SEEMOD Oxford July 5-6 2016

Talk titles and abstracts

Vahagn Aslanyan (Oxford)

Ax-Schanuel type theorems and geometry of strongly minimal sets in DCF_0

Let $(K; +, \cdot, ', 0, 1)$ be a differentially closed field. We explore the connection between Ax-Schanuel type theorems (predimension inequalities) for a differential equation $E(x, y)$ and the geometry of the set $U = \{y : E(t, y) \wedge y \neq 0\}$ where t is an element with $t' = 1$. We show that certain types of predimension inequalities will imply strong minimality and geometric triviality of U. Moreover, the induced structure on Cartesian powers of U is given by special subvarieties. If E has some special form then all fibres $U_s = \{y : E(s, y) \wedge y \neq 0\}$ (with s non-constant) have the same properties. In particular, since an Ax-Schanuel theorem for the j-function is known (due to Pila and Tsimerman), our results will give another proof for a theorem of Freitag and Scanlon stating that the differential equation of j defines a strongly minimal set with trivial geometry (which is not $\aleph_0$-categorical though).

Martin Bays (Muenster)

Pseudofinite-dimensional Schrödinger representations

Adapting recent work of Zilber to the setting of the continuous logic of uniform spaces, we obtain the infinitesimal Schrödinger representation of the (3-dimensional) Heisenberg algebra, acting on the space of tempered distributions, as an ultralimit of finite-dimensional "cyclic" representations of subgroups of the Heisenberg group. Moreover, we see that (parts of) the Weil representation (including the Fourier transform) can naturally be obtained this way.

This is joint work with Bradd Hart.

Elisabeth Bouscaren (Paris Sud)

Function Field Mordell-Lang and the model theory of finite rank groups

We will survey some joint work with Franck Benoist and Anand Pillay around the model theoretic proofs of the function field Mordell-Lang conjecture. In this talk, we will focus on the use of the model-theory of finite rank groups in these proofs, in Hrushovski's original proof as well as alternate recent proofs, in particular the use of the "Socle Theorem".

Thomas Coleman (UEA)

Permutation monoids and MB-homogeneous structures

Group embeddable monoids, by their nature, can be represented as a submonoid of permutations contained in some symmetric group $\mathrm{Sym}(X)$. As every finite group embeddable monoid is a group, we consider infinite submonoids B of the infinite symmetric group $\mathrm{Sym}(\mathbb{N})$ to avoid triviality; such a B is an *infinite permutation monoid.* Natural examples of these occur via the *bimorphism monoid* $\mathrm{Bi}(\mathcal{A})$ of a structure; that is, the collection of bijective endomorphisms of $\mathcal{A}$. It follows that every automorphism of $\mathcal{A}$ is a bimorphism of $\mathcal{A}$ but in general the converse is not true; and so we have that $\mathrm{Aut}(\mathcal{A}) \subseteq \mathrm{Bi}(\mathcal{A}) \subseteq \mathrm{Sym}(A)$, where A is the domain of $\mathcal{A}$.

Recent work in this field by Cameron and Nesetril [1] and Lockett and Truss [2] generalizes the idea of homogeneity to several notions of *homomorphism-homogeneity.* One such example is the property of *MB-homogeneity*: a structure $\mathcal{A}$ is MB-homogeneous if every monomorphism between finite substructures of $\mathcal{A}$ extends to a bimorphism of $\mathcal{A}$. Lockett and Truss completely classified homomorphism-homogeneous countable posets in [2].

In this talk, connections between permutation monoids and bimorphism monoids of structures are explored in order to develop a notion of oligomorphicity for infinite permutation monoids. In addition to this, a version of Fraissé's theorem is shown for MB-homogeneous structures, extending work of [1]. Finally, we construct $2^{\aleph_0}$ non-isomorphic examples of MB-homogeneous graphs and take steps towards a classification result. This is joint work with David Evans and Robert Gray during the course of my PhD studies.

[1] P. J. Cameron, J. Nesetril, *Homomorphism-homogeneous relational structures*, **Combinatorics, Probability and Computing**, vol. 15 (2006), no. (1-2), pp. 91-103.

[2] D. C Lockett, J. K. Truss, *Some more notions of homomorphism-homogeneity*, **Discrete Mathematics**, vol.336 (2014), pp.69-79.

Jamshid Derakhshan (Oxford)

Logic, Zeta Functions, and Representations

I will present a theorem on meromorphic continuation of Euler products of definable integrals in non-Archimedean local fields. I will then state three applications: to counting conjugacy classes in congruence quotients of algebraic groups over the integers (answering a question of Onn), to counting iso-twist classes of representations of finitely generated nilpotent groups (using and continuing work of Hrushovski-Martin-Rideau), and a proof of Manin's conjecture on counting rational points of bounded height in orbits of algebraic groups over the rationals (answeringquestions of Oh and Gorodnik).

The proof uses meromorphic continuation of Artin L-functions associated to representations of Galois groups of number fields. In fact, it gives a recipe to attach representations of Galois groups of certain number fields to such a definable Euler product such that the analytic properties of the Euler product reduces to that of the Artin L-functions of the representations. The results also apply to Euler products of motivic integrals and to definable integrals over the adeles.

Philip Dittmann (Oxford)

Valuation Theory of non-standard number fields

We investigate all valuations on elementary extensions $\mathbb{Q}*$ of $\mathbb{Q}$ and their finite extension fields, and work towards a classification of their residue fields. As part of this we can show that the arithmetically interesting valuations are those that we expect classically. This is part of an ongoing joint project with Sylvy Anscombe and Arno Fehm.

Alfonso Ruiz Guido (Oxford)

Zariski Geometries and Algebraic Spaces

In one direction I will talk about representability theorems of certain algebraic spaces using the strong version of trichotomy theorem for Zariski geometries. In the other, I will talk about a conjecture on realizability of certain Zariski Geometries as Algebraic Spaces.

Lubna Shaheen (Oxford)

On $\mathbb{F}_1$ geometry and model theory

(Joint with Boris Zilber and John Alexander Cruz Morales.) The geometric motivation of the object $\mathbb{F}_1$, "the field with one element" came from the work of Jack Tits in 1956, where he explained how one can define the Chevalley group of characteristic one to obtain some interesting geometries such that the symmetric groups happens to be the Weyl group of the corresponding Lie groups.

In the last two decades there have been a lot of development, with motivation coming from Arakelov theory and some ideas relating the notion to the Riemann zeta function.

We interpret fields of characteristic 1 and algebras over fields of characteristic 1 as **multiplicative monoids with a shadow addition** by which we mean structures of the form $(R; \cdot, 0, 1, D_1^{m_1,p_1}, D_2^{m_2,p_2}, \ldots)$ where $(R; \cdot, 0, 1)$ is a multiplicative commutative monoid with 0 and $D_k^{m_k,p_k}$ is an m_k-ary relation, p_k a prime number or 0, which we will write in a more suggestive form as a divisor equality

$$n_1x_1 + \ldots + n_mx_m \equiv 0 \mod p$$

The initial object in the category of $\mathbb{F}_1$-algebras is $\mathbb{F}_1 = \{0, 1\}$ the field with one element. We define the cyclotomic extensions $\mathbb{F}_{1^n}$ of $\mathbb{F}_1$ as the $\mathbb{F}_1$-algebra where $R = 0 \cup \mu_n$. We set $\mathbb{F}_1^{alg} = \bigcup_{n\in\mathbb{N}} \mathbb{F}_{1^n}$ (with the universe $\mu_\infty \cup \{0\}$) and formulate a conjecture that such an object is ω-stable.