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Mathematical Billiards, Flat Surfaces and Dynamics

Corinna Ulcigrai

University of BRISTOL

Mathematical Billiards

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a *billiard table*: $P \subset \mathbb{R}^2$;

a *ball*: point particle with no-mass;

Billiard motion: straight lines, unit speed, elastic reflections at boundary: angle of incidende = angle of reflection.

No friction. Consider trajectories which never hit a pocket: trajectories and motion are *infinite*.

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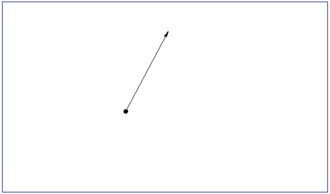

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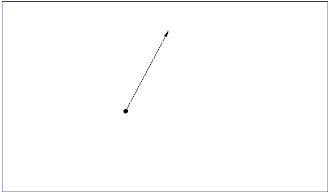

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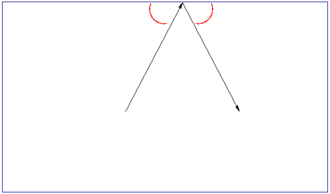

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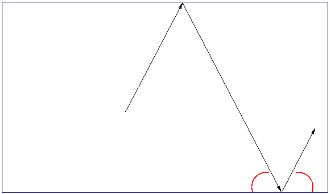

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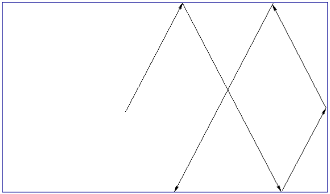

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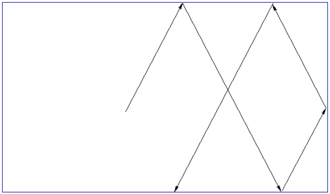

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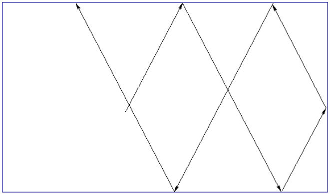

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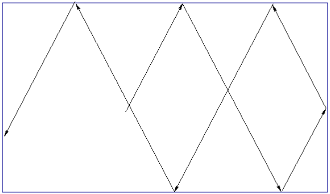

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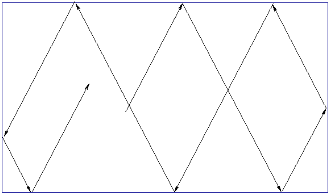

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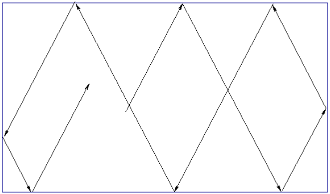

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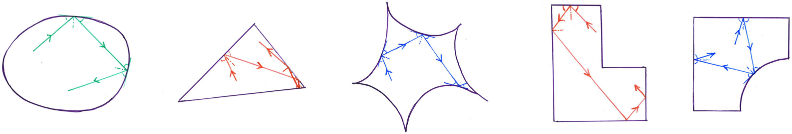

An L-shape billiard in real life:

[Credit: Photo courtesy of Moon Duchin]

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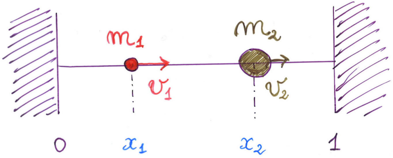

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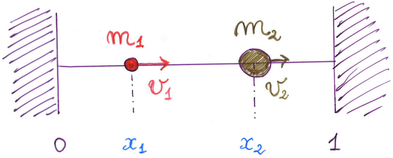

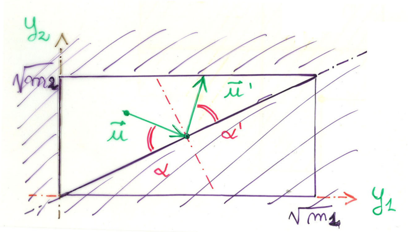

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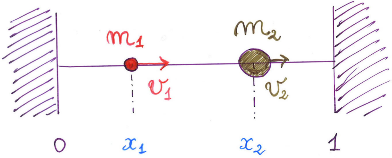

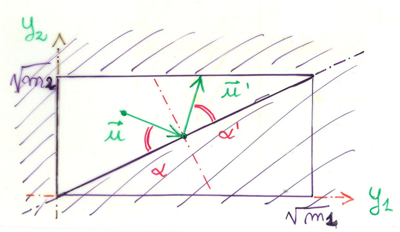

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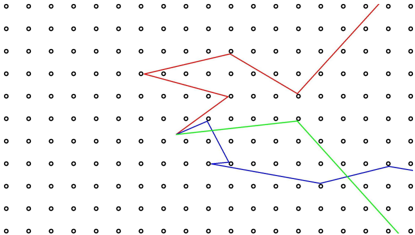

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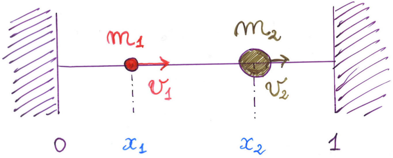

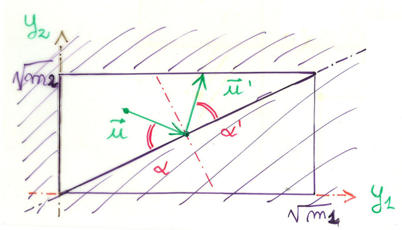

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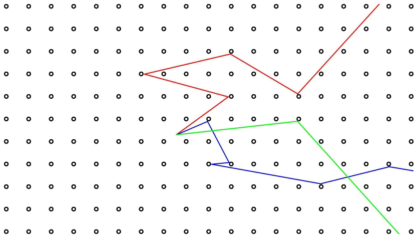

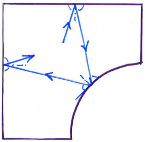

(Sinai billiard)

Billiards as Dynamical Systems

Mathematical billiards are an example of a *dynamical system*, that is a system that evolves in time.

Usually dynamical systems are *chaotic* and one is interested in determining the *asymptotic behaviour*, or long-time evolution of the trajectories.

Basic questions are:

- Are the *periodic trajectories*?
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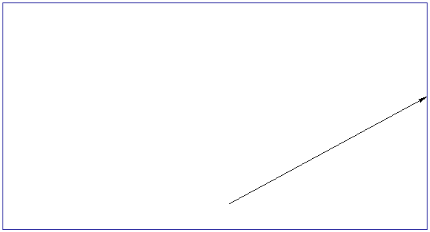

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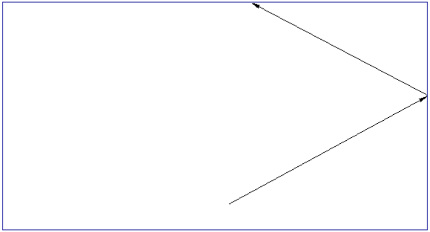

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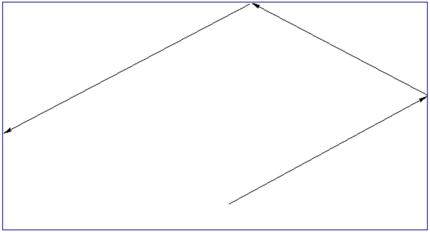

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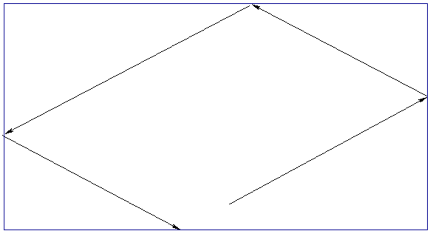

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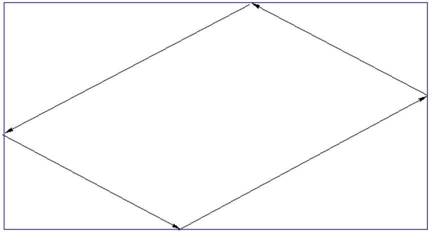

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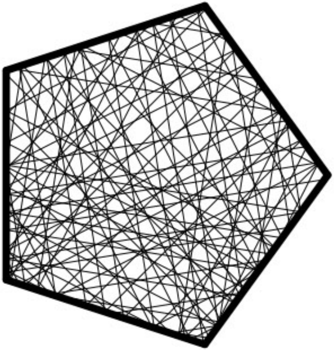

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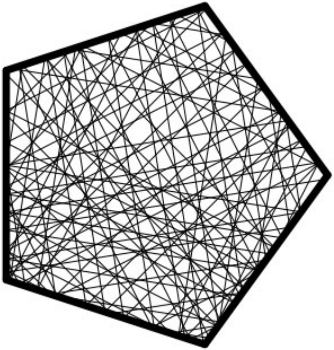

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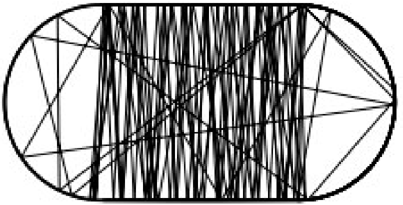

Shape of billiards and areas of dynamics

Integrable billiards
(convex boundary)

many periodic orbits

Hamiltonian Dynamics
variational methods

Polygonal billiards
(flat boundary)

Teichmüller Dynamics
very active area
(Kontsevitch, McMullen, Yoccoz)

Hyperbolic billiards
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Hyperbolic dynamics
very chaotic

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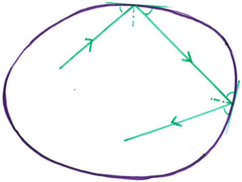

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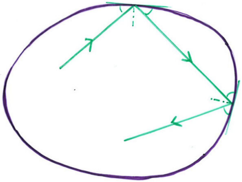

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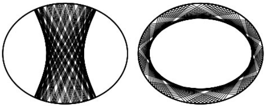

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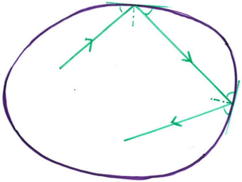

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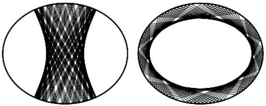

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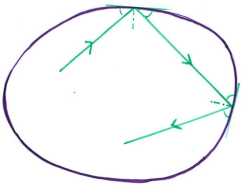

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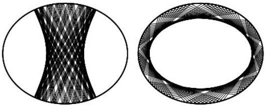

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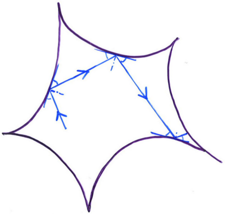

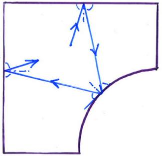

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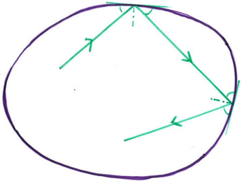

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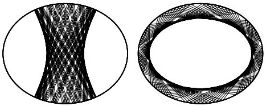

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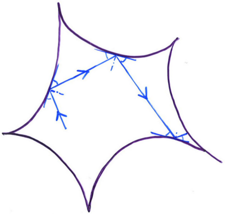

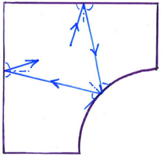

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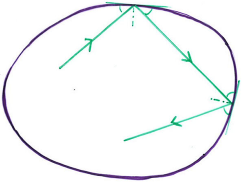

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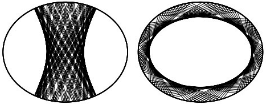

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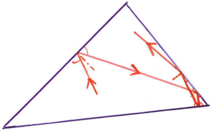
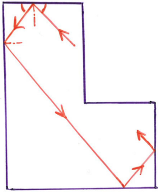
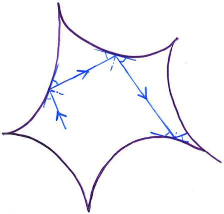
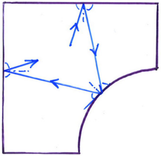




Teichmüller Dynamics
very active area
(Kontsevitch,
McMullen, Yoccoz)

Hyperbolic billiards
(convex boundary)

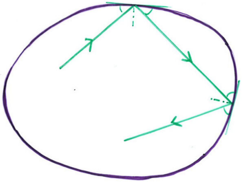




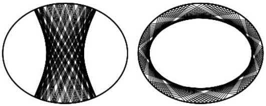
Hyperbolic dynamics
very chaotic

Shape of billiards and areas of dynamics

Integrable billiards
(convex boundary)



many periodic orbits



Hamiltonian Dynamics
variational methods

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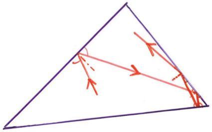

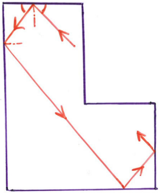

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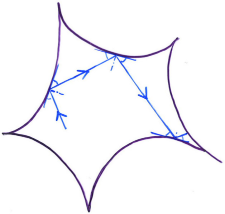

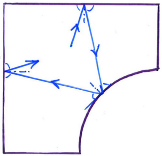

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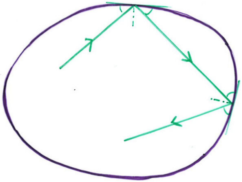

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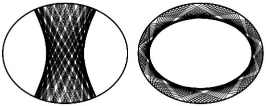

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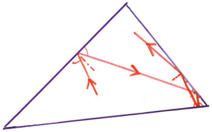

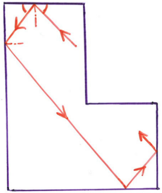

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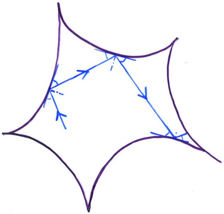

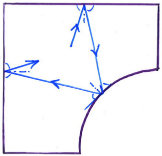

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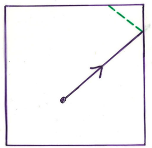
Unfolding a billiard trajectory: the square

From a billiard to a surface (Katok-Zemlyakov construction):



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Four copies are enough.

Glueing opposite sides one gets a torus

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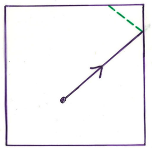

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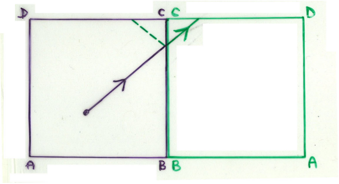

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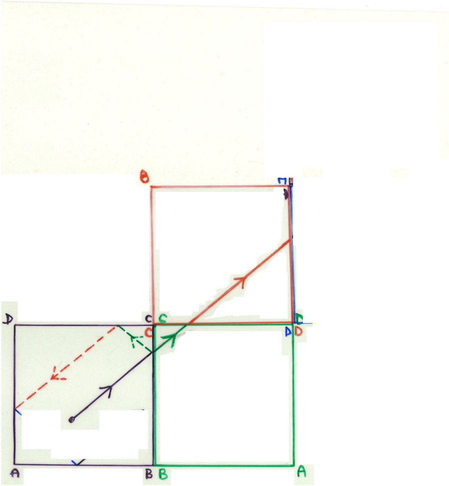

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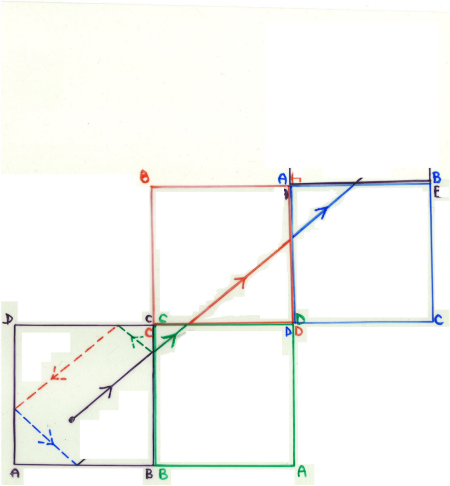

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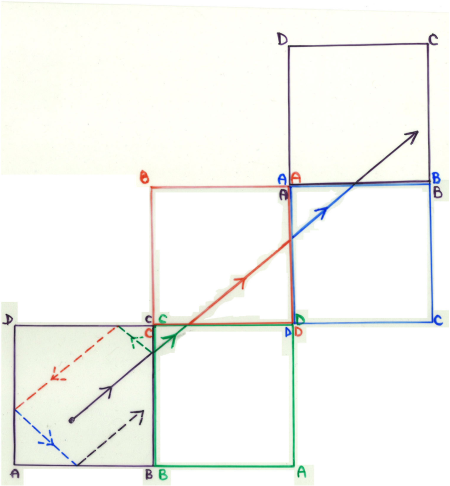

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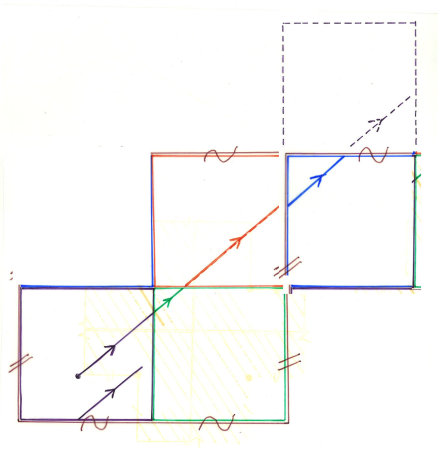

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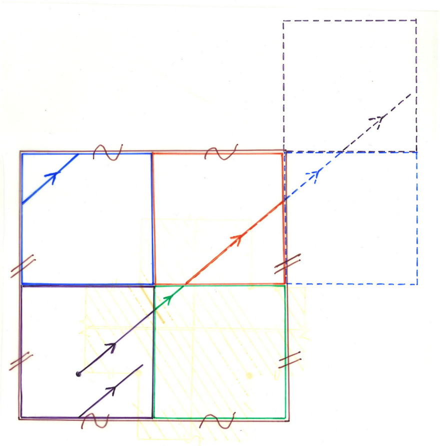

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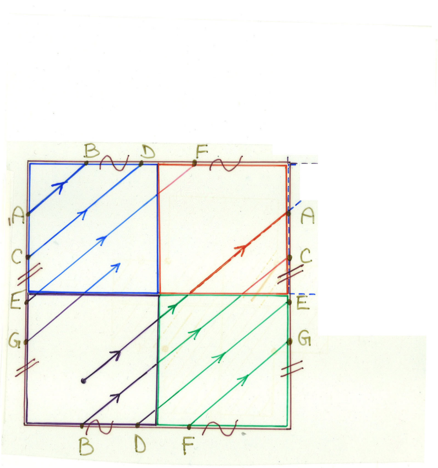

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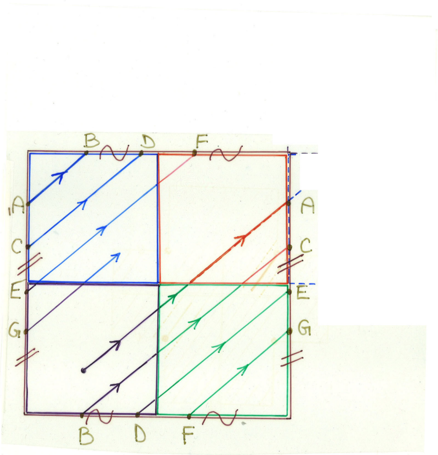

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Unfolding a billiard trajectory: higher genus

The unfolding construction works for any rational billiard, that is any polygonal billiard with angles of the form $\pi \frac{p_i}{q_i}$.

E.g.: billiard in a triangle with $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{\pi}{2}$ angles.
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If we glue opposite sides, this is a surface of genus 2. Linear trajectories have one saddle singularity.

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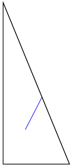

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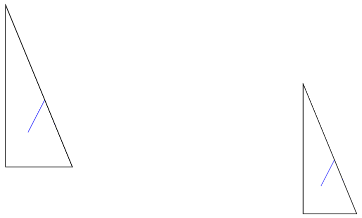

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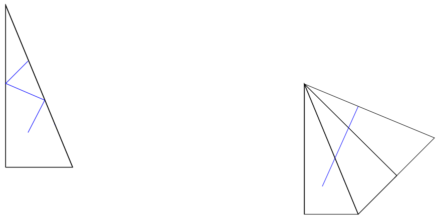

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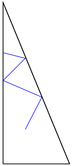

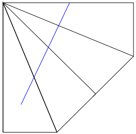

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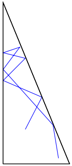

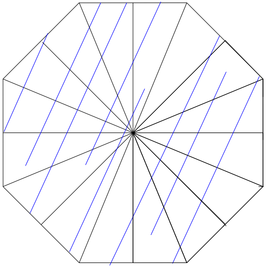

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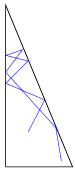

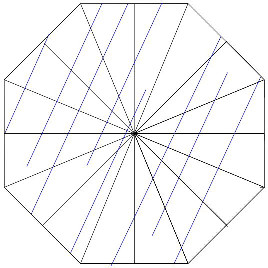

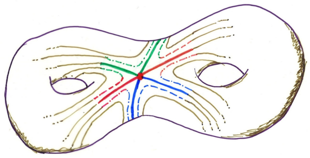

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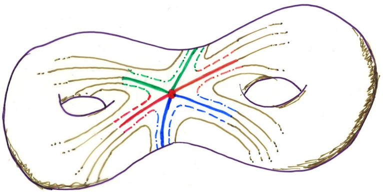
Flows on surfaces

We saw that billiard in rational polygons give rise to *flows on surfaces*.

More in general, trajectories of flows on surfaces also arise from solutions of differential equations:



The same trajectories can be obtained as local solutions of *Hamiltonian equations*:

$$\begin{cases} \frac{\partial x}{\partial t} = & \frac{\partial H}{\partial y} \\ \frac{\partial y}{\partial t} = & -\frac{\partial H}{\partial x} \end{cases}$$

$f_t(p)$ trajectory of p as t grows.

[Another motivation from *solid state physics*: locally Hamiltonian flows on surfaces describe the motion of an electron under a magnetic filed on the Fermi energy level surface in the semi-classical limit (Novikov).]

These flows $f_t : X \to X$ preserves the area. What are the dynamical properties of the trajectories?

Next: two of my recent works: mixing in area-preserving flows on surfaces and symbolic coding of trajectories in the octagon.

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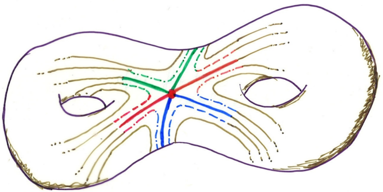

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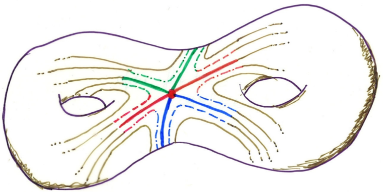

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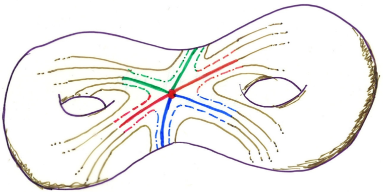

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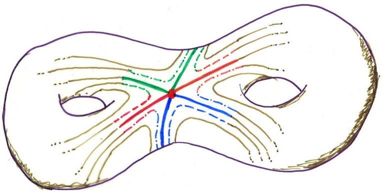

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Let A be a subset of the space X. Flow points in A for time t: does $f_t(A)$ spreads uniformly? E.g.

Definition

The flow f_t is *mixing* if for any two subsets A, B

$$\frac{Area(f_t(A) \cap B)}{Area(A)}$$

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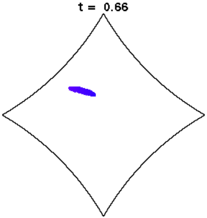

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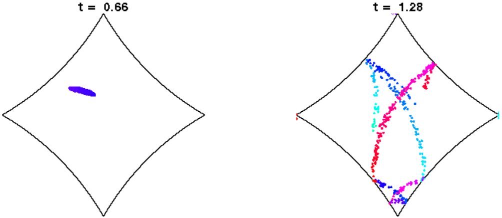

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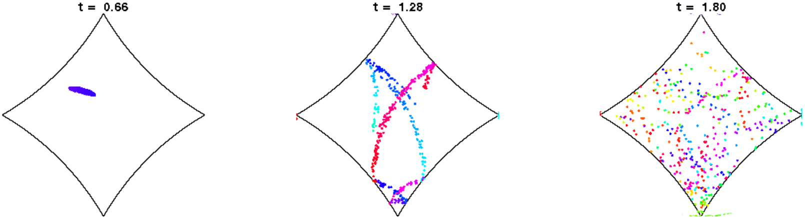

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The flow f_t is *mixing* if for any two subsets A, B

$$\frac{Area(f_t(A) \cap B)}{Area(A)}$$

Question (Arnold, 80s) are locally Hamiltonian flows on surfaces *mixing*?

Definition of Mixing

$f_t(p)$ describe the trajectory of p as t grows, each $f_t : X \to X$ flow preserves the area. Assume that the total area is 1.

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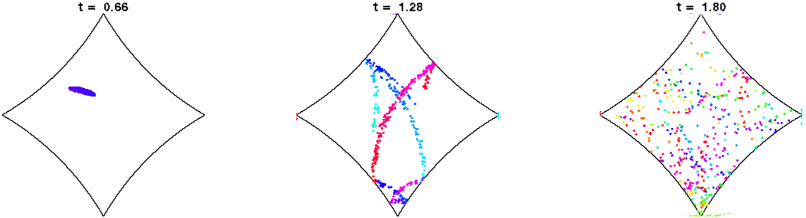

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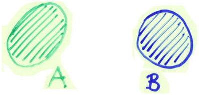

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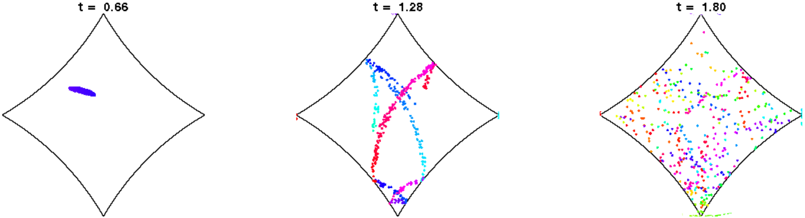

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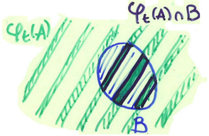

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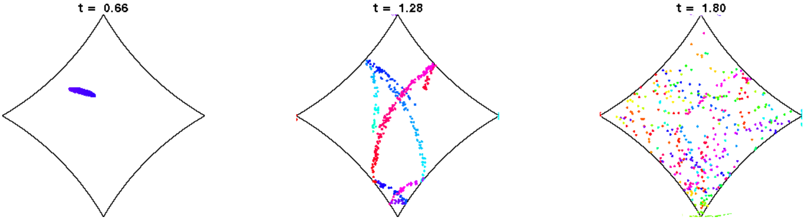

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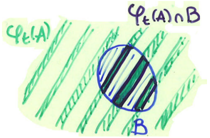

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A typical locally Hamiltonian flow on a surface that has traps is mixing in the complement of the traps.

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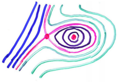

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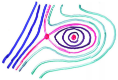

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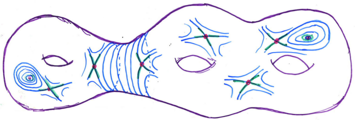

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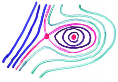

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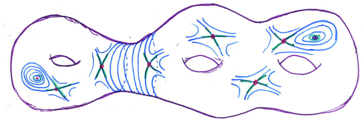

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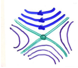

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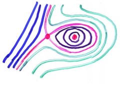

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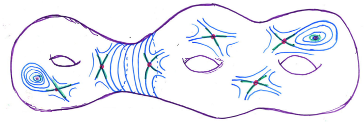

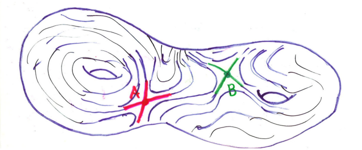

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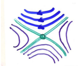

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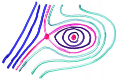

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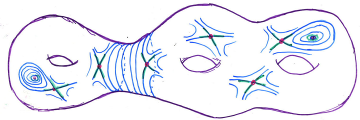

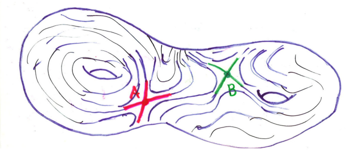

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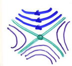

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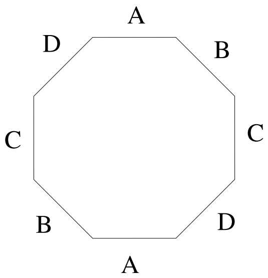

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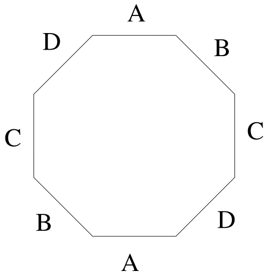

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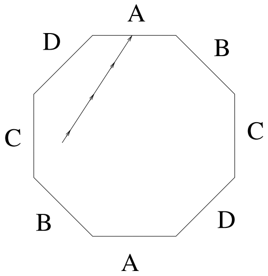

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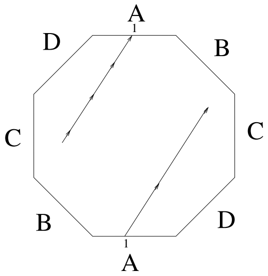

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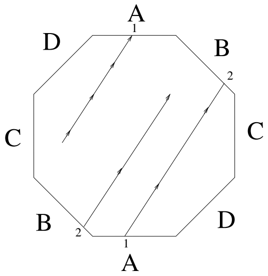

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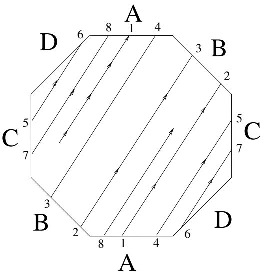

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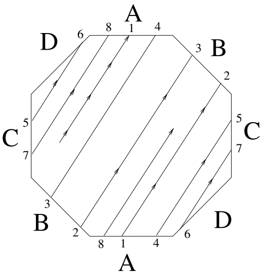

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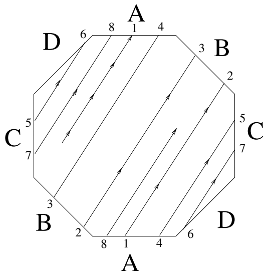

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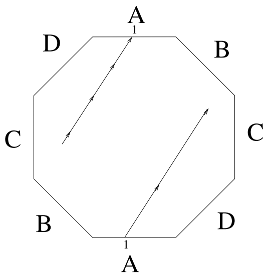

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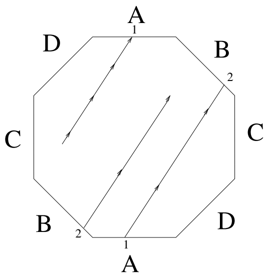

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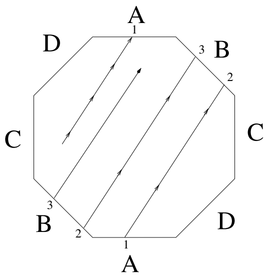

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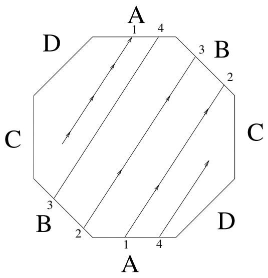

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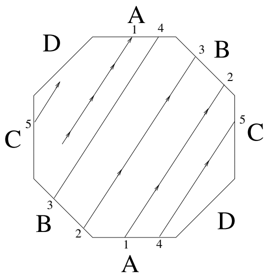

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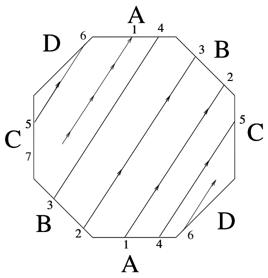

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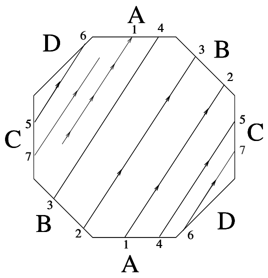

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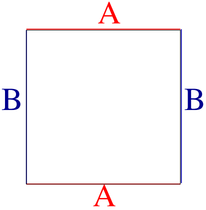
A classical case: Sturmian sequences

Consider the special case in which the polygon is a square.



In this case the cutting sequences are Sturmian sequences:

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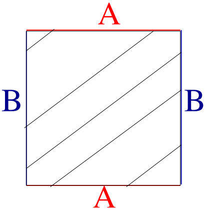

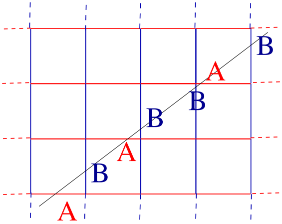

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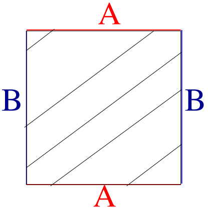

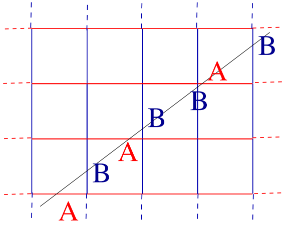

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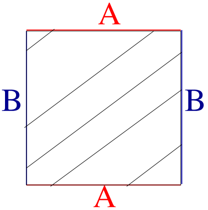

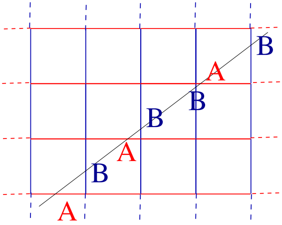

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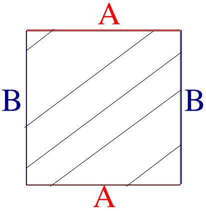

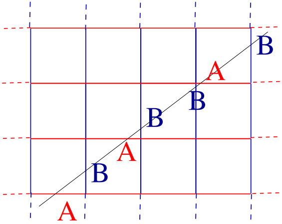

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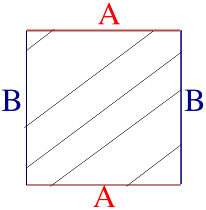

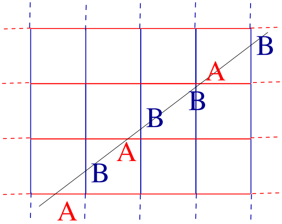

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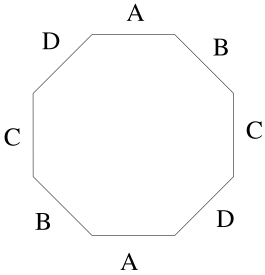

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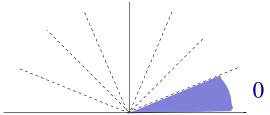

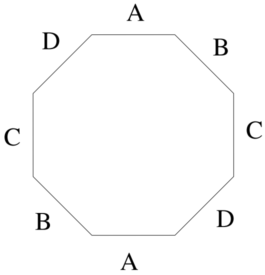

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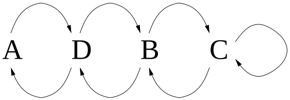

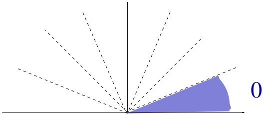

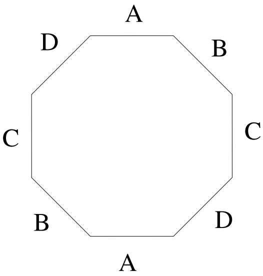

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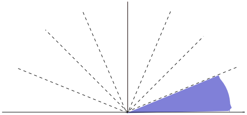

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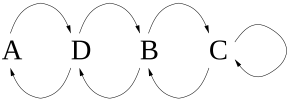

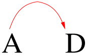

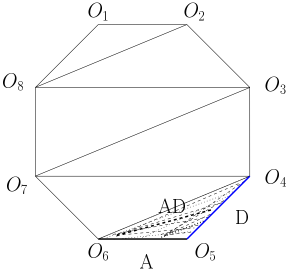

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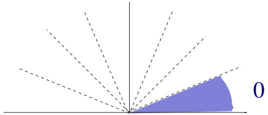

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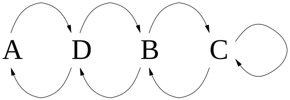

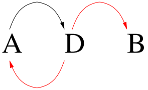

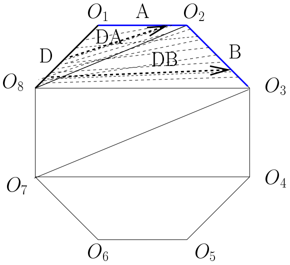

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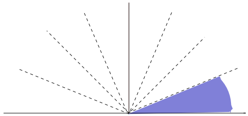

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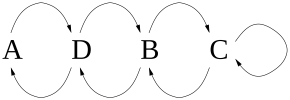

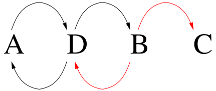

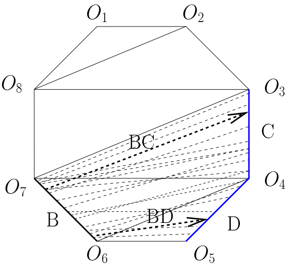

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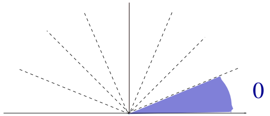

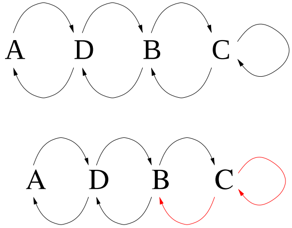

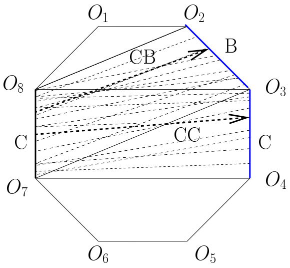

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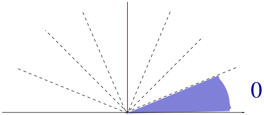

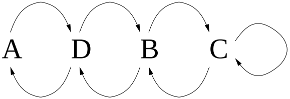

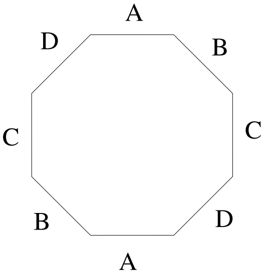

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A letter is sandwitched if it is preceeded and followed by the same letter, e.g. in D B B C B A A D the letter C is *sandwitched* between to Bs.

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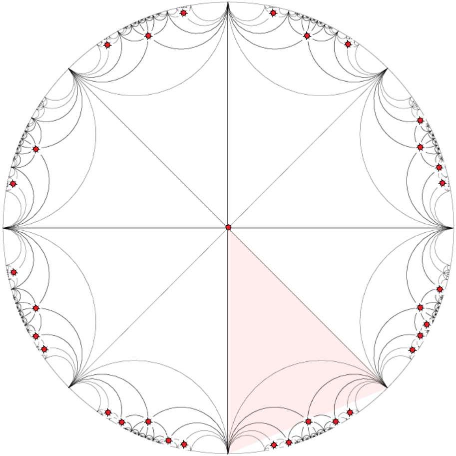
Renormalization and Teichmüller dynamics

The ideas in the proofs of the characterization of cutting sequences are inspired by tools in Teichmüller dynamics.



$SL(2,\mathbb{R})$ affine deformations of the octagon

$V_O < SL(2,\mathbb{R})$ Veech group of the octagon

Teichmueller disk

$\frac{SL(2,\mathbb{R})}{V(O)}$ $\frac{\text{affine deformations}}{\text{affini automorphisms}}$

Given θ, follow g_t^θ geodesic ray in direction θ to choose moves on a tree of affine deformations

Ergodic Theory and Dynamical Sytems in Bristol

The ergodic theory and dynamical systmes group in Bristol:

Jens Marklov

Alex Gorodnik

Corinna Ulcigrai

Thomas Jordan

Carl Dettmann

Post-docs: Felipe Ramirez, Shirali Kadyrov, Edward Crane, Alan Haynes (Heilbronn fellows)

Postgraduate students: Emek Demirci, Orestis Georgiou, Maxim Kirsebom, Riz Rahman, Henry Reeve, Oliver Sargent

Related groups: Number theory, Mathematical Physics, Quantum Chaos

Furthermore: Ergodic Theory and Dynamical Systems Seminar, Taught Course Center (with Bath, Imperial, Oxford, Warwick), Heibronn Institute

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Thomas
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Post-docs: Felipe Ramirez, Shirali Kadyrov,
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Dynamics, Number theory and quantum chaos

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Billiard with periodic scatterers of radius ϵ.

- What is the limit of the lenght of free flights as $\epsilon \longrightarrow 0$?
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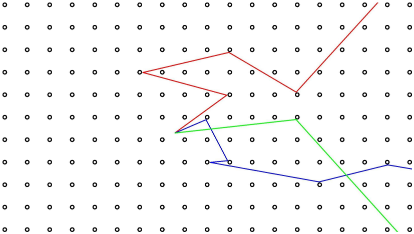

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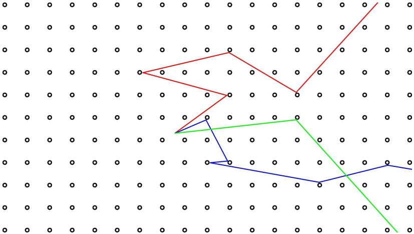

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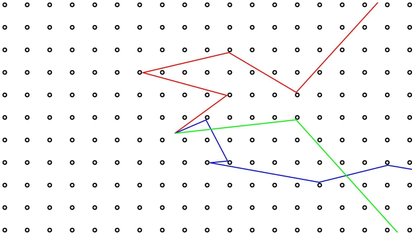

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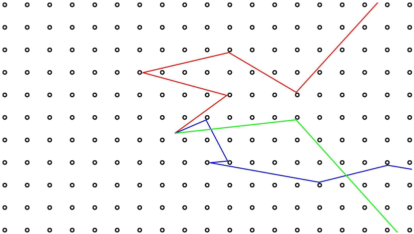

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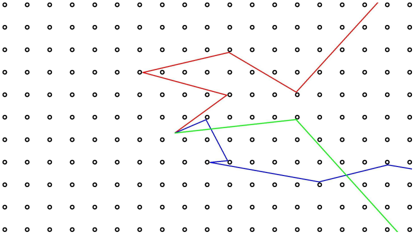

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Alex Gorodnik

Dynamics and Number theory

E.g. Counting Rational points on quadratic surfaces

Consider a quadratic equation $Q(x_1, x_2, x_3) = k$.

- Consider rational solutions, that is $Q\left(\frac{p_1}{q}, \frac{p_2}{q}, \frac{p_3}{q}\right) = k$.
- How many rational solutions with denominator $q \leq Q$, up to a given height H? how do they grow as Q and H grow?
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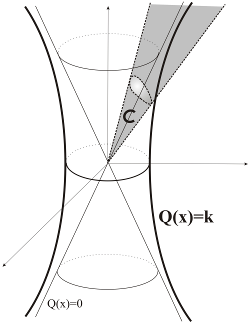

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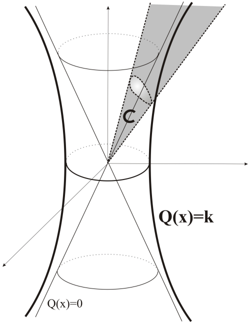

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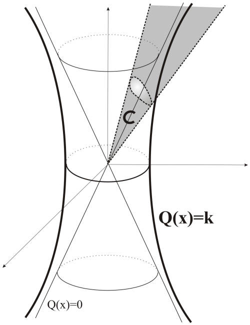

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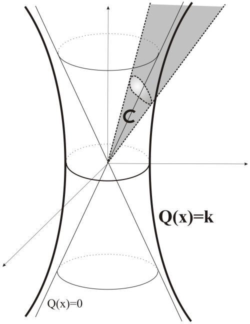

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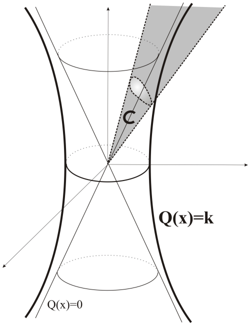

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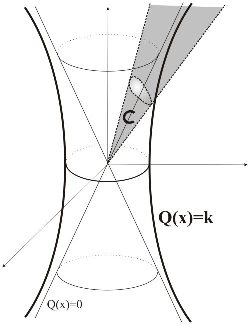

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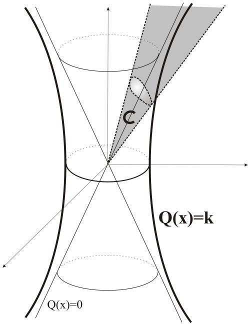

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Dimension theory of dynamical systems
(fractal sets and Hausdorff dimension)

E.g. Consider the fractal Bedford-McMullen carpet.

The (Hausdorff) fractal dimension is $\frac{\log(1+2(2^{\log 3/\log 5}))}{\log 3}$ (approximately 1.308)

- Q: What is the (Hausdorff) dimension of the orthogonal projections in different directions?

- A: 1 except for vertical projection when the dimension is $\log 4/\log 5$. [Joint work with Ferguson (Warwick) and Schmerkin (Manchester)]

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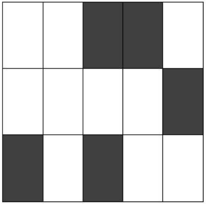

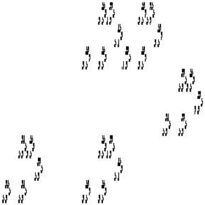

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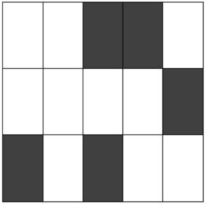

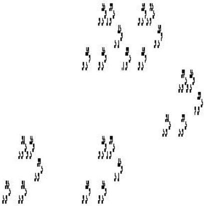

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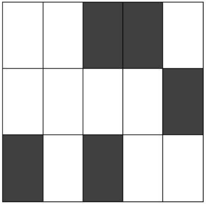

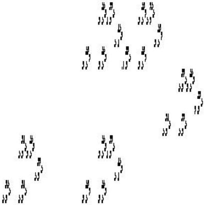

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Dynamical systems, billiards and simulations

E.g. Escape from billiards with holes

Consider a billiard with a *holes*.

- What is the escape rate of trajectories?
- What is the asymptotics as the hole size $\to 0$?

- In the case of the circle with two holes: the Riemann-Hypotheses is equivalent to the exact asymptotics in the small hole limit.

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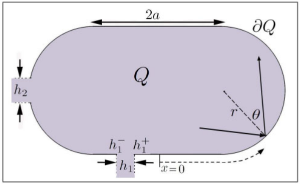

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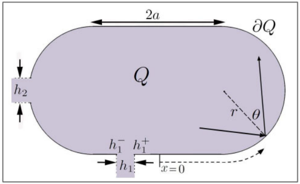

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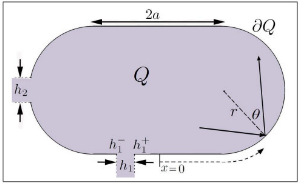

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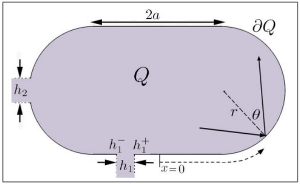

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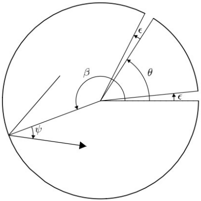

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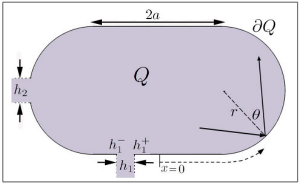

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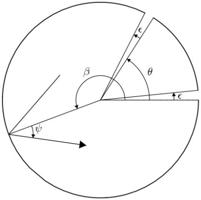

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University of Bristol

Thursday, 27th May 2010

- 1:30pm-2:30pm **Emmanuel Breuillard (Universite Paris-Sud 11)**

  Some applications of random matrix products

- 2:45pm - 3:45pm **Tom Ward (East Anglia)**

  Uniform distribution of periodic points

- 4:15pm - 5:15pm **Pablo Shmerkin (Manchester)**

  Thermodynamic formalism for the singular value function

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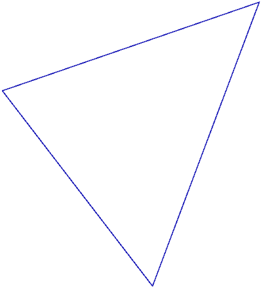
The following question, very simple to formule, has been long OPEN and has resisted many mathematicians' efforts:



Take an acute triangle,
there is a periodic trajectory
(the Fagnano trajectory)

Take an obtuse triangle:
is there a periodic trajectory?

Many questions about billiards with irrational angles are widely open ...

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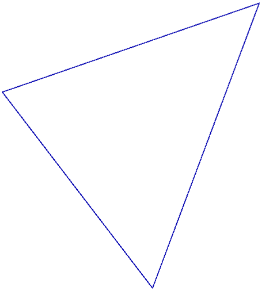

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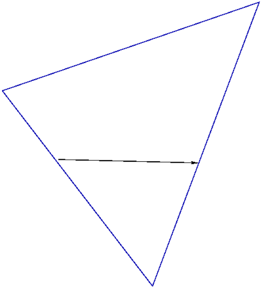

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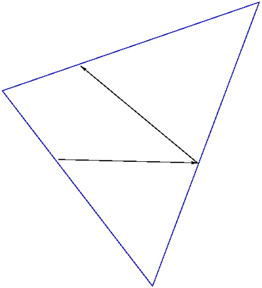

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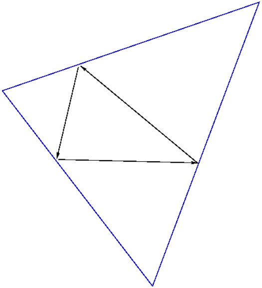

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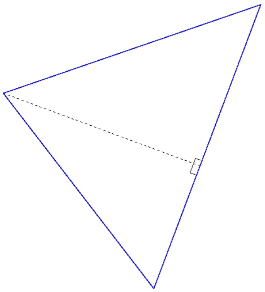

Take an acute triangle,
there is a periodic trajectory
(the Fagnano trajectory)

Take an obtuse triangle:
is there a periodic trajectory?

Many questions about billiards with irrational angles are widely open ...

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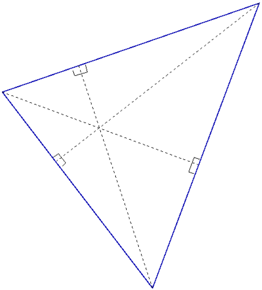

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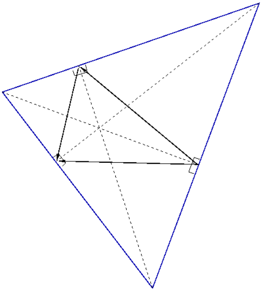

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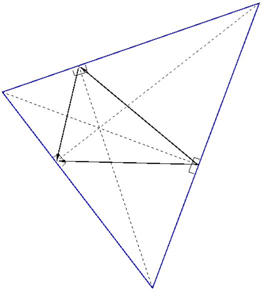

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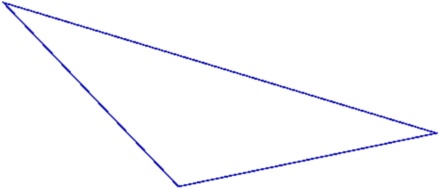

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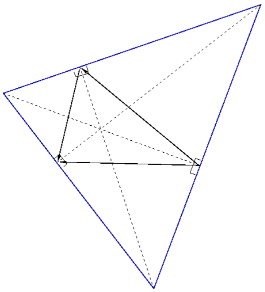

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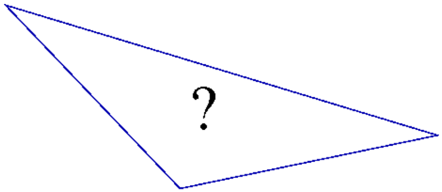

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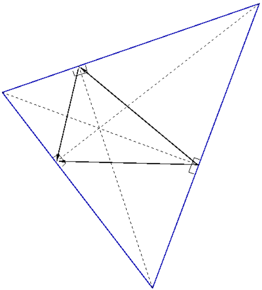

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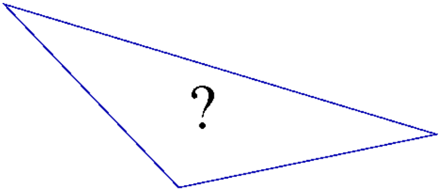

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