian options: a dynamic programming approach" |
| **14:45-15:45** | **Martin Huesmmann**<br><br>"Bass vs. Root" |
| **15:45-16:15** | *Coffee - conservatory* |
| **16:15-17:15** | **David Prömel**<br><br>"On Bass' Approach to the Skorohod Embedding Problem" |
| **17:15-18:15** | **Julien Claisse**<br><br>"Skorokhod embedding and robust hedging with local time" |
| **19:00** | *Workshop Dinner at St Anne's College* |

## Wednesday 16th March – All talks held in AHL Lecture Theatre

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|---|---|
| **9:15-9:45** | **Mathias Beiglböck**<br><br>"Causal transport in discrete time and applications" |
| **9:45-10:15** | **Julio Backhoff**<br><br>"On the summetry between the Root and Rost embeddings and their connection to certain optimal stopping problems" |
| **10:15-10:45** | *Coffee - conservatory* |
| **10:45-11:45** | **Xiaolu Tan**<br><br>"On the monotonicity principle of optimal Skorokhod embedding problem " |
| **11:45-12:45** | Open Problem session and Discussion |
| **12:45-13:45** | *Lunch - conservatory* |

## Abstracts

**David Hobson** (University of Warwick) "*On the value of being American*"

The virtue of an American option is that it can be exercised at any time. This right is particularly valuable when there is model uncertainty. Yet almost all the extensive literature on American options assumes away model uncertainty. This paper quantifies the potential value of this flexibility by identifying the supremum on the price of an American option when no model is imposed on the data, but rather any model is required to be consistent with a family of European call prices. The bound is enforced by a hedging strategy involving these call options which is robust to model error.

(Joint work with Anthony Neuberger (Cass Business School))

**Yan Dolinsky** (Hebrew University) "*Super-Replication in Extremely Incomplete Markets*"

In this work we introduce the notion of extremely incomplete markets. We prove that for these markets the super-replication price coincide with the model free super-replication price. Namely, the knowledge of the model does not reduce the super-replication price. We provide two families of extremely incomplete models: stochastic volatility models and rough volatility models. Moreover, we give several computational examples. Our approach is purely probabilistic.

(Joint work with Ariel Neufeld)

**Zhaoxu Hou** (University of Oxford) "*On robust pricing-hedging duality in continuous time*"

We pursue robust approach to pricing and hedging in mathematical finance. We consider a continuous time setting in which some underlying assets and options, with continuous paths, are available for dynamic trading and a further set of European options, possibly with varying maturities, is available for static trading. Motivated by the notion of prediction set in Mykland, we include in our setup modelling beliefs by allowing to specify a set of paths to be considered, e.g. super-replication of a contingent claim is required only for paths falling in the given set. Our framework thus interpolates between model-independent and model-specific settings and allows to quantify the impact of making assumptions or gaining information. We obtain a general pricing-hedging duality result: the infimum over superhedging prices is equal to supremum over calibrated martingale measures. In presence of non-trivial beliefs, the equality is between limiting values of perturbed problems. In particular, our results include the martingale optimal transport duality of Dolinsky and Soner and extend it to multiple dimensions and multiple maturities.

(Joint work with Jan Obłój)

**Gaoyue Guo** (École Polytechnique, Paris) "*An optimal embedding problem by Brownian motion*"

Motivated by the computation of model-independent bounds of exotic derivatives in practice, we consider an optimization problem similar to the optimal Skorokhod embedding problem (SEP) , where the embedded Brownian motion needs only to reproduce a finite number of prices of Vanilla options. We show a stability result, i.e. when more and more Vanilla options are given, the optimization problem converges to an optimal SEP. In addition, by means of different metrics on the space of probability measures, a convergence rate analysis is provided under suitable conditions.

**Tongseok Lim** (University of British Columbia) "*On the structure of optimal Skorokhod embedding between radially symmetric marginal*"

We determine the fine structure of optimal stopping time for Skorokhod embedding problem when the initial and terminal laws are radially symmetric in arbitrary dimension. Here optimality means that the embeddings maximize / minimize the cost E|B_0 - B_T|^p, p>0.

**Paul Gassiat** (Université Paris Dauphine) "*On Root's solution to the Skorokhod embedding problem*"

Given a target probability measure $\mu$, the classical Skorokhod Embedding Problem consists in finding a stopping time $\tau$ such that the stopped Brownian motion $B_\tau$ has distribution $\mu$. In 1968, Root showed that there exists a subset of time-space such that its hitting time by Brownian motion gives a solution to this problem. Root's proof was nonconstructive, leaving open the question of how this barrier can be computed in practical cases. We will report on recent progress in this direction, applications to numerical simulations, as well as extensions to general Markov processes.

(Based on joint works with A. Mijatovic, H. Oberhauser and G. dos Reis)

**Nizar Touzi** (École Polytechnique, Paris) "*The Root solution to the multi–marginal embedding problem: an optimal stopping and time–reversal approach*"

We provide a complete characterisation of the Root solution to the Skorohod embedding problem (SEP) by means of an optimal stopping formulation. Our methods are purely probabilistic and the analysis relies on a tailored time–reversal argument. This approach allows to address the long–standing question of a multiple marginals extension of the Root solution of the SEP. Our main result provides a complete characterisation of the Root solution to the n–marginal SEP by means of a recursive sequence of optimal stopping problems. Moreover, we prove that this solution enjoys a similar optimality property to the one-marginal Root solution.

(Joint work with Alexander Cox and Jan Obłój)

**Tiziano De Angelis** (University of Leeds) "*Rost's barriers and optimal stopping boundaries*"

We provide a new probabilistic proof of the (known) connection between Rost's solution of the Skorokhod embedding problem and a suitable family of optimal stopping problems for Brownian motion with finite time-horizon.
We associate to a target law mu a family of optimal stopping problems parametrised by the length T > 0 of their (finite) time horizons. By using probabilistic methods we prove key regularity results for the value function V^T and for the boundary of the stopping set D_T in each one of such problems. We then perform a time reversal and a suitable pasting of the regions {D_T , T > 0}, and prove that the resulting set is the Rost barrier which embeds the law mu.
Other existing proofs of the connection between Rost's solution of the Skorokhod embedding problem and optimal stopping rely upon PDE theory and/or viscosity solutions of variational problems. Here we use a different approach entirely based on stochastic calculus and probability with specific emphasis on the role of the optimal stopping boundary.
To conclude we also discuss a method for the numerical evaluation of Rost's barriers based on non-linear equations of Volterra type.

**Sigrid Källblad** (École Polytechnique, Paris) "*Model-independent bounds for Asian options: a dynamic programming approach*"

We consider the problem of finding model-independent bounds on the price of an Asian option, when the call prices at the maturity date of the option are known. Our method differ from most approaches to model-independent pricing in that we consider the problem as a dynamic programming problem, where the controlled process is the conditional distribution of the asset at the maturity date. By formulating the problem in this manner, we are able to determine the model-independent price through a PDE formulation. Notably, this approach does not require specific constraints on the payoff function (e.g. convexity), and would appear to be generalisable to many related problems.

(Joint work with A.M.G. Cox.)

**Martin Huesmann** (University of Bonn) "*Bass vs. Root*"

The Root embedding is a valuable tool for many applications, e.g. numerical simulations or the recent counterexample for the Cantelli conjecture. However, as soon as one needs quantitative knowledge on the solution to the SEP the Root embedding is no option and one often takes the Bass solution instead.
We will show that - viewed as a measure valued martingale - the Bass embedding can be seen as a Root solution in a slightly deformed geometry illustrating the close connection between these two solutions. Moreover, we will show that the Bass solution enjoys a simple and natural optimality property.

(Joint work with M. Beiglböck, A.Cox, and S.Kallblad)

**David Prömel** (ETH Zürich) "*On Bass' Approach to the Skorokhod Embedding Problem*"

In 1983 Richard F. Bass developed a method to solve the Skorokhod embedding problem (SEP) for Brownian motion based on martingale representation and time-change techniques. In the talk we deal with the SEP for more general stochastic processes as certain Gaussian processes with non-linear drift or Levy processes. While the time-change techniques naturally extend to these processes, the martingale representation completely breaks down. In order to replace it, our approaches relies either on solving a strongly coupled system of forward backward stochastic differential equations or on uniqueness results for Fokker-Planck equations.

**Julien Claisse** (École Polytechnique, Paris) "*Skorokhod embedding and robust hedging with local time*"

In this talk, we are interested in robust hedging of options on local time when one or more marginals of the underlying price process are known. By using the stochastic control approach initiated by Galichon, Henry-Labordère and Touzi, we identify the optimal hedging strategies and the corresponding prices in the one-marginal case. Then we extend the analysis to the two-marginal case, where we provide candidates for the optimal hedging strategies. To this end, we construct a new solution to the two-marginal SEP as a generalization of the Vallois embedding. Finally, a special multi-marginal case is studied, where the stopping times given by Vallois are well-ordered. In the full marginal setting, we construct a remarkable Markov martingale and compute its generator explicitly.

(Based on a joint work with Gaoyue Guo and Pierre Henry-Labordère)

**Mathias Beiglböck** (TU Vienna) "*Causal transport in discrete time and applications*"

Loosely speaking, causal transport plans are a relaxation of adapted processes in the same sense as Kantorovich transport plans extend Monge-type transport maps. The corresponding causal version of the transport problem has recently been introduced by Lassalle. Working in a discrete time setup, we establish a dynamic programming principle (DPP) that links the causal transport problem to the transport problem for general costs recently considered by Gozlan et al. Based on this DPP, we give conditions under which the Knothe-Rosenblatt coupling can be viewed as a causal analogue to the Brenier map. As an application we establish functional inequalities for the random walk. These estimates provide Talagrand-type inequalities for the nested distance of stochastic processes and are asymptotically equalities as the number of steps tends to $\infty$. (based on joint work of J. Backhoff, Y. Lin, A. Zalashko)

**Julio Backhoff** (TU Vienna) "*On the symmetry between the Root and Rost embeddings and their connection to certain optimal stopping problems*"

Recent works by Cox & Wang and Cox, Obloj & Touzi have highlighted the connection between the Rost/Root solutions to the Skorokhod Embedding Problem and certain optimal stopping problems. In this talk we present a simple probabilistic argument for this connection (as opposed to the original analytical ones) which furthermore implies an interesting relationship/symmetry between the Rost and Root solutions. This is joint work with M. Beiglböck, A. Cox and M. Huesmann.

**Xiaolu Tan** (Université Paris-Dauphine) "*On the monotonicity principle of optimal Skorokhod embedding problem*"

We provide an alternative proof of the monotonicity principle for the optimal Skorokhod embedding problem established by Beiglbock, Cox and Huesmann. This principle presents a geometric characterization that reflects the desired optimality properties of Skorokhod embeddings. Our proof is based on the adaptation of the Monge-Kantorovich duality in our context together with a delicate application of the optional cross-section theorem.

| Participant List | |
|---|---|
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Please see  below the location of St Anne's College, where you will be having dinner on Tuesday evening. There will be canapés and pre-dinner drinks in Seminar Room 9 from 7pm and the reception dinner will be served in Foyer B, Ruth Deech Building from 7.30pm.

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