A NOTE ON CYCLE LENGTHS IN GRAPHS

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Abstract. We prove that for every $c > 0$ there exists a constant $K = K(c)$ such that every graph G with n vertices and minimum degree at least cn contains a cycle of length t for every even t in the interval $[4, ec(G)-K]$ and every odd t in the interval $[K, oc(G)-K]$, where $ec(G)$ and $oc(G)$ denote the length of the longest even cycle in G and the longest odd cycle in G respectively. We also give a rough estimate of the magnitude of K.

1. Introduction

In this note we will study the set of distinct cycle lengths in graphs. For a graph G, we define the *cycle spectrum* $CS(G)$ of G as the sequence $\ell_1 < \ldots < \ell_r$ of lengths of cycles in G. The study of cycles in graphs has long been fundamental, and many questions about properties of graphs that guarantee some particular range of cycle lengths have been considered. For example, a graph G with n vertices is said to be *pancyclic* if $CS(G) = [3, n]$. It was proved by Bondy [5] that if G is a hamiltonian graph of order n with $|E(G)| \geq \frac{n^2}{4}$, then either G is pancyclic or n is even and $G = K_{n/2,n/2}$.

Brandt [7], [8] introduced the idea of weakly pancyclic graphs, that is, graphs with cycles of all lengths from the girth to the circumference. Here the *girth* $g(G)$ is the length of the shortest cycle in G, and the *circumference* $c(G)$ is the length of the longest cycle. Brandt showed that if $|E(G)| > \lfloor (n-1)^2/4 + 1 \rfloor$ then G is weakly pancyclic. Bollobás and Thomason [4] proved that if G is a nonbipartite graph of order n and size at least $\lfloor n^2/4 \rfloor - n + 59$, then G contains a cycle of length ℓ for $4 \leq \ell \leq c(G)$. Degree conditions for weakly pancyclic graphs were considered by Brandt, Faudree and Goddard [9], who showed in particular that if G is a non-bipartite 2-connected graph with minimum degree $\delta(G) \geq n/4 + 250$ then G is weakly pancyclic unless the shortest odd cycle in G has length 7.

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In graphs with fewer edges, it is still the case that a reasonably large density can force a large range of cycle lengths. Bondy and Simonovits [6] showed the very general result that if $|E(G)| > 100kn^{1+1/k}$ then G contains the cycle C_{2m} for every $m \in [k, kn^{1/k}]$, answering a conjecture of Erdős ([11], [13]). (For a recent improvement of this result see Verstraëte [19].) Others considered the question of how many different cycle lengths were present in G. Proving a conjecture of Erdős and Hajnal [12], Gyárfás, Komlós and Szemerédi showed that for suitable positive constants a, b, if the minimum degree $\delta(G) \geq b$ then

$$\sum_{i \in CS(G)} \frac{1}{i} \geq a \log\ \delta(G).$$

This implies that a large number of distinct cycle lengths exist in G. As part of the difficult and intricate proof of this result, they showed that most even cycles were present over a certain interval dependent on the minimum degree (most meaning with the exception of multiples of $2t$ for some integer $t \geq 2$). See Bollobás ([2] and [3]) for other results in this general area.

Faudree suggested the question of measuring the maximum gap in the cycle spectrum for graphs under various edge density conditions. For a graph G and a positive integer s, we say that the cycle spectrum $CS(G)$ is *s-dense in the interval* $[k, m]$ if for every $\ell \in [k, m]$, at least one of $\ell, \ell - 1, \ldots, \ell - s + 1$ is in $CS(G)$. For example, the theorem of Bondy and Simonovits [6] implies that if $|E(G)| > 100kn^{1+1/k}$ then $CS(G)$ is 2-dense in the interval $[2k, 2kn^{1/k}]$. This approach was considered in [15] for graphs with minimum degree $\frac{n-k}{2}$ where k is a constant and also for graphs that are hamiltonian and have at least one pair of adjacent vertices with high degree sum.

In this note we also consider a minimum degree condition. Note that Bondy's theorem [5] (together with Ore's classical theorem [18] that a graph with minimum degree at least $n/2$ is hamiltonian) implies that the cycle spectrum of any graph G with $\delta(G) \geq n/2$ is 2-dense in the interval $[4, n]$. Note that in this case n is the circumference $c(G)$ of G. Results of Fan [14] and Verstraëte [19] (improving on [17]) show that if G is a graph with minimum degree δ then the cycle spectrum of G contains $c\delta$ consecutive even integers, for a positive constant c. Our goal is to prove the following theorem, which implies that for each $c > 0$ there exists K such that all sufficiently large graphs G with $\delta(G) \geq cn$, the cycle spectrum $CS(G)$ is 2-dense in the interval $[4, c(G) - K]$. Below $oc(G)$ and $ec(G)$ denote, respectively, the length of the longest odd cycle and the length of the longest even cycle in G.

Theorem 1. *For every real number $c > 0$ there exists a constant $K = K(c)$ depending only on c such that the following holds. Let G be a graph with $n \geq 45K/c^4$ vertices and minimum degree at least cn. Then G contains a cycle of length t for every even integer $t \in [4, ec(G) - K]$ and every odd integer $t \in [K, oc(G) - K]$.*

The simpler statement that G contains cycles of all even lengths up to $c(G) - K$ is not true, as the following example shows. Let $m \geq 3$ be an odd integer and suppose $n = 2ms$ where $s \geq 2$ is an integer. We form a graph H from a disjoint union of m copies $K_1[X_1, Y_1], \ldots, K_m[X_m, Y_m]$ of the complete bipartite graph $K_{s,s}$ by adding m vertex-disjoint edges $e_1, \ldots, e_m$ as follows. For $1 \leq i \leq m - 1$ we let e_i join a vertex y_i of Y_i to a vertex of X_{i+1}, and we let e_m join a vertex of Y_m to a vertex of Y_1 different from y_1. Then $\delta(H) = s = n/2m$ and $c(G) = n - 1$ but the longest even cycle has length only $2s = n/m$. This example also shows that, for $c = 1/2m$, Theorem 1 is best possible up to the error term K, since H contains cycles of all even lengths in $[4, n/m]$ and all odd lengths in $[2m + 2, n - 1]$ and there are no other cycle lengths. For general c there is also the simpler example of the complete bipartite graph with $\lceil cn \rceil$ vertices in one class and $n - \lceil cn \rceil$ vertices in the other.

In our approach to proving Theorem 1, we show that $K(c) = O(c^{-5})$. However, we emphasize that this is only a very rough estimate and we do not undertake to find the smallest possible value of K here. We remark that the proof can be made somewhat simpler if we do not attempt to bound K by a reasonable function of c.

All graphs considered here are finite simple graphs. For terms not defined here see [10].

2. Proof of Theorem 1

We begin by collecting a number of useful facts into a lemma. Parts (1) and (3) are immediate. Results essentially the same as (2) appear in the work of many authors, see eg. Beck [1]. For completeness we give its short proof in Section 3.

Lemma 2. *Let $B[U, W]$ be a bipartite graph with vertex classes U and W. Let $\bar{d}(U)$ and $\bar{d}(W)$ denote the average degree of vertices in U and W respectively. Then*

(1) *W has a subset W' with $|W'| \geq |W|(\bar{d}(W)/(2|U| - \bar{d}(W)))$ such that every vertex w in W' has $d_U(w) \geq \bar{d}(W)/2$.*
(2) *There exist nonempty subsets $U'' \subset U$ and $W'' \subset W$ such that the subgraph $B[U'', W'']$ of B induced by $U'' \cup W''$ has $d_{W''}(u) \geq \bar{d}(U)/2$ for all $u \in U''$ and $d_{U''}(w) \geq \bar{d}(W)/2$ for all $w \in W''$.*

(3) *$B[U, W]$ has a path of length at least $2\delta(B)$-1 starting from any vertex x of B, where $\delta(B)$ denotes the minimum degree of B.*

We shall also need the following elementary technical lemma. We include its very standard proof in Section 3 for completeness. Here $\Gamma(v)$ denotes the neighborhood of the vertex v.

Lemma 3. *Let $c > 0$ be given, and let G be a bipartite graph with vertex classes $V = \{v_1, \ldots, v_r\}$ and W, where $|W| = n$. Suppose $r = \lceil 2/c \rceil$ and $d(v_i) \geq cn$ for each i. Then for some $i \neq j$ we have $|\Gamma(v_i) \cap \Gamma(v_j)| \geq c^2 n/2$.*

Our first step in the proof of Theorem 1 will be to show that in a given graph G, there exists a subgraph H consisting of a bipartite subgraph H_0 of large minimum degree, whose number of vertices is a large constant, together with a long path that joins one vertex of H_0 to another, and is otherwise disjoint from H_0. To prove the theorem, we will show that a cycle of a given length t can be found by "shortening" the path until it is only slightly shorter than t, and then adding a path in H_0 of precisely the right length to form the cycle.

Lemma 4. *Let $c > 0$ be given, and let G be a graph with n vertices and minimum degree $\delta(G) \geq cn$. Let $K_0 \geq \lceil 50000c^{-4} \rceil$, and suppose $n \geq 5K_0c^{-1}$. Let C_0 be a cycle in G of length at least $5K_0$. Then G contains a subgraph H consisting of*

(1) *a bipartite graph H_0 with vertex classes $X \cup \{x_0\}$ and Y where $|X| \leq K_0$, $|Y| \leq K_0$, $\delta(H_0) = k \geq c^3K_0/4096$, and x_0 is adjacent to every vertex of Y,*
(2) *a path P of length at least $|C_0| - 4K_0$ and of the same parity as $|C_0|$ that joins x_0 to a vertex x_1 of X, and is otherwise disjoint from H_0.*

Proof of Lemma 4. Let G be a graph as described and let C_0 be a cycle in G with length at least $5K_0$. For convenience we fix an orientation of C_0. Let S be an interval of C_0 of order $2K_0$, in other words S is a set of $2K_0$ consecutive vertices on C_0. Let $S_0 \subseteq S$ be a subset of set K_0 obtained by taking alternate vertices in S (so that all pairs of vertices in S_0 are an even distance apart in S). Consider the bipartite subgraph $G_0 = G[S_0, V(G) \setminus S]$. We note that $e(S_0, V(G) \setminus S) \geq |S_0|(cn - |S|) \geq cnK_0/2$, and so $\bar{d}_{G_0}(V(G) \setminus S) \geq cK_0/2$. Thus applying Lemma 2(1) we obtain a subset W_1 of $V(G) \setminus S$ such that $|W_1| \geq \frac{c}{(4-c)}(n - 2K_0)$, in which every vertex of W_1 has degree at least $cK_0/4$ into S.

Our aim now is to identify a certain special subset W_2 of W_1. If W_1 contains at least $cK_0/2(4-c)$ vertices that are not on C_0, we let W_2

be a subset of $W_1 \setminus V(C_0)$ of size $\lceil cK_0/2(4-c)\rceil$. We refer to this case as Case A. Otherwise, $|W_1 \cap V(C_0)| \geq c(n-5K_0/2)/(4-c)$, so since $|V(C_0)\setminus S| \leq n-2K_0$ we can cover $C_0 \setminus S$ by $\lceil (n-2K_0)/2K_0\rceil \leq n/2K_0$ intervals of order $2K_0$. Thus some interval I in $C_0 \setminus S$ of order $2K_0$ contains at least $2cK_0(n-5K_0/2)/(4-c)n > cK_0/(4-c)$ vertices of W_1. In this case, which we call Case B, we let W_2 be a subset of $I \cap W_1$ of size $\lceil cK_0/2(4-c)\rceil$ such that all vertices of W_2 are an even distance apart in I. In either case each vertex of W_2 still has degree at least $cK_0/4$ into S_0.

Now we apply Lemma 2(2) to the bipartite subgraph $G_2 = G[S_0, W_2]$. The result is an induced bipartite subgraph $B[S_3, W_3]$, where each $w \in W_3$ has $d_{S_3}(w) \geq \bar{d}_{G_2}(W_2)/2 \geq cK_0/8$, and each $s \in S_3$ has $d_{W_3}(s) \geq \bar{d}_{G_2}(S_0)/2 \geq (|W_2|/|S_0|)\bar{d}_{G_2}(W_2)/2 \geq c_2K_0$, where $c_2 = c^2/16(4-c)$. Then $cK_0/8 \leq |S_3| \leq K_0$ and $c_2K_0 \leq |W_3| \leq \lceil cK_0/2(4-c)\rceil$.

First we consider Case A. We choose x_0 to be the first vertex of S_3 on the interval S (in our fixed orientation). Let M be a set of size $\lceil c_2K_0\rceil$ contained in the neighbourhood of x_0 in W_3, and let G^* be the subgraph of $B[S_3, W_3]$ induced by $M \cup (S_3 \setminus \{x_0\})$. Then each vertex of M has degree at least $cK_0/8 - 1$ into $S_3 \setminus \{x_0\}$, so G^* has at least $c_2K_0(cK_0/8-1)$ edges. Hence the average degree on both sides of G^* is at least $c_2(cK_0/8-1)$. Therefore, by Lemma 2(2) there exists a subgraph $H_0[X, Y]$ of G^* with $X \subseteq S_3 \setminus \{x_0\}$ and $Y \subseteq M$ with minimum degree at least $c_2(cK_0/8-1)/2 > c^3K_0/4096$. Then $H_0[X, Y]$ satisfies (1) of Lemma 4. For (2), we take x_1 to be the vertex of X that is farthest from x_0 on the interval S, and let the path P be the segment of C_0 that joins x_0 to x_1 and is disjoint from the rest of X (note that x_0 and x_1 are an even distance apart in S and so P has the same parity as C_0). Then P has length at least $|C_0| - 2K_0$. This proves Lemma 4 in Case A.

Now we turn to Case B. Let $W_3 = \{w_1, \ldots, w_p\}$ in the order in which they appear on the oriented cycle C_0 (recall they all fall into the interval I and are all even distances apart in I). Let l be the smallest index such that there exists $j < l$ where w_j and w_l have a common neighbourhood N of size $\lceil c^2K_0/128\rceil$ in S_3. By Lemma 3 applied to $B[S_3, W_3]$, we know that $l \leq \lceil 16/c\rceil$. Let s_1 and s_2 be elements of N that are farthest apart in the interval S, and such that the oriented path $C_0(w_l, s_2)$ in C_0 from w_l to s_2 is disjoint from the oriented path $C_0(s_1, w_j)$.

Consider the graph $G^* = G[N \setminus \{s_1, s_2\}, W_3 \setminus \{w_1, \ldots, w_j, w_l\}]$. Note that each vertex in $N \setminus \{s_1, s_2\}$ has degree at least $c_2K_0 - 16c^{-1} - 2$ in G^*. Further, we know $|N \setminus \{s_1, s_2\}| \geq c^2K_0/128 - 2$. Therefore the number of edges in G^* is at least

$$(\frac{c^2K_0}{128}-2)(c_2K_0-\frac{16}{c}-2)\geq\frac{c^2c_2K_0^2}{128}-\frac{cK_0}{8}-\frac{c^2K_0}{64}-2c_2K_0.$$

Thus, the average degree on both sides of G^* is at least this number divided by $\max\{|N\setminus\{s_1,s_2\}|,|W_3|\}<cK_0/6$, which is greater than

$$\frac{3cc_2K_0}{64}-\frac{3}{4}-\frac{c}{16}-\frac{12c_2}{c}>\frac{cc_2K_0}{32}>\frac{c^3K_0}{2048}.$$

Here the first inequality follows since $K_0\geq 50000c^{-4}$, and the second from the definition of c_2.

Applying Lemma 2(2) to G^* we obtain a graph $H_0[X,Y]$ with $X\subset W_3\setminus\{w_1,\ldots,w_j,w_l\}$ and $Y\subset N\setminus\{s_1,s_2\}$ with minimum degree at least

$$k\geq\frac{c^3K_0}{4096}.$$

Then, setting $x_0=w_j$, we see that H_0 satisfies (1) as claimed.

To verify (2), we choose x_1 to be the element of X that is farthest from x_0 on the interval I, that is, we let $x_1=w_b$ where $b\leq p$ is the highest index such that $w_b\in X$. Finally we let the path P be $C_0(x_1,s_2)w_lC_0(s_1,x_0)$. Note that s_1 and s_2 are an even distance apart in S and so P has the same parity as $C_0(x_1,x_0)$. Then P has length at least $|C_0|-4K_0$, satisfying (2). □

We are now ready to prove Theorem 1.

Proof of Theorem 1. Let $K=5K_0$, where $K_0=150000c^{-5}$. Let C_0 be a longest even or a longest odd cycle and let H_0 be the bipartite subgraph of G and P the path guaranteed by Lemma 4(1). Recall that the minimum degree k of H_0 satisfies $c^3K_0/4096\leq k\leq K_0$. We first note that the bipartite subgraph H_0 contains cycles of all even lengths between 4 and $2k$, by Lemma 2(3) and the fact that x_0 is adjacent to every vertex of Y. We therefore need only check that G contains a cycle of length t for every t in the interval $[2k,\ell(P)]$ with t the same parity as $\ell(P)$, where $\ell(P)$ denotes the length of the path P.

Let $t\in[2k,\ell(P)]$ be fixed. We now describe a sequence of paths $P_0,P_1,\ldots,P_f$ with the following properties.

(1) Each P_i joins x_0 to x_1 and is otherwise disjoint from H_0, and its length has the same parity as $\ell(P)$,
(2) $P_0=P$,
(3) $\ell(P_i)-Q\leq\ell(P_{i+1})\leq\ell(P_i)-1$ for each i, where $Q=15c^{-2}+12c^{-1}$,
(4) $\ell(P_f)\leq t-4<\ell(P_{f-1})$.

We begin by setting $P_0 = P$. Assume that paths $P_0, \dots, P_i$ have been constructed. If $\ell(P_i) \le t-4$ then we set $f = i$ and stop. Otherwise we select $r = \lceil 3/c \rceil$ vertices $Z = \{z_1, \dots, z_r\}$ on P_i spaced at distance 4 apart and let S be the smallest interval containing them. Note that this is possible since $\ell(P_i) \ge 2k \ge c^3K_0/2048 > 4r$ by definition of K_0. Then by Lemma 3 applied to the graph $G[Z, V(G) \setminus S]$ (with $c' = 2c/3$), some pair $z \ne z'$ in Z have at least $2c^2n/9$ common neighbours in $V(G) \setminus S$. If one such neighbour y is disjoint from $P_i \cup H_0$, then we let P_{i+1} be the path obtained by replacing the (z, z') segment of P_i by zyz'. Note that this shortens the path by an even length of at least 2 and at most $4r$. Otherwise at least $2c^2n/9 - 2K_0$ common neighbours of z and z' fall onto P_i. Since P_i has length less than n, and $n \ge 90K_0c^{-4}$, there is an interval I in P_i of length at most $15c^{-2}$ that is disjoint from S and contains three common neighbours and therefore two that are an even distance apart in I, say y_1 and y_2. We obtain P_{i+1} by removing the (z, z') and (y_1, y_2) segments of P_i and adding the edges zy_1 and $z'y_2$ (or $z'y_1$ and zy_2, whichever results in a connected path). This shortens P_i by at least 2 and at most $4r + 15c^{-2}$. This completes the definition of the paths P_i.

Having found the path P_f, which by (3) satisfies $t-Q \le \ell(P_f) \le t-4$, we then use Lemma 2(3) as above to complete it to a cycle of length t by adding an (x_0, x_1) path of the required even length in H_0. Note that this is possible since $K_0 \ge 150000c^{-5}$ implies $k \ge c^3K_0/4096 \ge Q/2$. This completes the proof. □

3. Proofs of Lemmas

Proof of Lemma 2(2). In fact we shall prove a stronger statement: that there exist $U'' \subseteq U$ and $W'' \subseteq W$ such that $|B[U', W'']| \ge \bar{d}(U)|U'|/2$ for every $U' \subseteq U''$, and $|B[U'', W']| \ge \bar{d}(W)|W'|/2$ for every $W' \subseteq W''$. Here $|G|$ denotes the number of edges in the graph G. Then Lemma 2(2) follows immediately by taking $U' = \{u\}$ and $W' = \{w\}$.

We let $\emptyset \ne U'' \subseteq U$ and $\emptyset \ne W'' \subseteq W$ be minimal such that

$$|B[U'', W'']| \ge \bar{d}(U)|U''|/2 + \bar{d}(W)|W''|/2.$$

Note that such a choice exists since the pair (U, W) itself satisfies this condition. We claim that (U'', W'') has the desired property. To see this, suppose on the contrary that there exists some $U_0 \subseteq U''$ such that $|B[U_0, W'']| < \bar{d}(U)|U_0|/2$. Note that $U_0 \ne U''$, so $U'' \setminus U_0$ is not empty. But then $|B[U'' \setminus U_0, W'']| = |B[U'', W'']| - |B[U_0, W'']| > \bar{d}(U)|U''|/2 + \bar{d}(W)|W''|/2 - \bar{d}(U)|U_0|/2 = \bar{d}(U)|U'' \setminus U_0|/2 + \bar{d}(W)|W''|/2$, which shows that the pair $(U'' \setminus U_0, W'')$ contradicts the choice of (U'', W''). Similarly we reach a contradiction if there exists some $W_0 \subseteq W''$ such that $|B[U'', W_0]| < \bar{d}(W)|W_0|/2$. Therefore the statement is true, which proves Lemma 2(2). □

Proof of Lemma 3. Note we may assume that $|W| = n$, and that each v_i has degree exactly cn in G, since adding edges cannot decrease the size of the largest common neighbourhood. We let $W = \{w_1, \ldots, w_n\}$, and for each v_i we let x_i denote the vector of length n that has 1 in the jth position if $w_j v_i$ is an edge of G, and 0 otherwise. Then note that $\langle x_i, x_j\rangle = |\Gamma(v_i) \cap \Gamma(v_j)|$ for each i and j, where $\langle, \rangle$ denotes the standard innner product in $\mathbb{R}^n$. Suppose on the contrary that $|\Gamma(v_i) \cap \Gamma(v_j)| < c^2 n/2$ for all $i \neq j$.

We consider the quantity $S = \langle \sum_{i=1}^r x_i, \sum_{i=1}^r x_i\rangle$. Then by definition $S = \sum_{i=1}^n d(w_i)^2$, and hence by the Schwarz inequality we find

$$S \geq (\sum_{i=1}^n d(w_i))^2/n.$$

On the other hand, $S = \sum_{i=1}^r |\Gamma(v_i)| + \sum_{i\neq j}\langle x_i, x_j\rangle < \sum_{i=1}^r |\Gamma(v_i)| + (r^2 - r)c^2 n/2$ by our assumption. But each v_i has exactly cn neighbours, so $\sum_{i=1}^r |\Gamma(v_i)| = \sum_{i=1}^n d(w_i) = cnr$. Hence we obtain

$$(cnr)^2/n \leq S < cnr + (r^2 - r)c^2 n/2.$$

Therefore $cr < 1 + (r-1)c/2$, so since $2/c \leq r < 2/c + 1$ we conclude $2 < 2$. This contradiction shows that the result holds. □

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