Computational Algebraic Topology

HT 2013

Overview: Ideas and tools from algebraic topology have become more and more important in computational and applied areas of mathematics. This course will provide an introduction to the main concepts of (co)homology theory, and explore areas of applications in data analysis and computing at the graduate level.

Learning outcomes: Students should gain a working knowledge of homology and cohomology of simplicial sets and sheaves, and improve their geometric intuition. Furthermore, they should gain an awareness of a variety of application in rather different, research active fields of applications.

Prerequisites: The course will provide a self-contained, rapid introduction to cohomology theory for simplicial sets and sheaves. However, some familiarity with concepts from topology and homological algebra will be of help. It should be noted that MFoCS students may offer both this course and the Section A course 'Algebraic Topology' in Michaelmas Term for examination. Those offering only this course and with no familiarity with these topics would be well advised to at least sit in on the Section A course 'Algebraic Topology'.

Lecturers: Ulrike Tillmann (Lectures 1-8: Core and Topic A), Samson Abramsky (Lectures 9-14: Topics B and C).

Synopsis:
The course has two parts. The first part will introduce students to the basic concepts and results of (co)homology, in particular sheaf cohomology. In the second part applied topics are introduced and explored.

Core:
Homology and cohomology of chain complexes. Chain homotopy. Snake Lemma. Simplicial set. Mayer-Vietoris sequence. Acyclic carriers. (2 lectures)
Sheaves, presheaves and axiomatic sheaf cohomology. Classical cohomology theories: Alexander-Spanier cohomology, de Rham cohomology, singular cohomology, Cech cohomology. The de Rahm theorem and multiplicative structures. (4 lectures)

Topic A: Persistent homology: barcodes and stability, applications to data analysis. (2 lectures)

Topic B: Applications to distributed computing. (3 lectures)

Topic C: Sheaf cohomology and applications to quantum non-locality and contextuality.

Sheaf-theoretic representation of quantum non-locality and contextuality as obstructions to global sections. Cohomological characterizations and proofs of contextuality. (3 lectures)

Learning support: There will be two problem session covering the core material.

Reading list:

F. Warner, Foundations of Differential Manifolds and Lie Groups, Springer GTM 94 (1971). Chapter 5.

H. Edelsbrunner and J.L. Harer, Computational Topology - An Introduction, AMS (2010).

T. Kaczynski, K. Mischaikow, M. Mrozek, Computational Homology, Springer (2004).

See also, U. Tillmann, Lecture notes for CAT 2012,
http://people.maths.ox.ac.uk/tillmann/CAT.html

Topic A:

G. Carlsson, *Topology and data*, Bulletin A.M.S. 46 (2009), 255-308.

H. Edelsbrunner, J.L. Harer, *Persistent homology: A survey*, Contemporary Mathematics 452 A.M.S. (2008), 257-282.

S. Weinberger, *What is ... Persistent Homology?*, Notices A.M.S. 58 (2011), 36-39.

Topic B:

M. Herlihy, S. Rajsbaum, *Algebraic topology and distributed computing a primer*, Computer science today (1995), 203–217.

M. Herlihy, S. Rajsbaum, *Algebraic spans*, Mathematical Structures in Computer Science **10** (2000), 549 – 573.

Topic C:

S. Abramsky and Adam Brandenburger, The Sheaf-Theoretic Structure Of Non-Locality and Contextuality. In *New Journal of Physics*, 13(2011), 113036, 2011.

S. Abramsky and L. Hardy, Logical Bell Inequalities, Phys. Rev. A 85, 062114 (2012).

S. Abramsky, S. Mansfield and R. Soares Barbosa, The Cohomology of Non-Locality and Contextuality, in *Proceedings of Quantum Physics and Logic 2011*, Electronic Proceedings in Theoretical Computer Science, vol. 95, pages 1–15, 2012.

Schedules of lectures and problem sessions

Week 1:
No lectures; please attend lectures on homological algebra.

Week 2:
Lectures: Thursday 12 in L3, Friday 10 in SR1, Friday 5 in L3

Week 3:
Lectures: Thursday 12 in L3

Week 4:
Lectures: Thursday 12 and Friday 5 in L3
Problem session: Friday 10 in SR1

Week 5:
Lectures: Thursday 12 and Friday 5 in L3

Week 6:
Lectures: Thursday 12 and Friday 5 in L3
Problem session: Friday 10 in SR1

Week 7-8:
Lectures: Thursday 12 and Friday 5 in L3