CMI-LMS Research School: Algebraic Topology of Manifolds

11-15 September 2017
Andrew Wiles Building, Mathematical Institute, Oxford, UK

Schedule

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
	Registration				
09:00-10:00	Embedding Calculus	Embedding Calculus	TQFT	Embedding Calculus	Embedding Calculus
coffee					
10:15-11:15	Moduli Spaces	Moduli Spaces	Moduli Spaces	Moduli Spaces	Moduli Spaces
coffee					
11:30-12:30	TQFT	TQFT	TQFT	Homology Stability	Homology Stability
lunch					
14:00-15:30	Problem Session	Problem Session	Problem Session	Problem Session	Problem Session
tea				Guest Lecture I	
16:00-17:00	TQFT	Participants Forum		tea	
17:00-18:00		Participants Forum		Guest Lecture II	
evening	*Supper in Somerville* *Ghost tour & drinks in pub*	*Reception*	*Punting and pub dinner*	*Supper in Somerville*	

Guest Lecturers: Soren Galatius (Stanford, USA, and Copenhagen, Denmark), Graeme Segal (Oxford, UK)

Tutors: Sam Nariman (NorthWestern, USA), Martin Palmer (Bonn, Germany), Claudia Scheimbauer (Oxford).
.
Organiser: Ulrike Tillmann

Rooms in the MI: All lectures will take place in L3. A break out room is available in C3.

Participants Forum: This is intended to serve primarily as a way of initiating mathematical discussions later during the week. Presentations will be kept brief (<5 min). We will start at 4pm.

Socials:
Ghost tour of Oxford at night with drinks in the Turf Tavern. Meet at 8pm at the Woodstock Road entrance to Somerville College for the tour. The tour will finish at the Turf Tavern at around 9:30pm. Punting or walk with dinner at the Victoria Arms (weather permitting).
Reception. This will take place immediately after the Participants Forum in the common room of the Mathematical Institute.
Punting and pub dinner. Meet the Mathematical Institute at 5pm to walk to the Cherwell Boat House where punts will be available. We will have dinner at the Victoria Arms at around 6:30pm.

Accommodation: Somerville College, Oxford OX2 6HD. Check-in from 2pm on Sunday and check-out by 10am on Friday.

Lunches: On Monday and Tuesday participants will receive vouchers to spend at the cafeteria in the Mathematical Institute; on Wednesday, Thursday and Friday a bag lunch will be provided.

Suppers: On Monday and Thursday night a buffet supper will be served in Somerville College 6pm-7pm. Please note that these times are strict.

Lecture Courses

Lecture course 1. Topological Quantum Field Theory (5 lectures)
Syllabus: Basic concepts of TQFT and examples, cobordism theory, Baez-Dolan cobordism hypothesis, invertible theories and anomalies, extended field theories.
Lecturer: **Dan Freed (**Austin, USA). Dan is well-know for his work at the interface of mathematics and physics and spoke at the 2002 ICM. He has received many honours and prizes, including the LMS Senior Berwick Prize in 2014 with Mike Hopkins and Constantin Teleman for their celebrated joint work on twisted K-theory and loop groups.
Preliminary reading/prerequesites:
Aiyah's original paper *Topological quantum field theories* (1988) is a good starting point. Students might also find it helpful to dip into Dan's survey article *The cobordism hypothesis*, https://arxiv.org/abs/1210.5100.

Lecture course 2. Characteristic classes and moduli spaces of manifolds (5 lectures)
Syllabus: The classifying space BDiff(M) of the group of diffeomorphisms of a manifold M classifies fibre bundles with fibre M, and hence its cohomology may be naturally interpreted as the ring of characteristic classes of such fibre bundles. There is a modern and quite oblique way to get access to the cohomology of such spaces after suitably stabilising the manifold M. (Homological stability theorems say that such stabilisation is innocuous, in a range of homological degrees.) The first example is the Madsen--Weiss theorem, which concerns the case where M is an orientable surface which is stabilised by connect-sum with tori. More recently, Galatius and I have completed a programme establishing an analogous result for manifolds of any even dimension.
In these lectures I will give an overview of this programme, including detailed formulations of the main results, worked examples, and the ideas behind some proofs.
Lecturer: **Oscar Randal-Williams** (Cambridge, UK). Despite his relative short career, Oscar has established himself as one of the leaders in the field. Galatius presented their joint work at the 2014 ICM.
Preliminary reading/prerequesites:
To follow the statements of the main results: familiarity with the basics of differential topology, and algebraic topology at the level of Hatcher's book.
To follow the worked examples: the above along with the Serre spectral sequence and characteristic classes of vector bundles at the level of Milnor--Stasheff.
To follow the proofs: further differential topology up to the notion of handle structures and doing surgery (but _not_ anything so involved as the surgery exact sequence), and some comfort with (semi-)simplicial constructions (everything we will use may be found e.g. in arXiv:1705.03774).

Lecture course 3. The Goodwillie-Weiss embedding calculus (4 lectures)
Syllabus: Linear functors, immersions, Smale-Hirsch theory. Application: sphere eversion theorem. Polynomial functors and polynomial approximations. The Taylor tower. Description of the layers. Connection with the little discs operad and an applications of its formality. Applications to (high-dimensional) long knots. Some open problems.
Lecturer: **Greg Arone** (Stockholm, Sweden). A former student of Goodwillie, Greg is one of a few experts worldwide who has published papers studying both the functor calculus and the embedding calculus.
Preliminary reading/prerequesites:
M. Weiss, *Embeddings from the point of view of immersion theory*: Part I. Geom. Topol. 3 (1999), 67–101.
P. Boavida de Brito and M. Weiss, *Manifold calculus and homotopy sheaves.* Homology Homotopy Appl. 15 (2013), no. 2, 361–383.
Topics that will be strongly present in the background, so familiarity with them is desirable: Homotopy limits, operads.

Lecture course 4. Homology stability (2 lectures)
Syllabus: Homology stability of groups, categorical frame work, examples.
Lecturer: **Nathalie Wahl** (Copenhagen, Denmark). Nathalie is well-known in particular for her work on homology stability. She has received many awards, including an ERC starting grant.
Preliminary reading/prerequesites:
A little bit of group homology and the spectral sequences associated to a double complex would be good to have looked at before my lectures. A good reference is Brown's book "Cohomology of groups".