A generic approach to switching reconstruction

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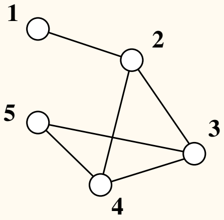

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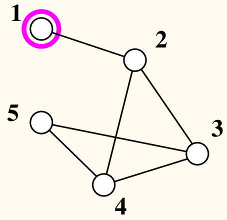

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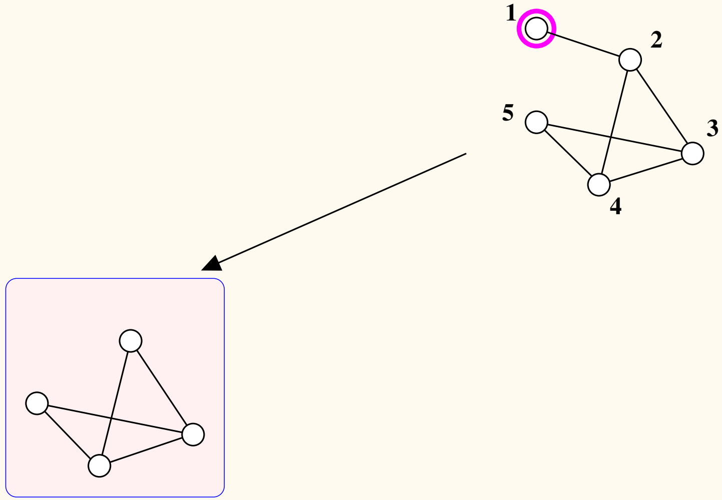

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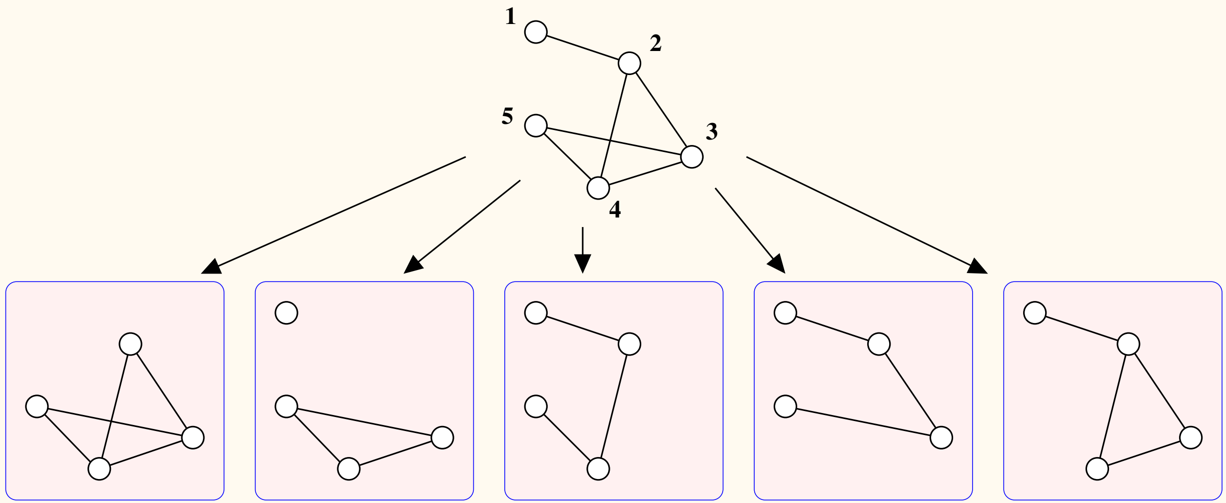

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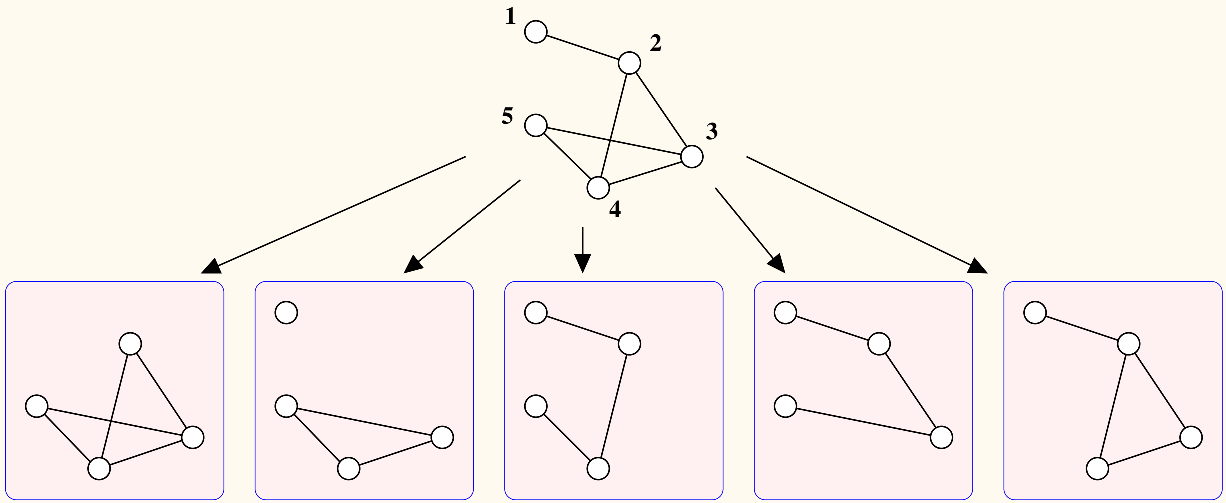

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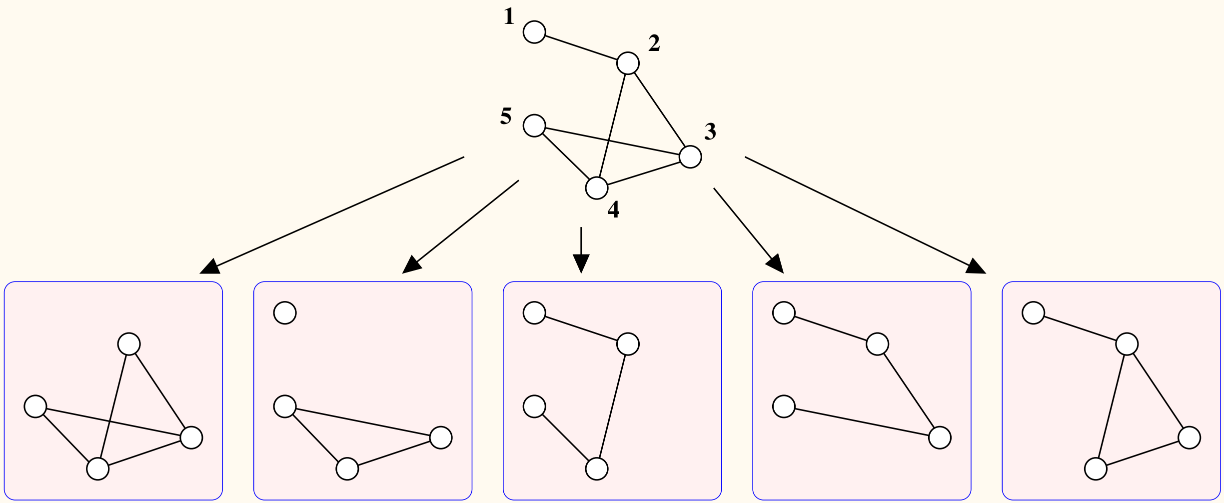

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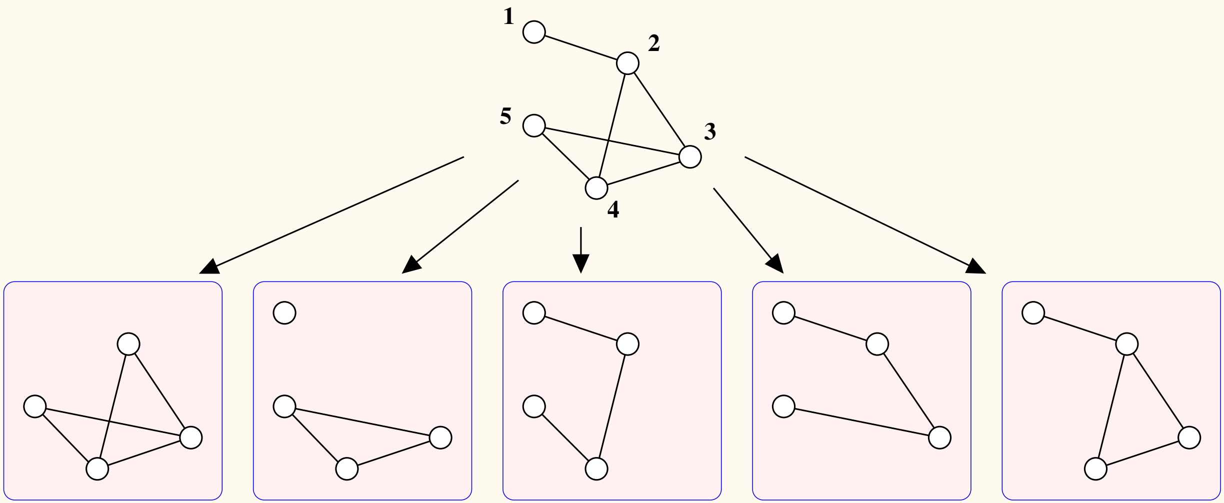

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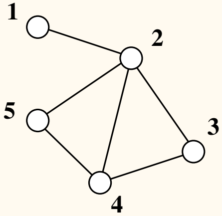

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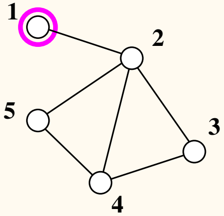

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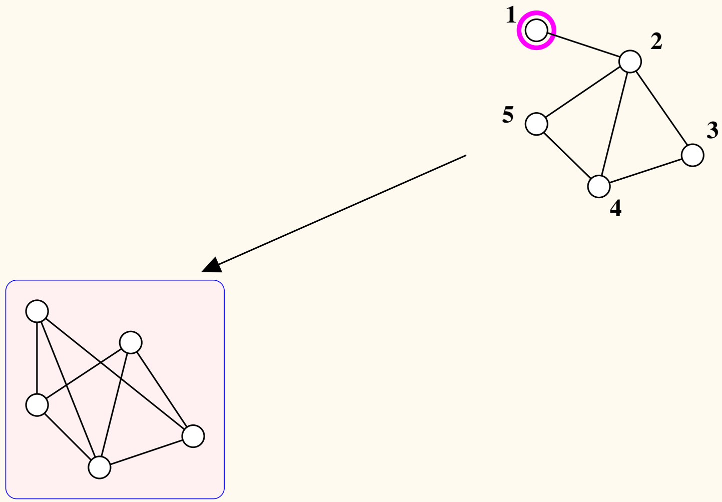

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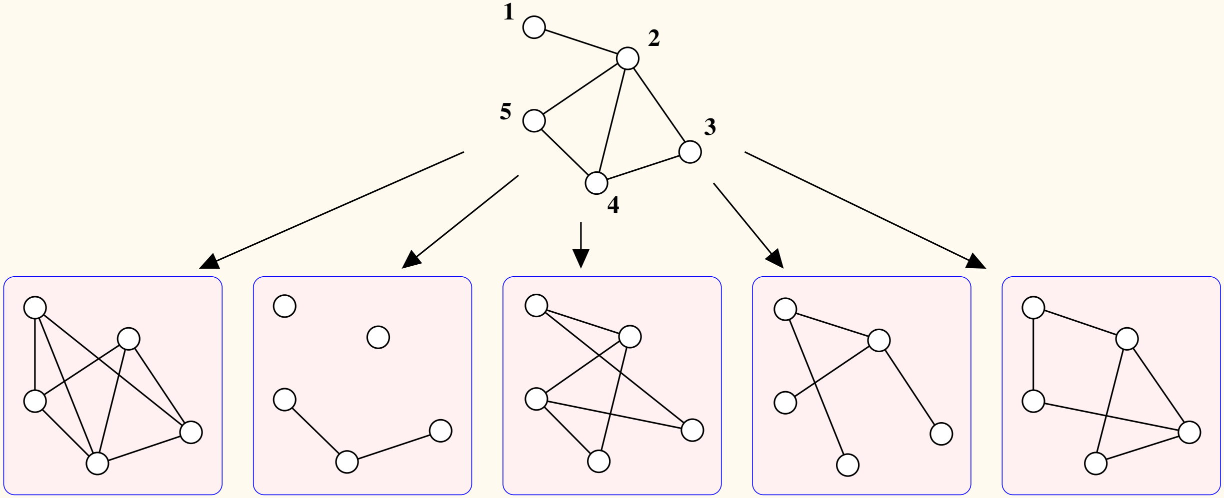

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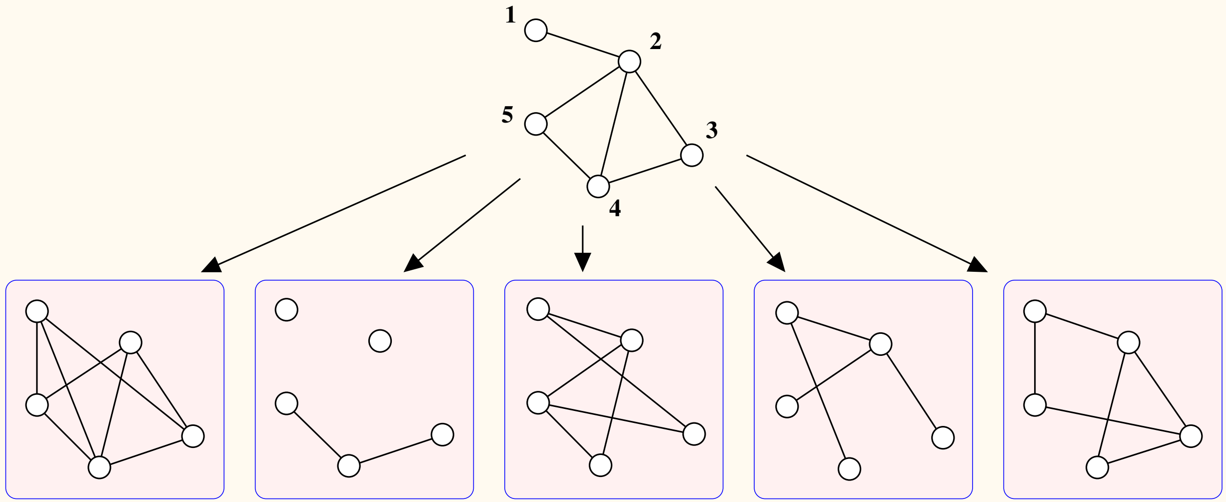

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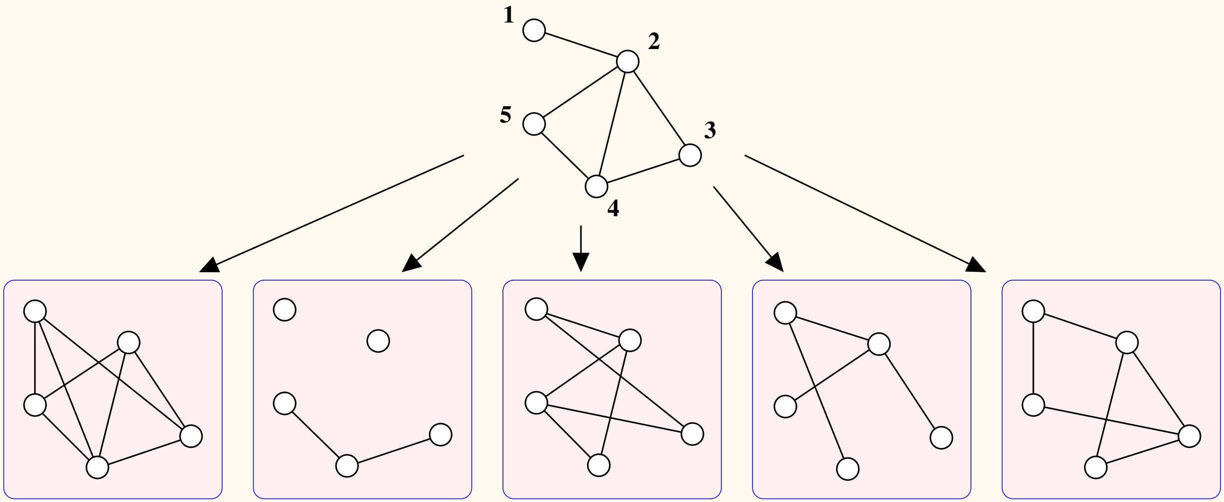

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Suppose G has order n. The following are reconstructible:

- Graphs with $n \not\equiv 0 \pmod 4$ (Stanley, 1985)
- Disconnected graphs, provided $n \neq 4$ (Krasikov, 1988)
- Graphs satisfying $\min\left(n\binom{n-1}{\Delta}, n\binom{n-1}{\delta}\right) < 2^{\frac{n}{2}-3}$ (Krasikov, 1988)
- Regular graphs, provided $n \neq 4$ (Ellingham & Royle, 1992)
- Triangle-free graphs, provided $n \neq 4$ (Ellingham & Royle, 1992)
- Graphs with $n = 8$ or $n = 12$ (Niesink, 2010)
- ...

Digraph switching reconstruction problem

Given an oriented simple graph (no loops, parallel edges, digons) G with N vertices, make a multiset $\mathcal{D}(G)$ with N *cards*. Each card G_v is obtained from G by reversing every arc adjacent to v in G (i.e. switching at v).

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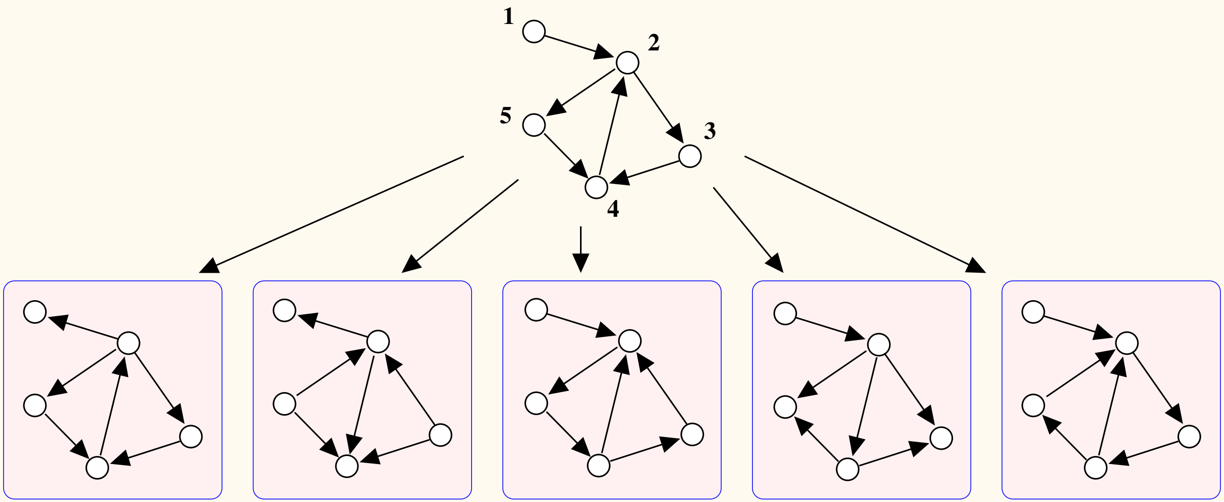

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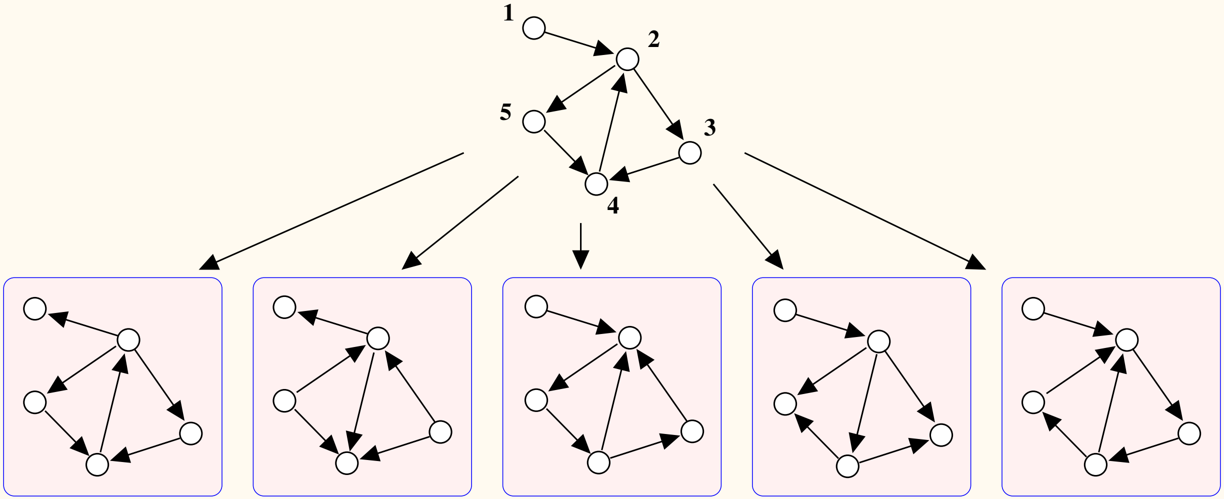

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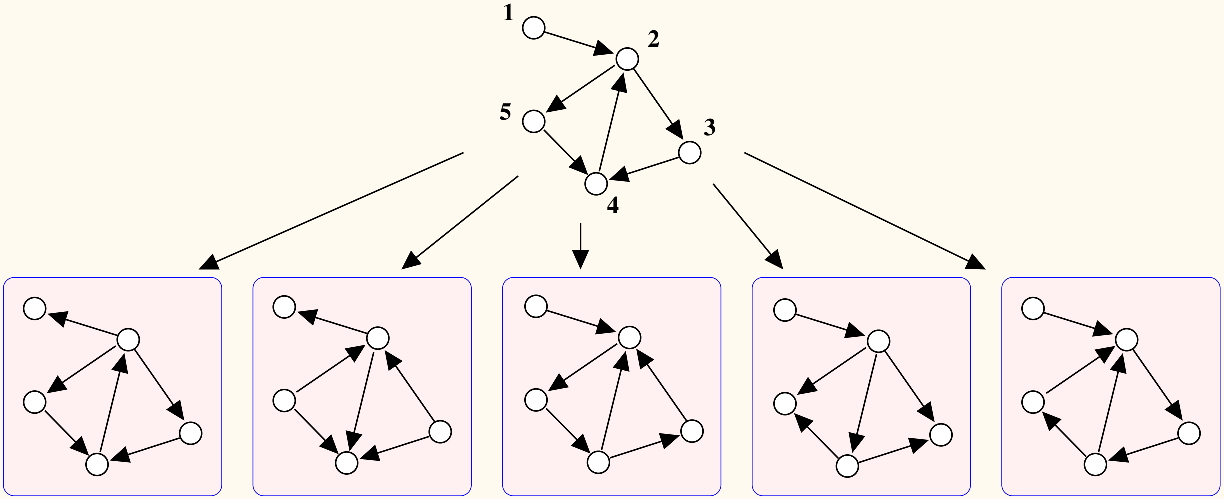

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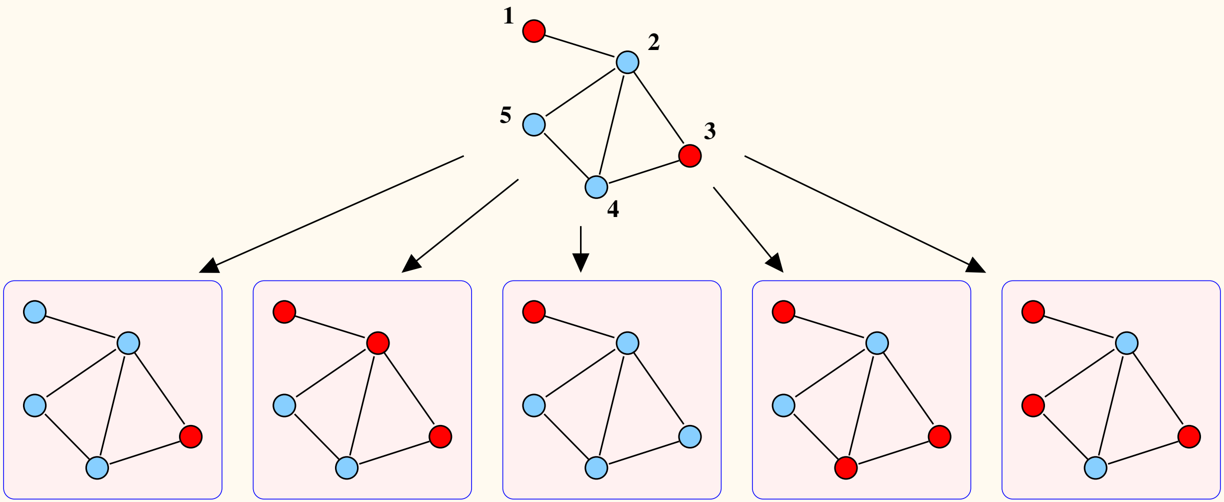
Digraph switching reconstruction problem

For this problem,

- digraphs with $n \not\equiv 0 \pmod 4$ are reconstructible (Bondy & Mercier, 2011)
- there are 35 non-reconstructible digraphs on 4 vertices (Bondy & Mercier, 2011),
- there are 5559 non-reconstructible digraphs on 8 vertices (McKay & Schweitzer, 2014) but no larger examples are known,
- we have a complete solution when the underlying graph has degree at most 2 (McKay & Schweitzer, 2014).

A toy switching reconstruction problem

Given an 2-vertex-coloured undirected graph G with n vertices, make a multiset $\mathcal{D}(G)$ with n *cards*. Each card is obtained from G by swapping the colour of one vertex.



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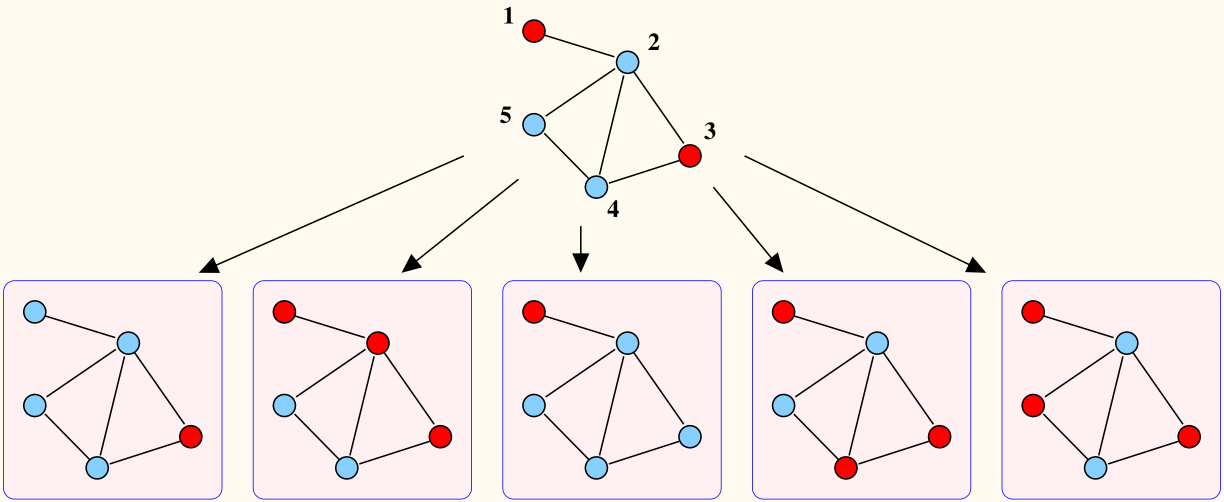

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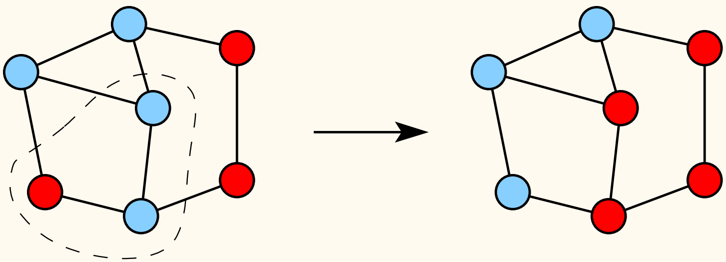

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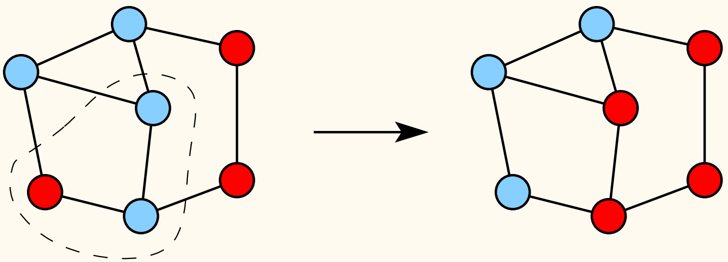

This allows us to generalise the notion of a deck. The k-deck of G is defined as

$$\mathcal{D}_k(G) = \sum_{W \subseteq Z : |W| = k} [G_W].$$

Generic formulation

To define a switching reconstruction problem, we need to know

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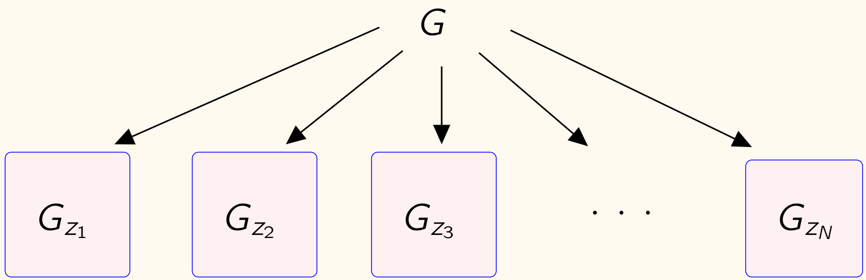

The deck $\mathcal{D}(G)$ is the 'multiset' $\mathcal{D}(G) = \sum_{z \in Z} [G_z]$.

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We call $(\phi, \sim)$ a switching operation if it satisfies these properties:

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S2. For $G, G' \in \mathcal{G}$, $G \sim G' \Rightarrow \mathcal{D}(G) = \mathcal{D}(G')$.

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We say G (technically, $[G]$) is reconstructible if it is determined uniquely within $\mathcal{G}$ up to equivalence by $\mathcal{D}(G)$.

A sneak peek

Given a switching operation $(\phi, \sim)$, the redundancy of G is

$$\rho(G) = \max_{G' \sim G} |\{W \subseteq Z : |W| \text{ is a multiple of 4 and } G_W = G'\}|,$$

Theorem. (Sufficient condition for reconstructibility) $G \in \mathcal{G}$ is reconstructible if

$$|\{W : W \subseteq Z, G_W \sim G, \text{and } |W| \text{ is divisible by 4}\}| < 2^{N/2-1}.$$

In particular, this is true if $\rho(G)\,|[G]| < 2^{N/2-1}$.

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$\mathcal{D}_k$ and T^k are closely related!

- $\mathcal{D}_k(G)$ is obtained by switching G on each set of k distinct vertices (contains $\binom{N}{k}$ cards)
- $T^k[G]$ is obtained by switching G on any k subsequent vertices (contains N^k cards)

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Some observations:

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T^0[G] &= [G] \\
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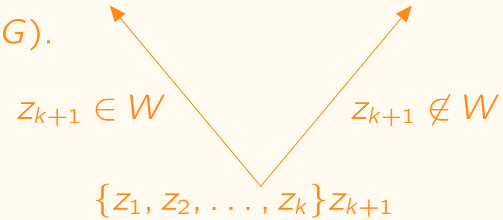

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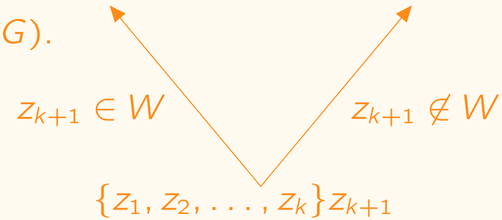

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From this, we can show

Lemma 2. $(N-k+1)\mathcal{D}_{k-1}(G) + (k+1)\mathcal{D}_{k+1}(G) = \alpha_k(T, N)\,\mathcal{D}_1(G)$. In particular, the left hand expression is reconstructible.

Lemma 3. Suppose $N = 2m$ is even. For $G \in \mathcal{G}$, $z \in Z$ and $j \geq 0$. Then

$$\mathcal{D}_{2j}(G) - (-1)^j \binom{m}{j} [G] = T^{-1}(\alpha_{2j}(T, N) - \alpha_k(0, N))\,\mathcal{D}_1(G)$$

and hence the left hand expression is reconstructible.

Some generic results

Upshot of lemmas.
$(N-k+1)\mathcal{D}_{k-1}(G)+(k+1)\mathcal{D}_{k+1}(G)$ and $\mathcal{D}_{2j}(G)-(-1)^j\binom{m}{j}[G]$ are reconstructible.

Theorem. Suppose that $N=2m$ and $G\in\mathcal{G}$ is not reconstructible.

- For each even j, there are at least $\binom{m}{j}$ sets $W\subseteq Z$ such that $|W|=2j$ and $G_W\sim G$.
- In total, there are at least 2^{m-1} sets $W\subseteq Z$ such that $G_W\sim G$.

Key idea: We can identify $[G]$ as the only negative term in $\mathcal{D}_{2j}(G)-(-1)^j\binom{m}{j}[G]$.

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Theorem. Suppose that $N = 2m$ and $G \in \mathcal{G}$ is not reconstructible.

- For each even j, there are at least $\binom{m}{j}$ sets $W \subseteq Z$ such that $|W| = 2j$ and $G_W \sim G$. Moreover, for any $z \in Z$, at least $\binom{m-1}{j-1}$ such sets include z and at least $\binom{m-1}{j}$ don't.
- In total, there are at least 2^{m-1} sets $W \subseteq Z$ such that $G_W \sim G$, including at least 2^{m-2} sets which include z and 2^{m-2} which don't.

Some generic results

Upshot of lemmas.
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Theorem. Suppose $G\in\mathcal{G}$. Then the following hold.

(a) If N is odd, then G is reconstructible.

(b) If $N\equiv 2 \pmod 4$, then $[G]\cup[G_Z]$ is reconstructible. In particular, if $G_Z\sim G$ then G is reconstructible.

(c) If $N\equiv 0 \pmod 4$, then G is reconstructible if $G_Z\not\sim G$.

Some generic results

Given a switching operation $(\phi, \sim)$, the redundancy of G is

$$\rho(G) = \max_{G' \sim G} |\{W \subseteq Z : |W| \text{ is a multiple of 4 and } G_W = G'\}|,$$

Theorem. (Sufficient condition for reconstructibility) $G \in \mathcal{G}$ is reconstructible if

$$|\{W : W \subseteq Z, G_W \sim G, \text{and } |W| \text{ is divisible by 4}\}| < 2^{N/2-1}.$$

In particular, this is true if $\rho(G)\,|[G]| < 2^{N/2-1}$.

To apply this directly for specific problems, it is helpful to have:

- Some constant underlying structure H of G $(\rho(G)\,|\mathrm{Aut}(H)| < 2^{N/2-1}|\mathrm{Aut}(G)|)$
- Low redundancy
- Lower bound on $|\mathrm{Aut}(G)|$ (we shall use 1)
- Upper bound on $|\mathrm{Aut}(H)|$

Applications

Digraph switching

$\mathcal{G}$: Oriented simple graphs with n vertices.

Z: Vertex set $V(G)$.

ϕ: Obtain G_v from G by reversing the direction of each arc incident with v.

$\sim$: Arc-preserving graph isomorphism

$\rho(G)$: If $G_W = G$, then W is a union of H-components, so redundancy is $2^{c(H)}$.

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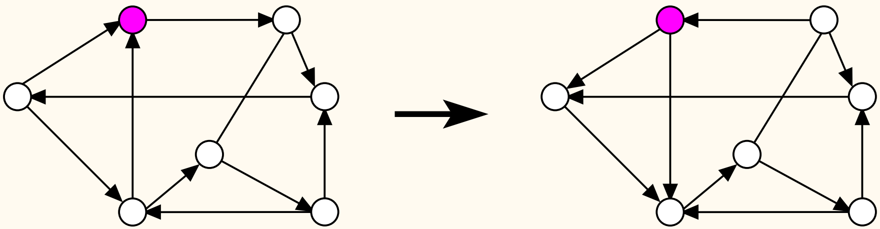

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- If H is a polygon then G is reconstructible for $n \geq 14$,
- If H is a 3-connected planar graph then G is reconstructible for $n \geq 16$, and
- If H is a vertex-transitive 3-connected cubic graph then G is reconstructible for $n \geq 28$.

Also, $G_W \sim G$, so G is reconstructible if $n \not\equiv 0 \mod 4$.

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- If H is a polygon then G is reconstructible for $n \geq 13$,
- If H is a 3-connected planar graph then G is reconstructible for $n \geq 13$, and
- If H is a vertex-transitive 3-connected cubic graph then G is reconstructible for $n \geq 25$.

Applications

Edge colour switching at a vertex

$\mathcal{G}$: Undirected simple graphs on n vertices, each edge coloured red or blue.

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ϕ: Obtain G_v from G by changing the colour of each edge incident with v.

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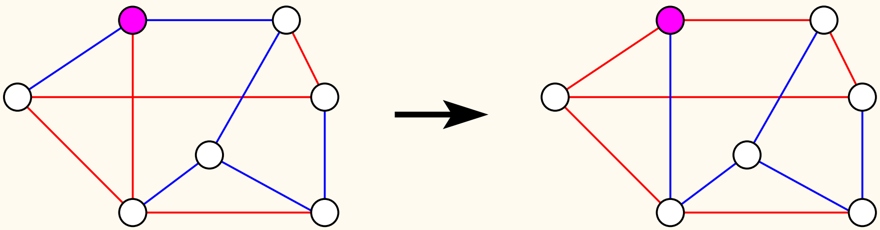

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- If H is a 3-connected planar graph then G is reconstructible for $n \geq 13$, and
- If H is a vertex-transitive 3-connected cubic graph then G is reconstructible for $n \geq 25$.

Here, $G_W \sim G$, so G is reconstructible if $n \not\equiv 0 \mod 4$.

Applications

Toy problem (vertex colour switching at a vertex)

$\mathcal{G}$: Undirected simple graphs on n vertices, each vertex coloured red or blue.

Z: Vertex set $V(G)$.

ϕ: Obtain G_v from G by changing the colour of vertex v.

$\sim$: Colour-preserving graph isomorphism

$\rho(G)$: The only set W such that $G_W = G$ is empty set, so $\rho(G) = 1$..

Applications

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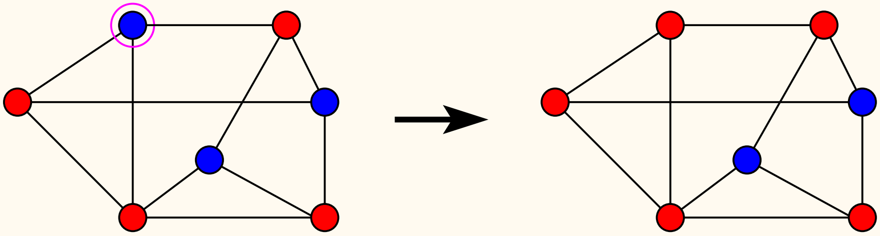

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- If H is a polygon then G is reconstructible for $n \geq 11$,
- If H is a 3-connected planar graph then G is reconstructible for $n \geq 13$, and
- If H is a vertex-transitive 3-connected cubic graph then G is reconstructible for $n \geq 22$.

Also, $G_W \not\sim G$, so G is reconstructible if n is odd.

Applications

Toy problem (vertex colour switching at a vertex)

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- If H is a polygon then G is reconstructible for $n \geq 11$,
- If H is a 3-connected planar graph then G is reconstructible for $n \geq 13$, and
- If H is a vertex-transitive 3-connected cubic graph then G is reconstructible for $n \geq 21$.

Applications

Embedding switching

$\mathcal{G}$: Combinatorial embeddings on an orientable surface of undirected simple graphs with n vertices.

Z: Vertex set $V(G)$.

ϕ: Obtain G_v from G by reversing the cyclic order at v.

$\sim$: Isomorphism of graphs preserving cyclic orders.

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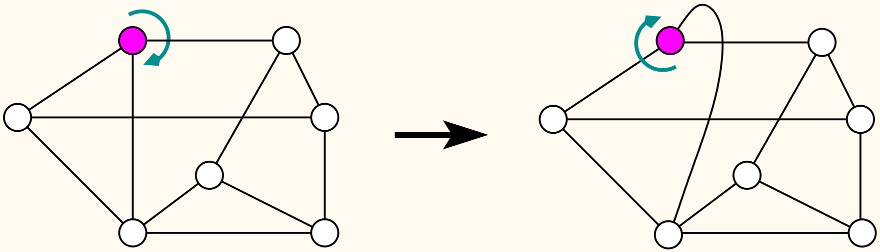

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- If H is a 3-connected planar graph then G is reconstructible for $n \geq 13$, and
- If H is a vertex-transitive 3-connected cubic graph then G is reconstructible for $n \geq 21$.

Here, $G_W \not\sim G$, so G is reconstructible if n is odd.