Matlab Billiards GUI Revision

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1 Introduction

This is an additional guide to the Matlab GUI for billiard simulations, as written by Steven Lansel [1]. Additional functionality was added to the GUI from June – September 2006, and the following is documentation on the changes made. Please note that these changes were made on a Windows XP Pro x86 using Matlab R2006a, and that there are known compatibility issues with the code base on other versions of Matlab (Matlab R2006a for Mac OS 10.4 for instance).

2 Revised Tables

The tables in this section were originally in the code for the GUI, but have been revised to include additional functionality as follows.

2.1 Stadium

Originally, the stadium as present in the GUI had semi-circles at the end, and it could be either a full, half, or quarter stadium as per the user's choice. This was changed to allow the ends to be either circular or elliptical, thus allowing a more general class of stadia. The ability to use a full, half, or quarter stadium remains just the same. In the case of an elliptical stadium, the user is prompted for two radii (the semi-minor and semi-major axes) instead of one.

3 New Tables

The tables in this section were added during the Summer of 2006, and represent new code, unless otherwise noted.

3.1 Double Mushroom

This table is very much like the familiar mushroom, except there are caps at both ends of the rectangular stem. These caps may be either circular (in which case the user is prompted for a radius for each of the top and bottom semi-circles) or elliptical (in which case the user is prompted for two radii (the semi-minor and semi-major axes) for each cap).

Additionally, the top and bottom may have unique width ratios. The width ratio is the quotient of the length of the right side of the neutral part of the cap to the length of the left side of the neutral part of the cap. Note that a width ratio of infinity means the cap is all the way on the left side of the stem, and a width ratio of zero means the cap is all the way on the right side of the stem.

3.2 Non-Concentric Circles

This table is fairly simple, and reminiscent of the Sinai billiard. There are, as the name implies, two non-concentric circles - one circle of a larger radius on the outside of a smaller circle, which do not share a common center. The user inputs the radius of each circle, and then the distance between the two centers. The inner circle is displaced along the vertical axis through the center of the larger circle.

3.3 Rounded Mushroom

This table is also very much like the familiar mushroom. Instead, however, at the corners where the stem meets the cap, there are quarter circles, to essentially round the corners out. This arises from a consideration of the mushroom with billiards of a non-zero finite size, because that case is related to the case of a slightly altered mushroom with a point particle instead (and one of the alteratios is the rounding of these corners). The inputs are the same as the mushroom, except the user also inputs a radius for the two quarter circles.

Note that the code for the rounded mushroom was distributed with the GUI in its earlier version, though it was not integrated into the GUI to function with it - that is, it was usable, but it was something of a hidden feature, as it was not accessible from the GUI previously.

3.4 Kaplan Billiard

This table is also reminiscient of the Sinai billiard, in that it can be similar to considering half of a Sinai billiard. This table consists of a square, with a semi-circle along part of the right side instead of the neutral boundary. Typically, this semi-circle is dispersive (it curves into the square), but the option has been added to allow for a focusing semi-circle (it curves out of the square). The user inputs include side length of the square, radius of the semi-circle, and how high along the side the semi-circle should be shifted (measured from the bottom of the side to the point where the semi-circle and neutral boundary meet).

3.5 Lemon Billiard

The lemon billiard is sometimes referred to as an ellipse hyperbola billiard [2], and is a billiard whose structure depends only on a number δ. δ may range from 0 to ∞, though it may not be 2. For $\delta > 2$, the limit of the billiard as $\delta \to \infty$ is a circular billiard. In the case of $\delta = 1$, the billiard is a square.

The lemon billiard is formed by the curves in the x-y plane given by $y = \pm\frac{\sqrt{1+\delta(\delta-2)(1-x^2)}-1}{\delta-2}$. The user input for the lemon billiard is simply δ.

References

[1] Steven Lansel and Mason A. Porter. *A Graphical User Interface to Simulate Classical Billiard Systems*. nlin.CD/0405033 (2004).

[2] V. Lopać, I. Mrkonjić, and D. Radić. Chaotic Behavior in Lemon-shaped Billiard with Elliptical and Hyperbolic Boundary Arcs *Physical Review E* Vol. 64 (2001).