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Lecture 5. Minimax, CF, and Hankel norm approximation

1. Minimax

"Best" = "minimax" = "Chebyshev" = L^∞

Degree m real polynomial approx on an interval: best ⇔ equioscillation of $(f-p)(x)$ between $\ge m+2$ extrema.
Type (m,n) real rational approx on an interval: best ⇔ equioscillation of $(f-r)(x)$ between $\ge m+n+2-\delta$ extrema.
Here r has exact type (μ,ν) and $\delta=\min(m-\mu,n-\nu)$ is the defect.

Padé approxs have analogous characterization based on $(f-r)(z) = O(z^{m+n+1-\delta})$
⇒ the Padé and Walsh tables (best approxs) break into square blocks of identical entries.

These approximations became important in engineering with the arrival of digital signal processing in the 1970s.
Polynomial = FIR = finite impulse response, rational = IIR = infinite impulse response.

Rational approxs much more powerful than polynomial for functions with singularities or near-singularities.

Computation: Exchange algorithm (Remez 1934 for polynomial, Werner 1962/Mahely 1963 for rational).
An alternative is differential correction (Cheney-Loeb 1961), slower but better theory. Or use CF approx.

For complex approximation, use Tang's Remez generalization for polynomial (Tang 1988) or AAA-Lawson
for rational (`Chebfun aaa(F,Z,'lawson',nsteps)`.

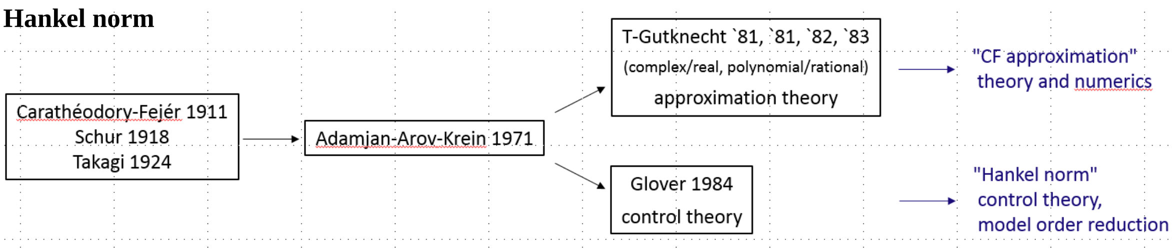
2. CF (= Carathéodory-Fejér)

Only near-best, in theory, but for smooth functions, often matches the true best approx to machine precision.
Derived from SVD of Hankel matrix of Taylor coeffs (complex, unit disk) or Chebyshev coeffs (real, $[-1,1]$).

Chebfun: `cf` (this code due to Joris van Deun).

See chap. 20 of *Approximation Theory and Approximation Practice* for an introduction and the notes below ✓.

3. Hankel norm



CF approximation	Hankel norm approximation
approximation theory	linear algebra
scalars	matrices
approximate a function f by a function r	approx a high-order transfer function by a low-order one
polynomial degree m or rational type (m,n)	rational type $(n-1,n)$ or (n,n)
complex or real (Taylor, Chebyshev)	complex (unit disk, left half-plane)
software: cf in Chebfun	software: hankmr in System & Control Toolbox; SLICOT
means to an end (namely, minimax)	an end in itself? (not always clear)
focus on fundamentals, mathematics problems	focus on applications, engineering problems
connections with Padé, Remez, LP	connections with balanced truncation, rational interpolation and least-squares, Lyapunov and Riccati equations
small literature	big literature
see T, *Approximation Theory and Approximation Practice*	see Zhou-Doyle-Glover, *Robust and Optimal Control* and Antoulas, *Approximation of Large-Scale Dynamical Systems*