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Elements of the group K_1 associated to abelian schemes

(joint work with Vincent Maillot)

In this short description of results, we shall use the general terminology of higher-dimensional Arakelov theory (cf. [7]).

Let B be a regular arithmetic variety over $\mathbb{Z}$ and let $\pi : A \to B$ be an abelian scheme over B. We choose a Kähler fibration structure ω on $A(\mathbb{C})$, such that the metrics induced on the fibers are translation invariant. We choose a line bundle L on A, which is rigidified along the 0-section and such that there exists $k \in \mathbb{N}^*$, such that there exists an isomorphism $L^{\otimes k} \simeq \mathcal{O}_B$ respecting the rigidification. We equip this line bundle with the unique hermitian metric $h_{L(\mathbb{C})}$, whose curvature form vanishes and such that the rigidification is an isometry. We shall write $T(A(\mathbb{C}), \omega, h_{L(\mathbb{C})}) \in \widetilde{A}(B)$ for the higher analytic torsion form of $L(\mathbb{C})$ over $B(\mathbb{C})$ (see [1]). We shall denote by reg the regulator map

$$\mathrm{reg} : K_1(B) \to \oplus_{p\geqslant 0} H^{2p-1}_{D,\mathrm{an}}(B(\mathbb{C}), \mathbb{R}(p)).$$

Here $H^{2p-1}_{D,\mathrm{an}}(B(\mathbb{C}), \mathbb{R}(p))$ is the p-th analytic Deligne cohomology of $B(\mathbb{C})$. There is a natural inclusion of groups $\oplus_{p\geqslant 0} H^{2p-1}_{D,\mathrm{an}}(B(\mathbb{C}), \mathbb{R}(p)) \subseteq \widetilde{A}(B)$ (see [2]).

The object of the talk was to present the following

Proposition 0.1. *Suppose that $L|_{A_b} \not\simeq \mathcal{O}_{A_b}$ for all fibers A_b of π. Then*

(1) *The element $T(A(\mathbb{C}), \omega, h_{L(\mathbb{C})})$ does not depend on the choice of ω. We shall thus henceforth write $T(A(\mathbb{C}), L)$ for $T(A(\mathbb{C}), \omega, h_{L(\mathbb{C})})$.*

(2) *We have*

$$T(A, L) \in \mathrm{image}(\mathrm{reg} \otimes \mathbb{Q}).$$

(3) *Let $n \in \mathbb{N}$ be such that $(n, k) = 1$. Suppose that the dual abelian scheme $A^\vee \to B$ has n^{2g} disjoint n-torsion sections. Let $M_1, \dots, M_{n^{2g}}$ be the corresponding rigidified line bundles on A. Then*

$$T(A, L^{\otimes n}) = \sum_{j=1}^{n^{2g}} T(A, L \otimes M_j).$$

In the case where $\dim(A/B)$ (elliptic fibrations), it is shown in [6] that the function part of $T(A, L)$ is a certain elliptic unit. The No. 2 in the Proposition 0.1 contains in particular the reciprocity law for this elliptic unit. The No. 3 is a generalisation of part of the distributivity law for (certain) elliptic units.

Sketch of proof of Proposition 0.1. No.1 is a consequence of the anomaly formula [1, Th. 3.10]. No. 2 is a direct consequence of the arithmetic Riemann-Roch theorem in all degrees proven in [3]. No. 3 results from a computation with the Fourier-Mukai transform and from the main result of [5].■

During the talk, G. Kings made the very interesting suggestion that the elements $T(A, L)$ coincide with the Hodge realisations of certain elements of $K_1(B) \otimes \mathbb{Q}$ constructed using the motivic polylogarithmic sheaf on abelian scheme (see [4]).

Since these elements are constructed using an (apparently) completely different method, the proof of such an identity would be of great interest.

References

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