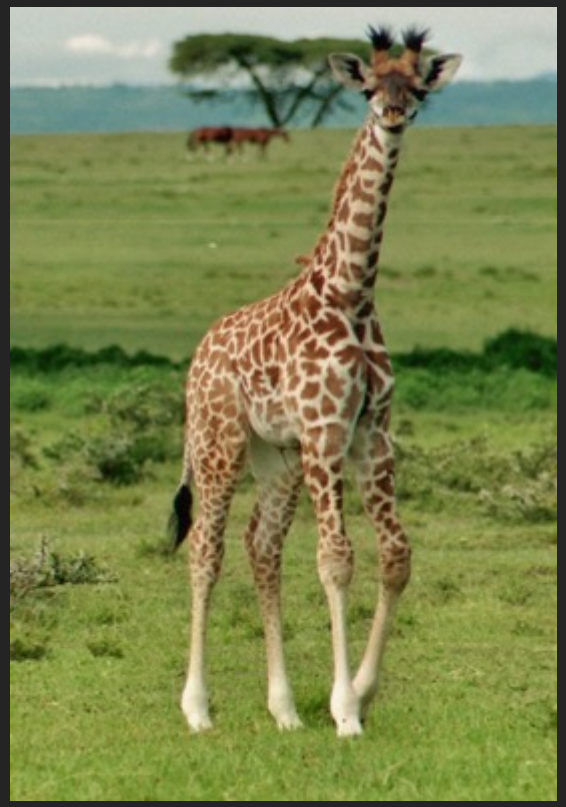


Adventures with Algebra and Geometry in East Africa

Balázs Szendrői

University of Oxford

Jornadas de Matemática

Gulbenkian 2021

Outline

1. Mathematics in Africa: challenges and opportunities
2. Algebraic geometry and applications
3. Activities in Africa to develop algebraic geometry and related areas

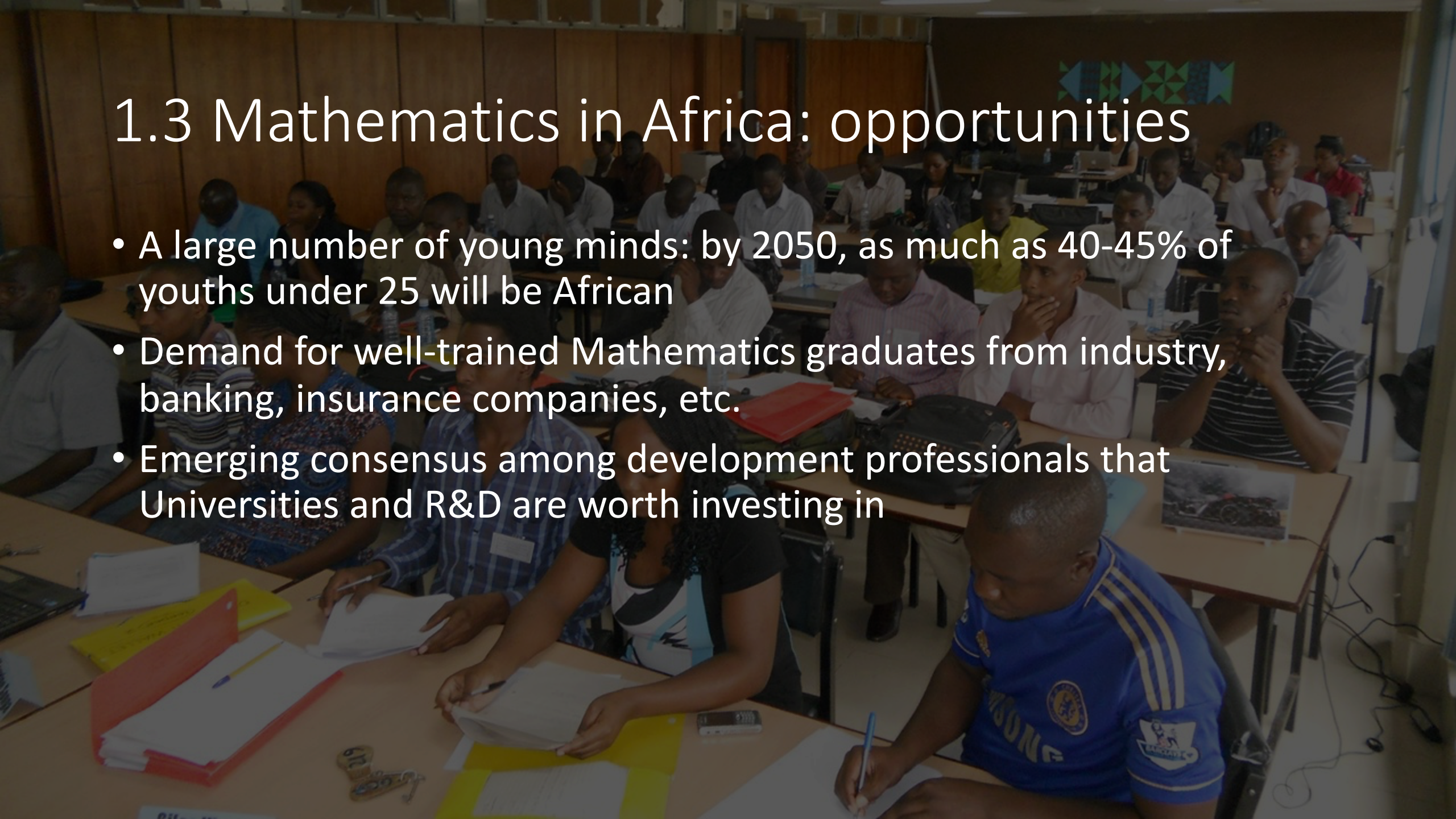
1.1 Mathematics in Africa: lofty beginnings

- Blombos Cave, South Africa: symbolic drawing from 70,000-100,000 years ago
- Ishango Bone, Congo: 20,000 years old.
  Tally of objects? Diary? Basic arithmetic calculations?
- Nabtha Playa, Sudan: early stone circle from cca. 5000BCE

1.2 Mathematics in Africa: challenges

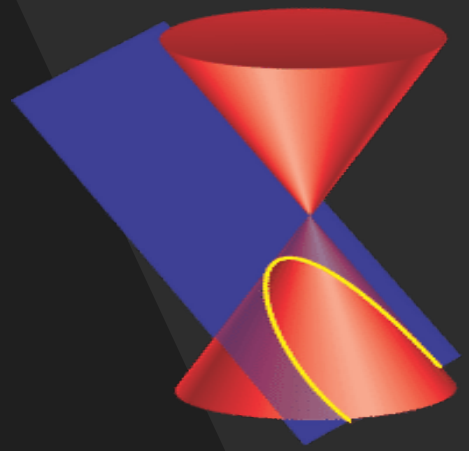
- At many Universities, there is no critical mass of active researchers
- For young graduates, difficulty of funding PhD studies abroad; local PhD programmes may be weak
- Recent PhD's are given large teaching loads and immediate responsibilities (Head of Dept, etc)
- Mathematical level of incoming undergraduates moderate
- Lack of funds to initiate international collaborations – especially "South-South" collaborations

1.3 Mathematics in Africa: opportunities



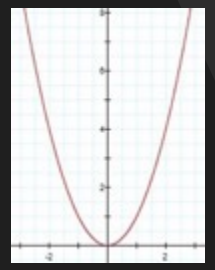
- A large number of young minds: by 2050, as much as 40-45% of youths under 25 will be African
- Demand for well-trained Mathematics graduates from industry, banking, insurance companies, etc.
- Emerging consensus among development professionals that Universities and R&D are worth investing in

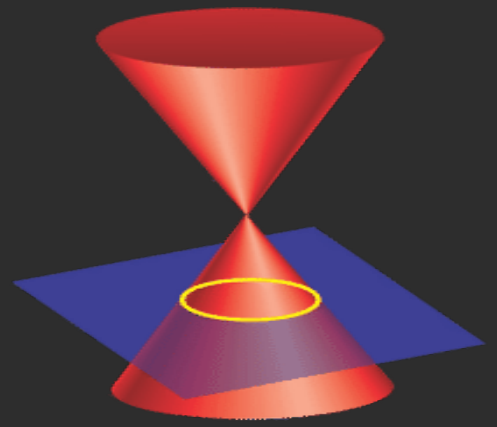
2.1 The beginning of algebraic geometry: conics



Parabola

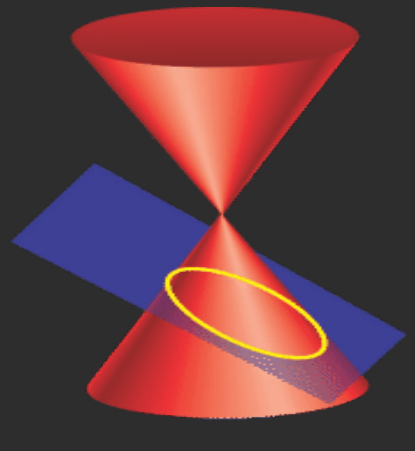
$y = ax^2$





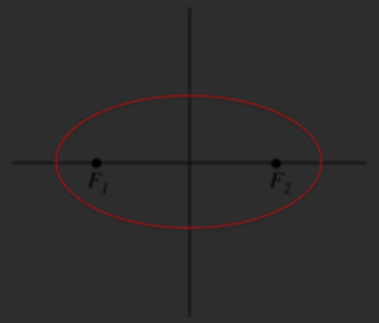
Circle

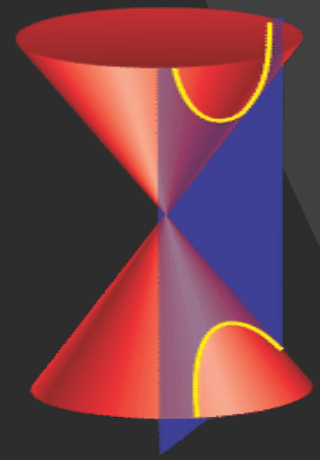
$x^2+y^2 = b^2$



Ellipse

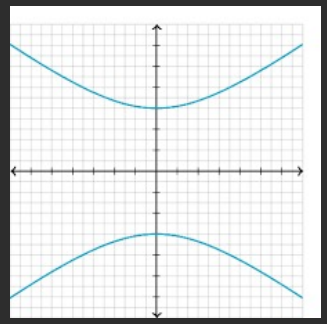
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$





Hyperbola

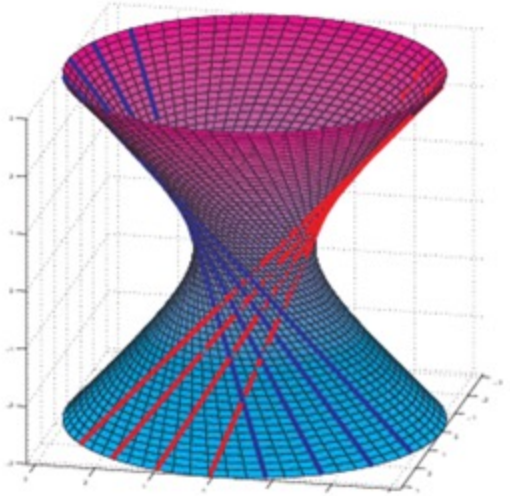
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

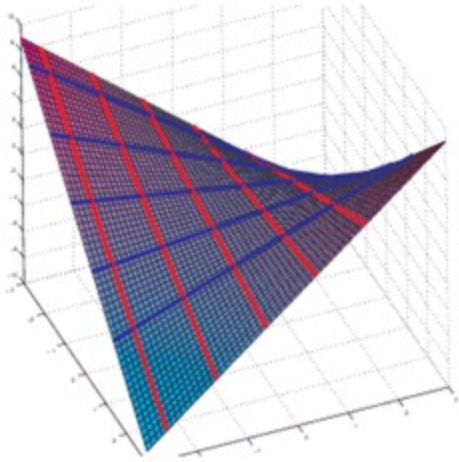


2.2 Surfaces given by algebraic equations

$$x^2 + y^2 - z^2 - 1 = 0$$

$$z - x^2 + y^2 = 0$$





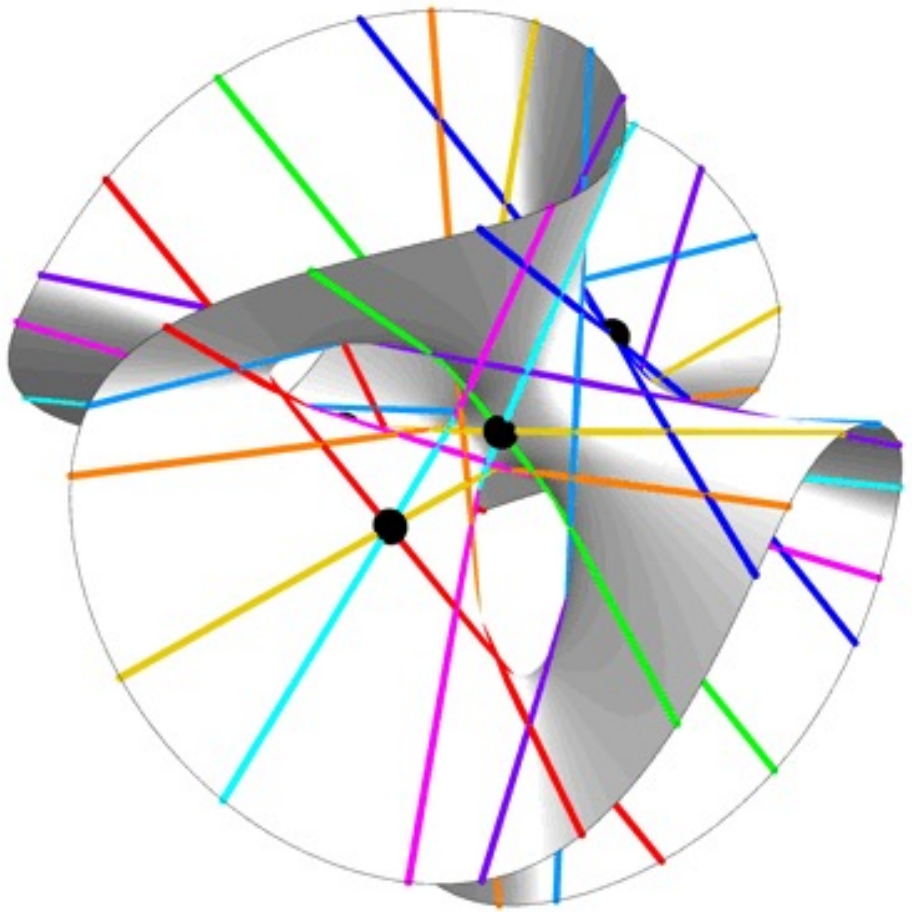
2.3 A cubic surface

- A cubic surface is described by degree 3 equation:

$$x^3 + y^3 + z^3 - 1 = 0$$

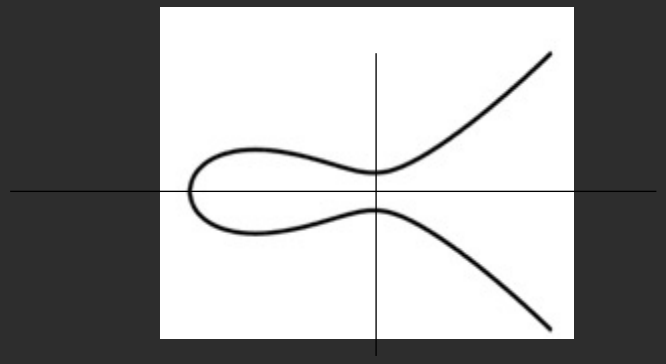
- This contains *exactly* 27 lines.

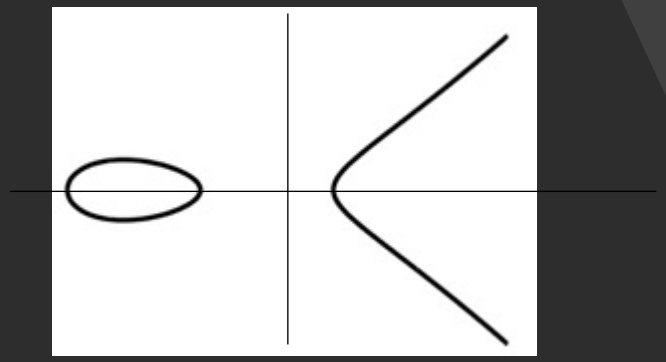
- A mathematical curiosity, but very pretty!



2.4 Elliptic curve cryptography

- Cubic elliptic curves: described by degree 3 equations





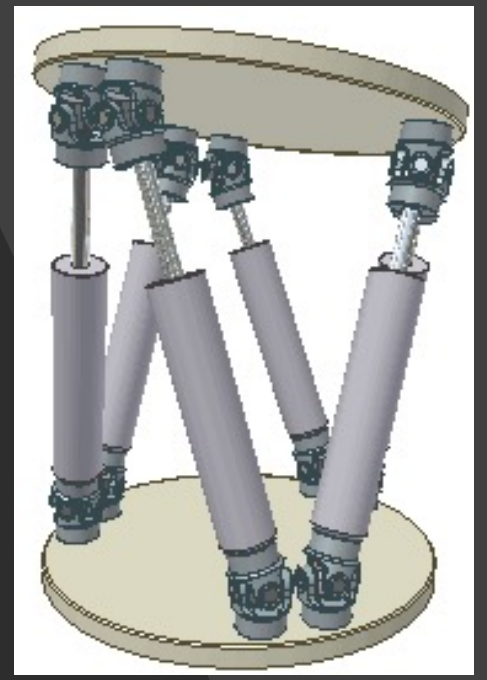
- Information can be stored in the geometric structure of *cubic* curves: *Elliptic Curve Cryptography*
- One concrete example:

$$y^2 + xy = x^3 - 1895782483362476188247825431x + 428107465551850284688462121997629913671 45$$

- Need computer software to handle such equations!
- Used by Bitcoin for transaction validation.

2.5 Algebraic robotics

- Kinematics of rigid moving apparatuses
- One example: Stewart platform
- Interesting motion patterns



- Idealized picture: triangular platform
- Kinematics of this system described by algebraic equations
- Ultimate aim: industrial applications

From Seigal, arXiv:1603.08546
Picture created by Charles Wampler and Douglas Arnold

2.6 Algebraic equations in systems biology

- There are exactly 9 isolated stationary points in the Wnt shuttle model (Harrington et al)

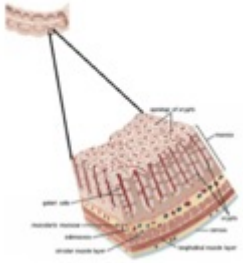
- Biological interpretation: The first model that explains multiple cellular decisions (division, movement, specialization)

From Harrington et al, arXiv:1502.03188

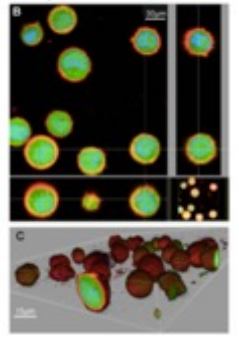
Wnt shuttle model

$$
\begin{aligned}
\dot{x}_1 &= -k_1x_1 + k_2x_2 \\
\dot{x}_2 &= k_1x_1 - (k_2 + k_{26})x_2 + k_{27}x_3 - k_3x_2x_4 + (k_4 + k_5)x_{14} \\
\dot{x}_3 &= k_{26}x_2 - k_{27}x_3 - k_{14}x_3x_6 + (k_{15} + k_{16})x_{15} \\
\dot{x}_4 &= -k_3x_2x_4 - k_9x_4x_{10} + k_4x_{14} + k_8x_{16} + (k_{10} + k_{11})x_{18} \\
\dot{x}_5 &= -k_{28}x_5 + k_{29}x_7 - k_6x_5x_8 + k_5x_{14} + k_7x_{16} \\
\dot{x}_6 &= -k_{14}x_3x_6 - k_{20}x_6x_{11} + k_{15}x_{15} + k_{19}x_{17} + (k_{21} + k_{22})x_{19} \\
\dot{x}_7 &= k_{28}x_5 - k_{29}x_7 - k_{17}x_7x_9 + k_{16}x_{15} + k_{18}x_{17} \\
\dot{x}_8 = -\dot{x}_{16} &= -k_6x_5x_8 + (k_7 + k_8)x_{16} \\
\dot{x}_9 = -\dot{x}_{17} &= -k_{17}x_7x_9 + (k_{18} + k_{19})x_{17} \\
\dot{x}_{10} &= k_{12} - (k_{13} + k_{30})x_{10} - k_9x_4x_{10} + k_{31}x_{11} + k_{10}x_{18} \\
\dot{x}_{11} &= -k_{23}x_{11} + k_{30}x_{10} - k_{31}x_{11} - k_{20}x_6x_{11} - k_{24}x_{11}x_{12} + k_{25}x_{13} + k_{21}x_{19} \\
\dot{x}_{12} = -\dot{x}_{13} &= -k_{24}x_{11}x_{12} + k_{25}x_{13} \\
\dot{x}_{14} &= k_3x_2x_4 - (k_4 + k_5)x_{14} \\
\dot{x}_{15} &= k_{14}x_3x_6 - (k_{15} + k_{16})x_{15} \\
\dot{x}_{18} &= k_9x_4x_{10} - (k_{10} + k_{11})x_{18} \\
\dot{x}_{19} &= k_{20}x_6x_{11} - (k_{21} + k_{22})x_{19}
\end{aligned}
$$

Biological system



Experimental data



2.7 Topological data analysis

- Many data sets arising in nature come with geometric structure
  - Spatial data
  - Cyclicity
  - Hidden structure
- Persistent homology: method to find algebraic shadow of geometric structure

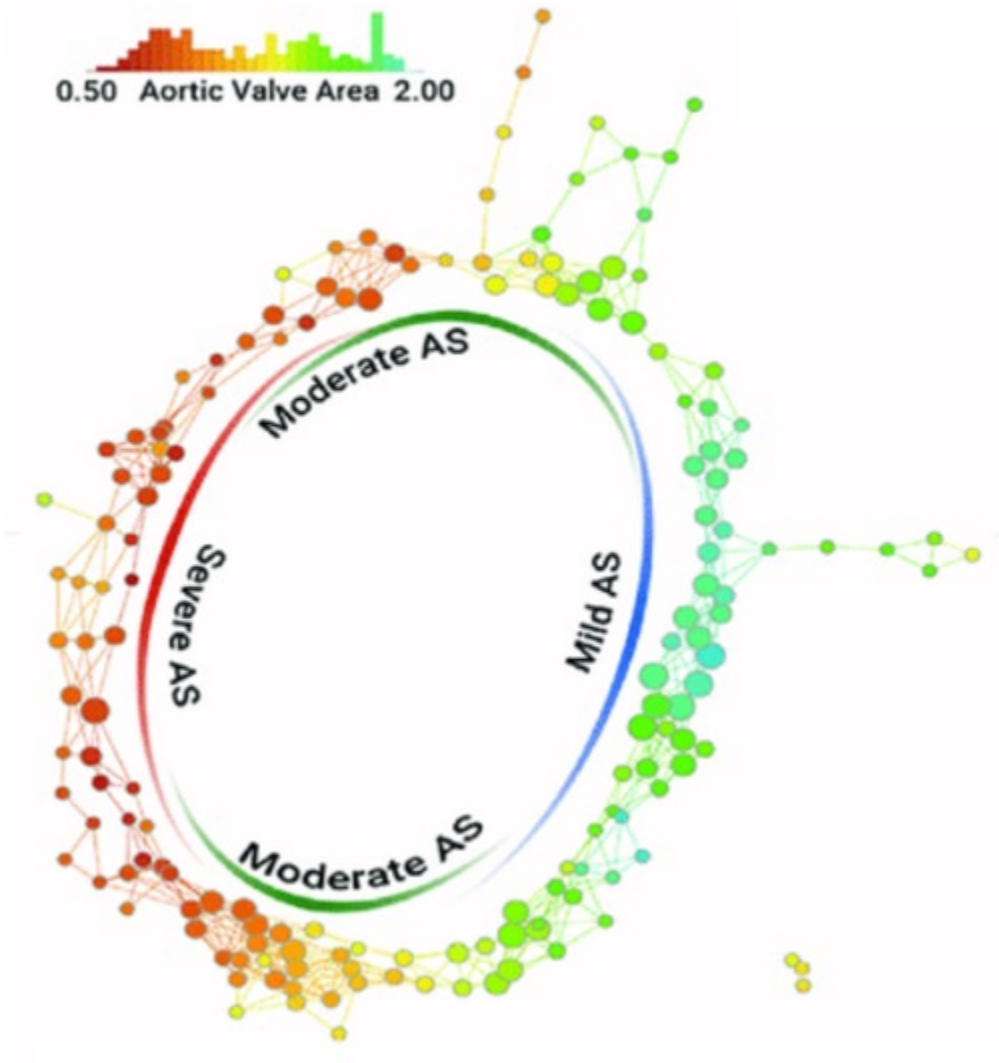


Image from Casaclang-Verzosa et al 2019

3.1 The MARM scheme of the LMS

- MARM: Mentoring African Research in Mathematics, by the London Mathematical Society
- Relatively small grants (£10k) to individuals in the UK and elsewhere
- My first link to Africa: Department of Mathematics, University of Maseno

3.2 EAUMP summer schools

- Summer school series on pure mathematics, organized together with
  - EAUMP: East African Universities Mathematics Programme
  - ISP: International Science Programme of Uppsala University and Sida, Sweden
  - ICTP: International Centre for Theoretical Physics, Trieste, Italy
  - CIMPA / African Mathematical Union *Ecole Mathematique Africaine* programme
- Usually a 3-week school with 25-40 student participants and 4-6 lecturers

3.2 EAUMP summer schools

- Since 2013:
  - 2013 School on Algebraic Geometry, Mombasa, Kenya
  - 2014 School on Representation Theory, Arusha, Tanzania
  - 2015 School on Experimental Mathematics, Makerere University, Uganda
  - 2016 School on Number Theory, University of Rwanda

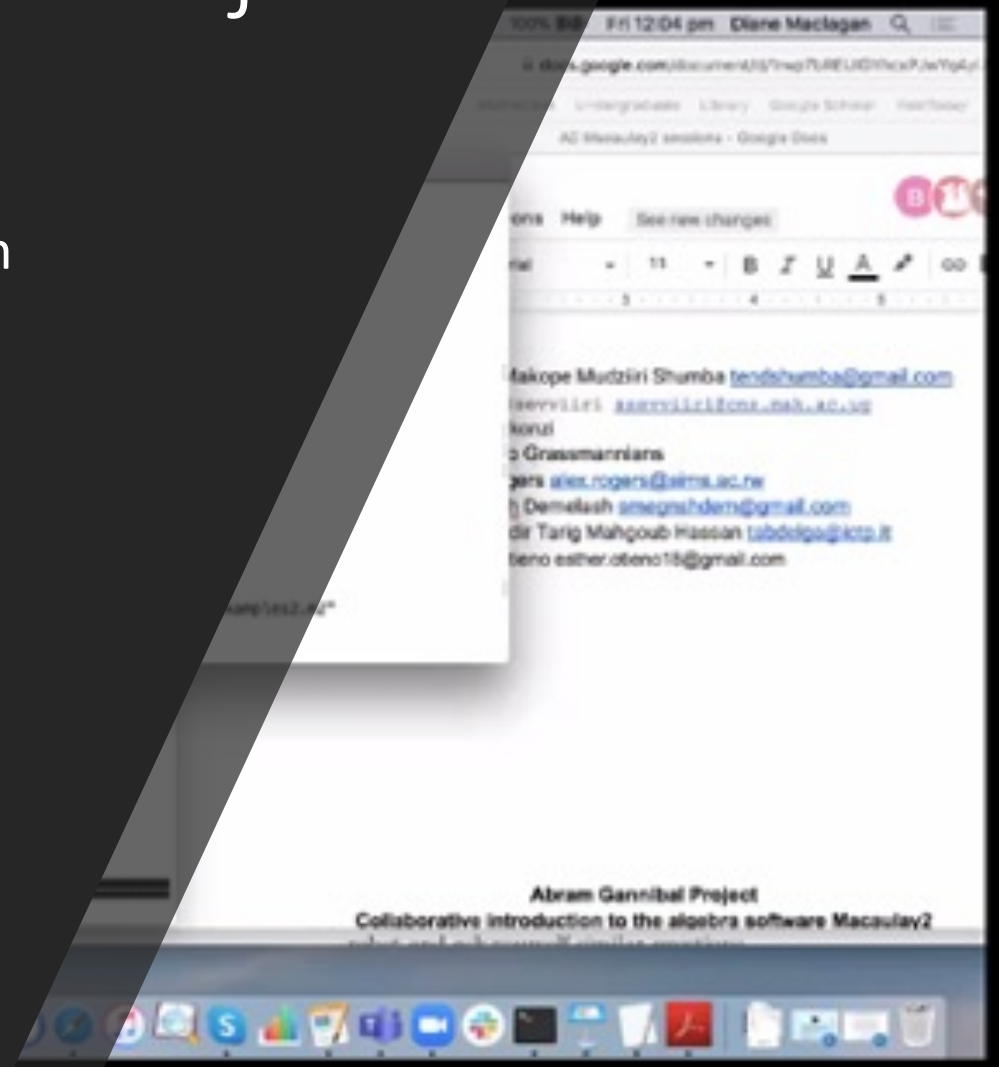
- 2017 School on Modern Functional Analysis, University of Nairobi, Kenya
- 2018 School on Homological Methods in Algebra and Geometry, Dar es Salaam, TZ
- 2019 School on Algebraic Topology and its Applications, Makerere University, UG
- 2021 School Topics in Concrete Mathematics (remote school)

3.3 Nairobi workshops in algebraic geometry

- Organized together with Jared Ongaro, University of Nairobi, 2014-2021
- Topics included the study of graded rings, algebraic curves, computer algebra, differential geometry, etc.
- Main role: provide ideas for Masters projects
- Successful! 11 students graduated with Masters theses in algebraic geometry and related areas since 2014.
- 6 of these obtained international PhD positions in China, Germany, UK, Sweden

3.4 Abram Gannibal Project

- This was supposed to be an ambitious series of collaborative workshops in Uganda and Nigeria in spring 2020, followed by visits to the UK.
- Workshops cancelled because of Covid-19, also no bilateral visits possible
- Remote workshops on computer algebra packages Macaulay2, Sage, Polymake, also systems biology (including Iara Goncalves, Universidade Eduardo Mondlane, Mozambique)
- Bilateral visits to the UK in early 2022



3.5 Outreach activities

- Activities mostly run by friends David Stern, Emily Fleming, Jeff Goodman, Zach Mbasu, and others
- AMI: African Maths Initiatives (Kenya), SAMI: Supporting African Maths Initiatives (UK)
- Math camps in 7 African countries, aimed at 13-17 year olds
- “Virtual Math Camp” card deck: deck of cards with mathematical problems