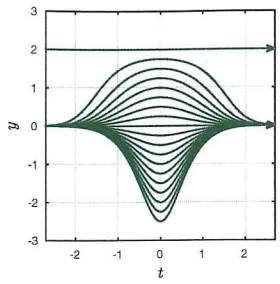
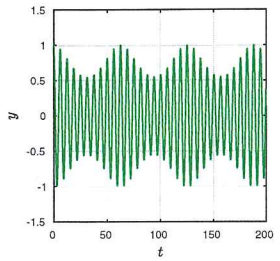
Appendix B. 100 more examples



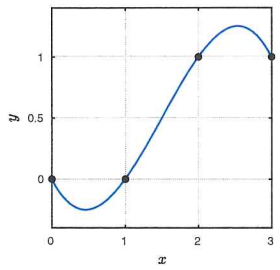
1. Equation with two steady states (Davis, p. 30) [Chap. 3]

```
d = [0 2.7]; N = chebop(@(t,y) diff(y)-t*y*(y-2),d);
t = chebfun('t',d);
for a = -2.5:.25:2
  N.lbc = a; y = N\0;
  plot(-flipud(t),flipud(y)), hold on
  arrowplot(t,y+1e-10)
end
```



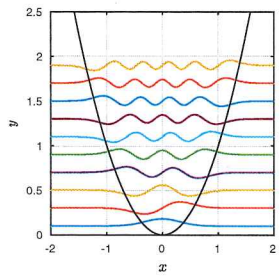
2. Alternating growth and decay [Chap. 4]

```
L = chebop(@(t,y) diff(y,2)+.06*sin(t/10)*diff(y)+y,[0 200]);
L.lbc = [1;0]; y = L\0; plot(y)
```



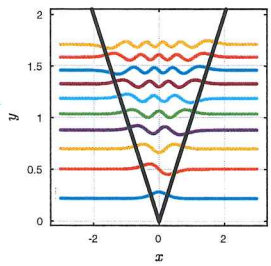
3. Beam with four interpolation points [Chap. 5]

```
L = chebop(@(y) diff(y,4),[0 3]);
L.bc = @(x,y) [y(0); y(1); y(2)-1; y(3)-1];
y = L\0; plot(y)
hold on, plot(0:3,[0 0 1 1],'.k')
```



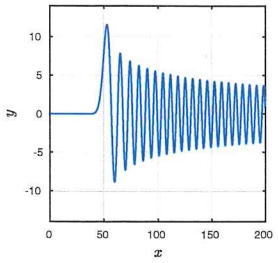
4. Harmonic oscillator for Schrödinger equation [Chap. 6]

```
x = chebfun('x',[-3 3]); V = x^2; h = 0.1;
L = chebop(@(y) -h^2*diff(y,2)+V*y,[-3 3]);
L.bc = 0; [W,D] = eigs(L,10);
for k = 1:10, plot(D(k,k)+0.06*W{k}), hold on, end
plot(V,'k')
```



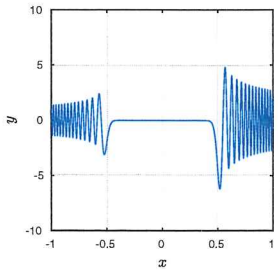
5. Harmonic oscillator with V-shaped potential [Chap. 6]

```
x = chebfun('x',[-3 3]); V = abs(x); quantumstates(V)
```



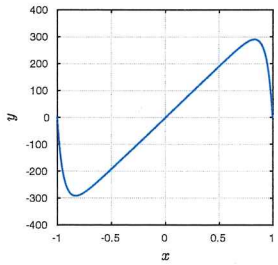
6. Bessel equation [Chap. 7]

```
nu = 50;
L = chebop(@(x,y) x^2*diff(y,2)+x*diff(y)+(x^2-nu^2)*y,[0 200]);
L.lbc = 0; L.rbc = 1; y = L\0; plot(y)
```



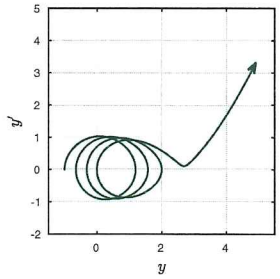
7. Example from Lee and Greengard [Chap. 7]

```
L = chebop(@(x,y) 1e-5*diff(y,2)+(x^2-0.25)*y,[-1,1]);
L.lbc = 1; L.rbc = 2; y = L\0; plot(y)
```



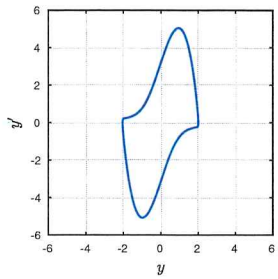
8. Nearly ill-posed problem [Chap. 7]

```
x = chebfun('x',[-1 1]);
L = chebop(@(x,y) -.05*diff(y,2)+x*diff(y)-y,[-1 1]);
f = chebfun('exp(x)'); L.lbc = 0; L.rbc = 0; y = L\f;
plot(y)
```



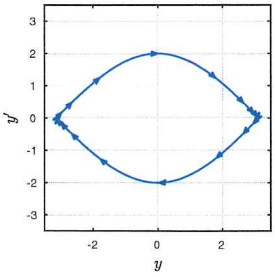
9. Duffing equation (Davis p. 399) [Chap. 8]

```
d = [0 30]; t = chebfun('t',d);
N = chebop(@(y) diff(y,2)+y-.088*y^3,d); N.lbc = [-1; 0];
y = N\sin(0.05*t); arrowplot(y,diff(y))
```



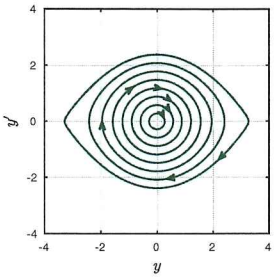
10. Van der Pol limit cycle [Chap. 9]

```
T = 16;
N = chebop(@(y) diff(y,2)-3*(1-y^2)*diff(y)+y,[0,T]);
N.lbc = [1;1];
y = N\0; y = y{4,T}; plot(y,diff(y))
```



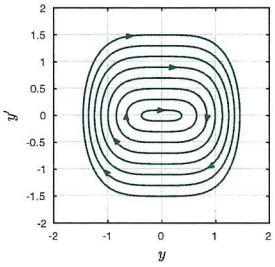
11. Nonlinear pendulum heteroclinic paths [Chap. 9]

```
N = chebop(@(y) diff(y,2)+sin(y),[0 21]);
N.lbc = [-3.1;0];
y = N\0; arrowplot(y,diff(y),'multi',20)
```



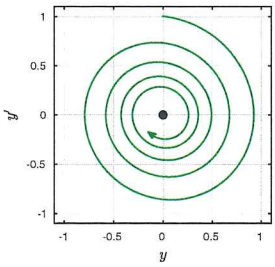
12. Unforced Duffing equation, phase portrait [Chap. 9]

```
N = chebop(@(y) diff(y,2)+y-.088*y^3,[0 20]);
for a = 0.28:.3:2.6
  N.lbc = [0; a]; y = N\0;
  arrowplot(y,diff(y)), hold on
end
```



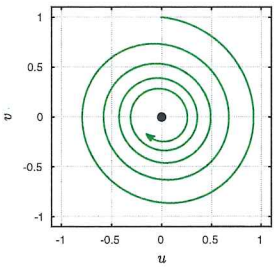
13. Cubic oscillator [Chap. 9]

```
N = chebop(@(t,y) diff(y,2)+y^3,[0 20]);
for a = 0.1:.2:1.5
  N.lbc = [0; a];
  y = N\0; arrowplot(y,diff(y)), hold on
end
```



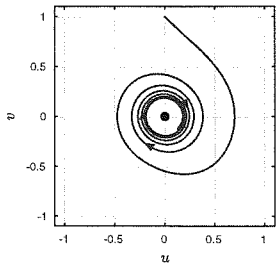
14. Stable spiral point (scalar second-order) [Chap. 9]

```
L = chebop(@(y) diff(y,2)+.1*diff(y,1)+y,[0 29]);
L.lbc = [0; 1]; y = L\0; arrowplot(y,diff(y))
ylim([-1 1.1]), hold on, plot(0,0,'.k')
```



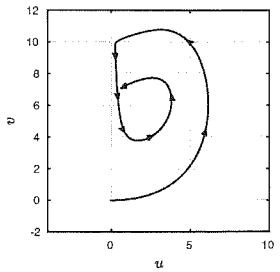
15. Stable spiral point (first-order system) [Chap. 10]

```
L = chebop(@(t,u,v) [diff(u)-v; diff(v)+u+.1*v],[0 29]);
L.lbc = @(u,v) [u;v-1]; [u,v] = L\0;
arrowplot(u,v)
ylim([-1 1.1]), hold on, plot(0,0,'.k')
```



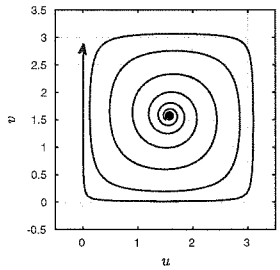
16. Cubic damping (Duan p. 6) [Chap. 10]

```
L = chebop(@(t,u,v) [diff(u)-v; diff(v)+u+v^3],[0 40]);
L.lbc = @(u,v) [u;v-1]; [u,v] = L\0;
arrowplot(u,v,'multi',4)
ylim([-1 1.1]), hold on, plot(0,0,'.k')
```



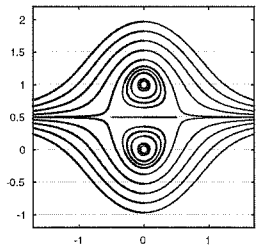
17. Oscillatory system (Hirsch-Smale-Devaney p. 229) [Chap. 10]

```
a = 10; b = 1;
N = chebop(@(t,u,v) [diff(u)-a+u+4*u*v/(1+u^2); ...
                     diff(v)-b*u*(1-v/(1+u^2))],[0 10]);
N.lbc = @(u,v) [u; v]; [u,v] = N\0;
arrowplot(u,v,'multi',8)
```



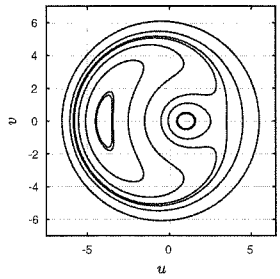
18. Square limit cycle (Hirsch-Smale-Devaney p. 215) [Chap. 10]

```
N = chebop(@(t,u,v) [diff(u)+sin(u)*(.1*cos(u)+cos(v)); ...
                     diff(v)+sin(v)*(.1*cos(v)-cos(u))],[0,60]);
N.lbc = @(u,v) [u-1.5; v-1.5]; [u,v] = N\0;
arrowplot(u,v)
hold on, plot(pi/2,pi/2,'.k')
```



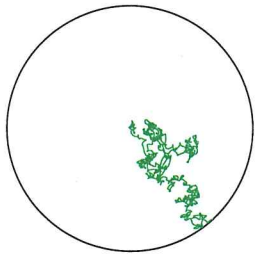
19. Four fixed points (Davis p. 345) [Chap. 10]

```
N = chebop(@(t,u,v) [diff(u)+v-u^2-v^2; diff(v)-u+2*u*v],[0 5]);
N.maxnorm = [1.7 3];
for v0 = .5 + .147*(-10:10)
  N.lbc = @(u,v) [u; v-v0]; [u,v] = N\0;
  plot(u,v), hold on, plot(-u,v)
end
```



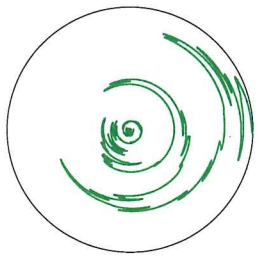
20. Hamiltonian system (Jordan-Smith, p. 79) [Chap. 10]

```
N = chebop(0,2);
N.op = @(t,u,v) [diff(u)-v*(13-u^2-v^2); diff(v)-12+u*(13-u^2-v^2)];
for a = -4.5:5.5
  N.lbc = @(u,v) [u-a; v-.2];
  [u,v] = N\0; plot(u,v), hold on
end
```



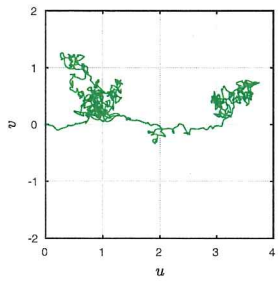
21. Smooth Brownian motion to the circle [Chap. 12]

```
d = [0 2]; lam = 2e-3;
rng(0), f = randnfun(lam,d,'norm') + 1i*randnfun(lam,d,'norm');
L = chebop(@(y) diff(y),d); L.lbc = 0; L.maxnorm = 1;
plot(chebfun('exp(1i*pi*x)'),'k'), hold on
y = L\f; plot(y)
```



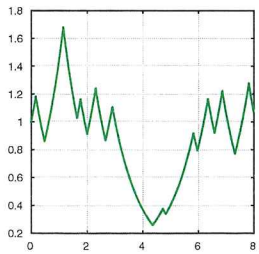
22. Complex decay with sidewise randomness [Chap. 12]

```
d = [0 40]; lam = 0.1;
rng(0), f = randnfun(lam,d,'norm');
L = chebop(@(y) diff(y) + (.3 + 3i*f)*y,d); L.lbc = 1;
plot(chebfun('exp(1i*pi*x)'),'k'), hold on
y = L\0; plot(y)
```



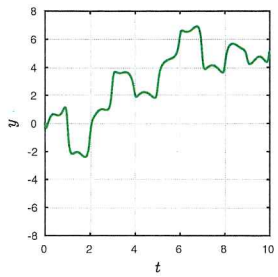
23. Conveyor belt [Chap. 12]

```
d = [0 4]; lam = 0.01;
rng(0), f = randnfun(lam,d,'norm'); g = randnfun(lam,d,'norm');
L = chebop(@(t,u,v) [diff(u)-16*exp(-100*v^2); diff(v)],d);
L.lbc = @(u,v) [u; v];
[u,v] = L\[f;g]; plot(u,v)
```



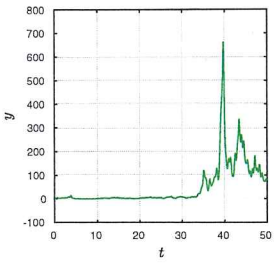
24. Equation with random ±1 forcing [Chap. 12]

```
rng(0), f = sign(randnfun(.5,[0,8],'norm'));
L = chebop(@(t,y) diff(y)-f*y,[0 8]);
L.lbc = 1; y = L\0; plot(y)
```



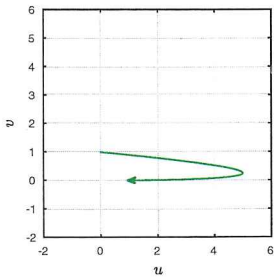
25. Equation with intermittent kicks [Chap. 12]

```
dom = [0,10]; t = chebfun('t',dom);
rng(0), f = randnfun(.5,dom,'norm')/(1.1-cos(2*pi*t));
L = chebop(@(t,y) diff(y)-f,dom);
L.lbc = 0; y = L\0; plot(y)
```



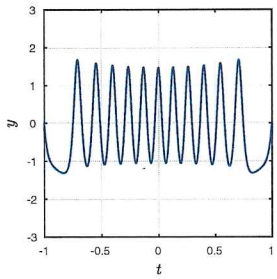
26. Smooth geometric Brownian motion [Chap. 12]

```
d = [0 50]; lam = 0.5;
rng(0), f = randnfun(lam,d,'norm');
L = chebop(@(y) diff(y)-f*y,d); L.lbc = 1;
y = L\f; plot(y)
```



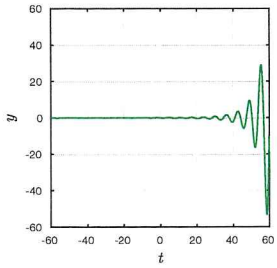
27. Linear transient growth [Chap. 15]

```
L = chebop(@(t,u,v) [diff(u)+u-20*v; diff(v)+2*v],[0 3]);
L.lbc = @(u,v) [u;v-1];
[u,v] = L\0; arrowplot(u,v)
fs = 8; lw = 0.7;
```



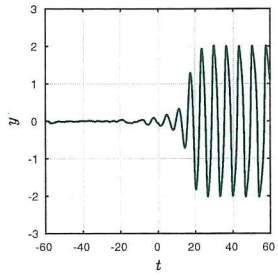
28. Carrier equation [Chap. 16]

```
N = chebop(@(x,y) 0.001*diff(y,2)+2*(1-x^2)*y+y^2,[-1 1]);
N.lbc = 0; N.rbc = 0; x = chebfun('x');
y = N\1; plot(y)
```



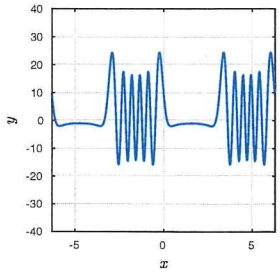
29. Hopf bifurcation linear [Chap. 17]

```
T = 60; dom = [-T,T];
L = chebop(@(t,y) diff(y,2)-0.4*(t/T)*diff(y)+y,dom);
L.lbc = [0;0]; rng(0), f = 0.05*randnfun(dom,'norm');
y = L\f; plot(y)
```



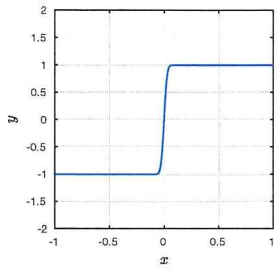
30. Hopf bifurcation van der Pol eq. [Chap. 17]

```
T = 60; dom = [-T,T];
N = chebop(@(t,y) diff(y,2)-2*(t/T)*(1-y^2)*diff(y)+y,dom);
N.lbc = [0;0]; rng(0), f = 0.05*randnfun(dom,'norm');
y = N\f; plot(y)
```



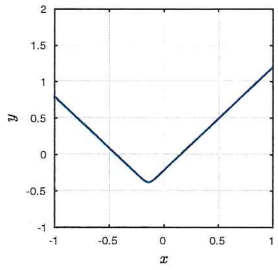
31. Periodic variant of Fig. 7.5 [Chap. 19]

```
L = chebop(@(x,y) 0.005*diff(y,2)-sin(x)*y,[-2*pi 2*pi]);
L.bc = 'periodic'; y = L\1; plot(y), ylim([-40 40])
```



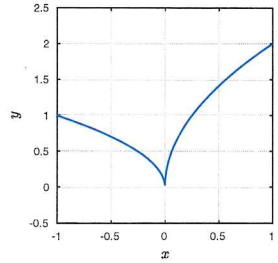
32. Linear shock from Lee & Greengard [Chap. 20]

```
L = chebop(@(x,y) 0.001*diff(y,2) + 2*x*diff(y),[-1 1]);
L.lbc = -1; L.rbc = 1; y = L\0; plot(y)
```



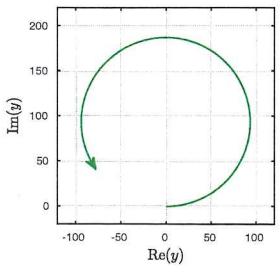
33. Nonlinear interior layer [Chap. 20]

```
N = chebop(@(y) 0.05*diff(y,2)+diff(y)^2-1,[-1 1]);
N.lbc = 0.8; N.rbc = 1.2; y = N\1; plot(y)
```



34. Cusp example from Lee and Greengard [Chap. 20]

```
L = chebop(@(x,y) 1e-6*diff(y,2)+x*diff(y)-.5*y,[-1,0,1]);
L.lbc = 1; L.rbc = 2; y = L\0; plot(y)
```



35. Blowup eq. with complex perturbation [Chap. 21]

```
d = [0 1.01]; N = chebop(@(y) diff(y)-y^2,d);
N.lbc = 1; rng(0); f = 0.01i*randnfun(d,'norm');
y = N\f; arrowplot(y)
```