

OAC-manifolds

Inaugural Meeting
Oxford, 22 February 2016



Time	Speaker	Location
	Registration	Reception
09:00-10:00	Oscar Randal-Williams (Cambridge)	L6
	Coffee	
10:15-10:40	Csaba Nagy (Aberdeen)	L6
10:45-11:10	Mark Penney (Oxford)	L6
11:15-11:40	Nina Friedrich (Cambridge)	L6
	Coffee	
12:00-13:00	Andre Henriques (Oxford)	L1
	Lunch	
14:15-15:15	Diarmuid Crowley (Aberdeen)	L4
	Tea	
15:45-16:45	Richard Hepworth (Aberdeen)	L6

Postgraduate Morning

Oscar Randal-Williams: *Characteristic classes of fibre bundles*

Abstract: For a smooth manifold M , the classifying space $B\text{Diff}(M)$ of its topological group of diffeomorphisms is an important object of study, both within differential topology and for applications to geometry. In particular, the cohomology of this space may be identified with the ring of characteristic classes for smooth fibre bundles with fibre M . I will give an overview of the techniques which go into the study of such spaces, and of the types of results which have so far been obtained.

Csaba Nagy: *Classifying certain 8-manifolds*

Abstract: I will consider simply-connected spin 8-manifolds M with the homology of an r -fold connected sum of $S^2 \times S^6$. The set of diffeomorphism classes of such manifolds forms a finitely generated abelian group, $\Theta(r)$.

In this talk I will sketch a proof that $\Theta(r)$ is isomorphic to other groups of geometric interest, specifically mapping class groups of certain 7-manifolds and also groups of framed links studied by Haefliger. I will also report on the computation of the rank of $\Theta(r)$ and properties of its torsion subgroup.

Mark Penney: *Hopf categories from categorified Hall algebras.*

Abstract: It has been known since the early '90s that Hopf algebras determine 3D TFTs. In that same decade Crane and Frenkel proposed that 4D TFTs may be determined by so-called Hopf categories, which are linear categories having compatible monoidal and comonoidal structures. Unfortunately this proposal faltered due to the lack of examples of such categories, a problem which has not been remedied until recently.

In this talk I will present a construction which takes as input an abelian category (satisfying certain finiteness properties) and produces a Hopf category. It is based on a categorification of the Hall algebra of an abelian category. I will conclude the talk by describing the Hopf category resulting from the abelian category of finite dimensional vector spaces over a finite field.

Nina Friedrich: *Homological Stability of Moduli Spaces of High Dimensional Manifolds*

We will first explain how to translate homological stability in the geometric setting of moduli spaces of high dimensional manifolds to the algebraic setting of quadratic forms. For simply-connected manifolds, Galatius and Randal-Williams have shown that certain simplicial complexes arising on the algebraic side are highly connected, and hence deduced homological stability theorems for moduli spaces of simply-connected manifolds. We generalise this to a much larger class of manifolds (those having virtually polycyclic fundamental group).

Afternoon Programme

Andre Henriques: *A model for $K(\mathbb{Z},4)$ and a conjecture about elliptic curves*

Abstract: The group $PU(H)=U(H)/U(1)$ of projective unitary transformations of a Hilbert space is a $K(\mathbb{Z},2)$, and its classifying space is therefore a $K(\mathbb{Z},3)$. That group is best understood as the automorphism group of the algebra $B(H)$ of bounded operators on a Hilbert space. Its action on the subset $\text{Fred}(H) \subset B(H)$ of Fredholm operators is responsible for the existence of twistings of K -theory by classes in $H^3(-, \mathbb{Z})$.

I will present a higher categorical analogue of $B(H)$, called $\text{Bim}(R)$. $\text{Bim}(R)$ is a certain tensor category and it is an instance of what I call a bicommutant category.

The automorphism group of $\text{Bim}(R)$ is, I conjecture, a $K(\mathbb{Z},3)$, which would make its classifying space a $K(\mathbb{Z},4)$. I will present some compelling evidence for this claim. This picture fits well with the fact that elliptic cohomology admits twists by classes in $H^4(-, \mathbb{Z})$, and an idea of mine that there should be a model of elliptic cohomology involving certain "categorified Fredholm operators" (this is totally speculative: I can define these categorified Fredholm operators, but I don't know how to prove anything about them).

I will finish by presenting a concrete conjecture that comes out of these speculations, about elliptic curves over number fields, and which does not involve any topology or analysis.

Diarmuid Crowley: *The Gromoll filtration, Toda brackets and positive scalar curvature*

Abstract: An exotic $(n+1)$ -sphere has disc of origin D^k if k is the smallest integer such that some clutching diffeomorphism of the n -disc which builds the exotic sphere can be written as an $(n-k)$ -parameter family of diffeomorphisms of the k -disc. In this talk I will present a new method for constructing exotic spheres with small disc of origin via Toda brackets.

This method gives exotic spheres in all dimensions $8j+1$ and $8j+2$ with disc of origin 6 and with Dirac operators of non-zero index (such spheres are often called "Hitchin spheres").

I will also briefly discuss implications of our results for the space of positive scalar curvature metrics on spin manifolds of dimension 6 and higher, and in particular the relationship of this project to the work of Botvinnik, Ebert and Randal-Williams.

This is part of joint work with Thomas Schick and Wolfgang Steimle.

Richard Hepworth: *Homological Stability for Families of Coxeter Groups*

Abstract: A series of groups and homomorphisms $G_0 \rightarrow G_1 \rightarrow G_2 \dots$ is said to satisfy homological stability if in any fixed degree d the sequence of homology groups

$H_d(BG_0) \rightarrow H_d(BG_1) \rightarrow H_d(BG_2) \rightarrow \dots$ eventually consists of isomorphisms. Homological stability holds for symmetric groups, general linear groups, mapping class groups, and many many more. In almost all cases we do not know the homology groups themselves. In this talk I will explain a stability result for certain families of Coxeter groups. Examples include the A_n , BC_n and D_n families (all finite), the superideal simplex reflection groups (all hyperbolic) and many others.

Organiser: Ulrike Tillmann



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