Basal processes and geomorphology

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Sediments and sliding

- Till rheology
- Deformation

Drainage in sediments

- Darcy flow
- Canals

Geomorphology

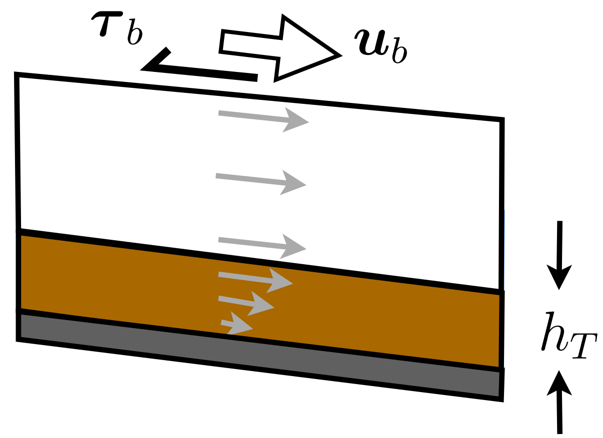
- Meltwater deposits
- Deformational deposits

Sediments and sliding

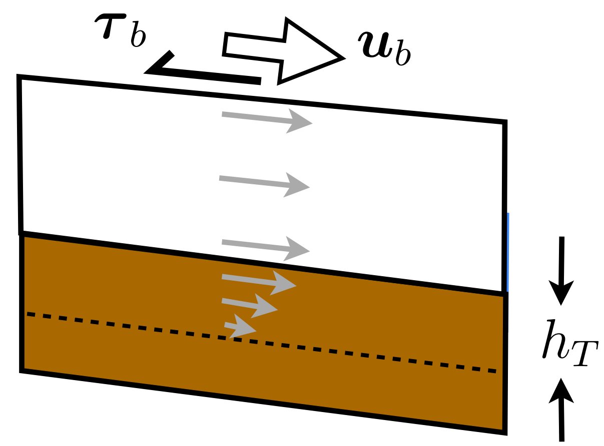
Sliding over sediments

'Sliding' could involve:

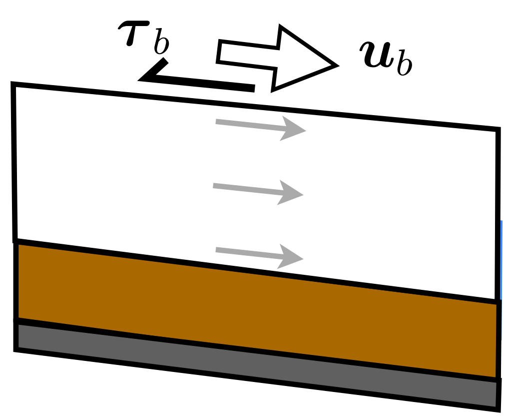
- Shear deformation of sediment layer



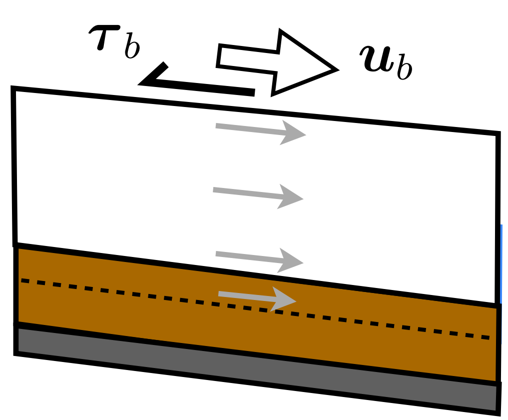
- Shear of a finite horizon of the sediment

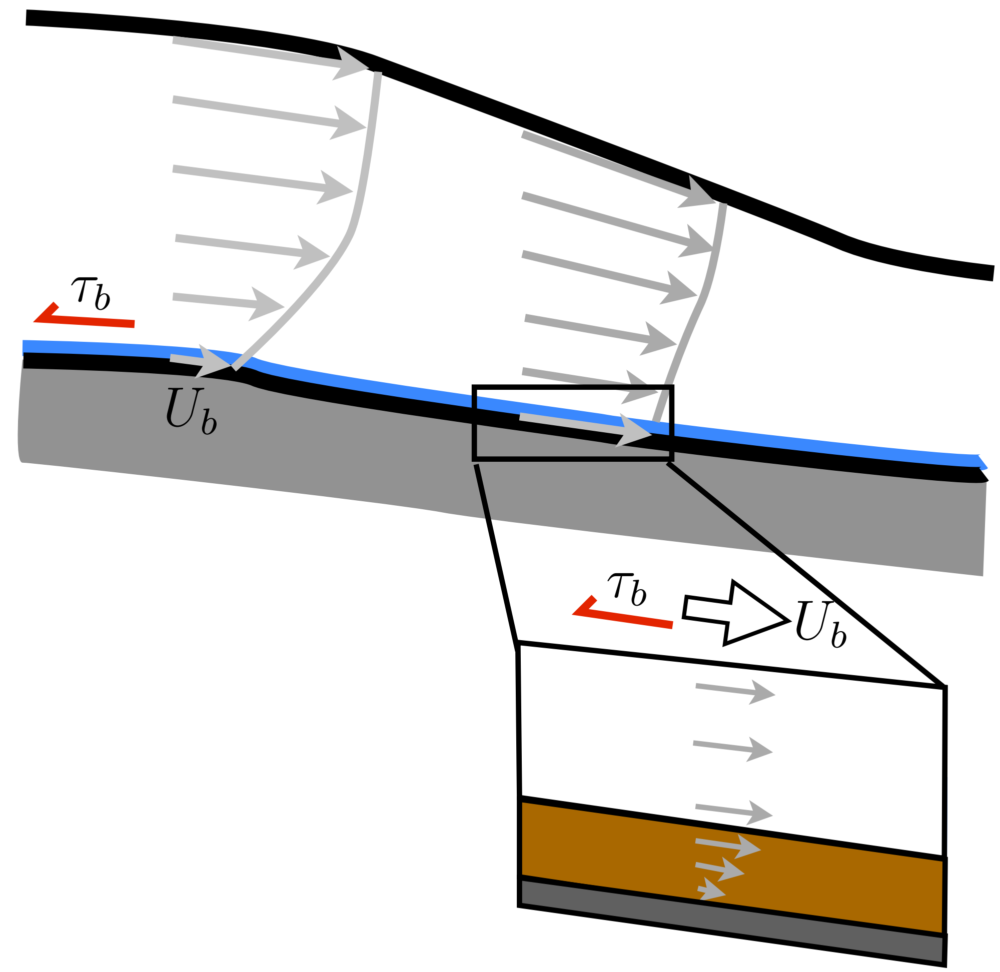


- Slip at the ice-till interface

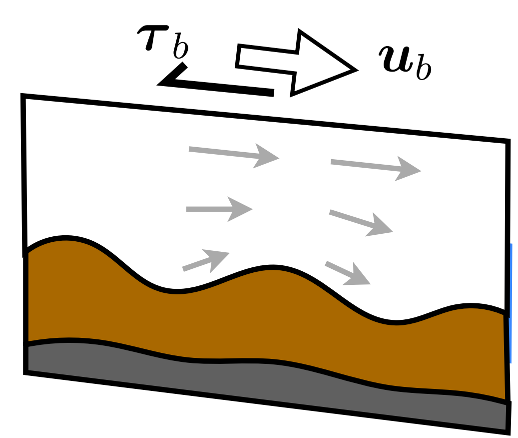


- Slip on slip-planes within the sediment layer





Macroscopic resistance may come from flow around sediment landforms



Till rheology

Hooke & Iverson 1998, Kamb 1991, Iverson 2011, Iverson & Zoet 2015

Laboratory experiments on samples show that till has a **yield stress**

$$\tau_f = \cancel{c_0} + \mu\sigma_e$$

Yield stress depends on **effective stress**

$$\sigma_e = P - p_w \approx N$$

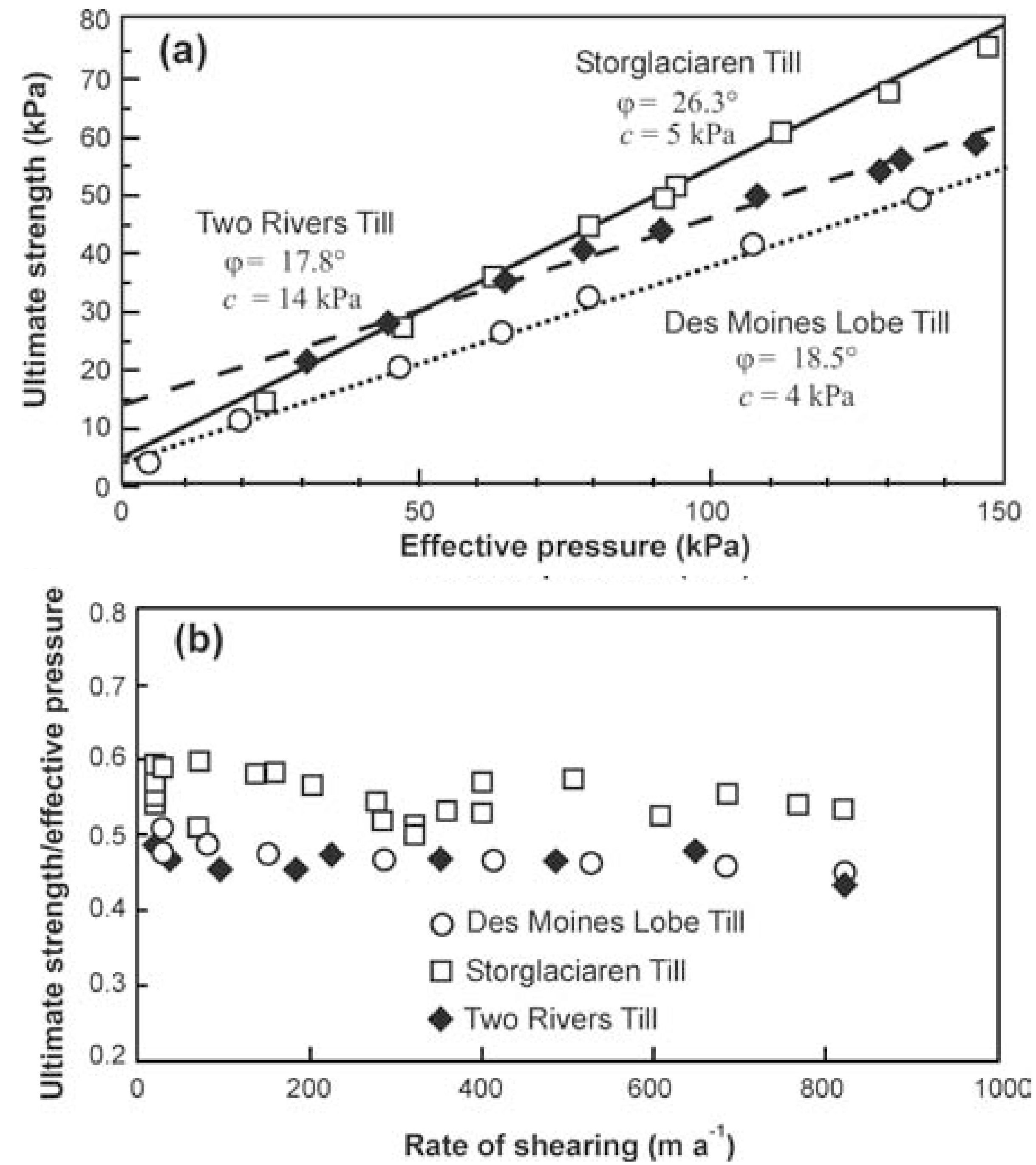
↑ Effective pressure at ice-till interface

(effective stress increases with depth into the till - it is weakest at the top).

$\mu = \tan\psi \approx 0.4$ Coefficient of friction

$c_0 \approx 3\,\mathrm{kPa}$ Cohesion

Experiments suggest **stress is almost independent of strain rate** (i.e. perfectly plastic).

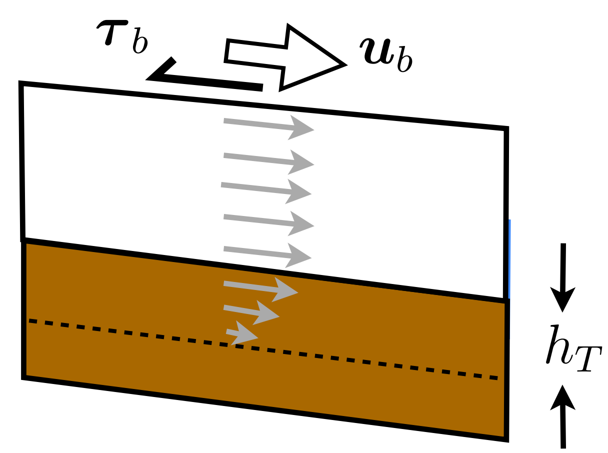


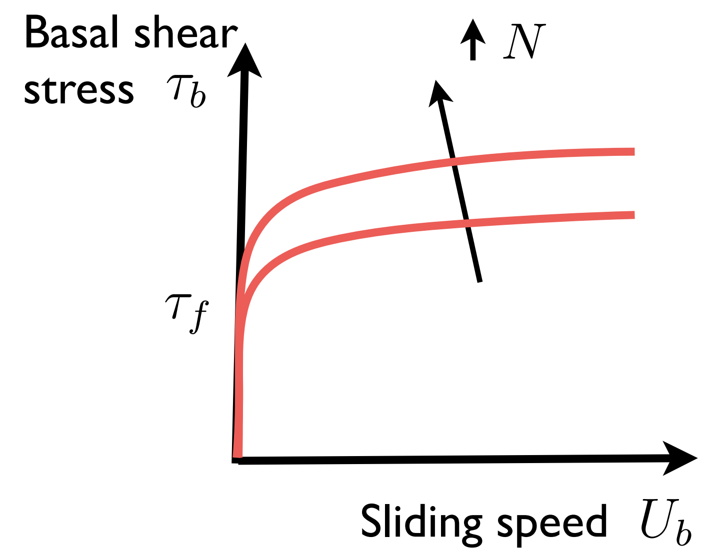
Iverson et al 1998, Clarke 2005

Sliding over till

Viscous rheology

$$\dot{\varepsilon} = A(\tau - \tau_f)^a \sigma_e^{-b} \qquad \tau \geq \tau_f = \mu \sigma_e$$



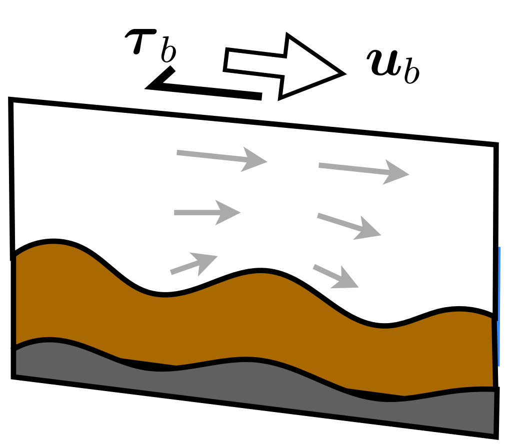


Pore water pressure roughly hydrostatic

⇨ Effective stress increases with depth through till $\sigma_e = N + \Delta\rho_{sw} g (Z_b - z)$

Deformation only if $\sigma_e \leq \tau_b/\mu$ ⇨ Deforming horizon $h_T = [\tau_b - \mu N]_+ / \mu \Delta\rho_{sw} g$

⇨ Sliding law $\tau_b = \mu N + C U_b^{1/a} N^{b/a}$

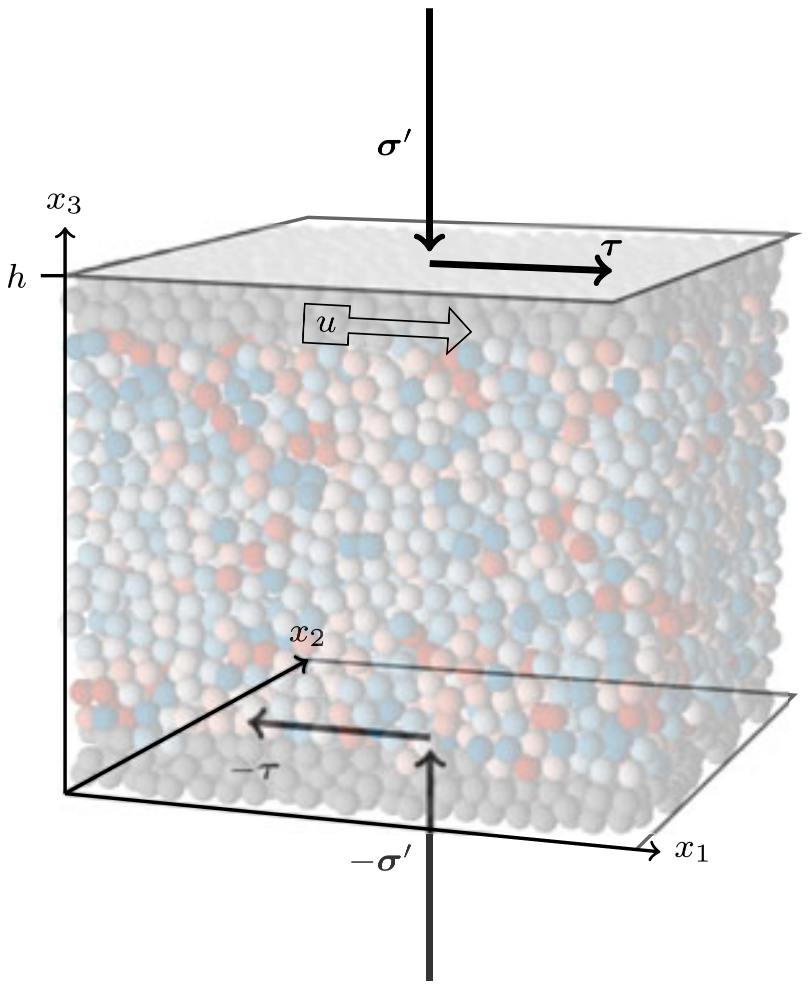


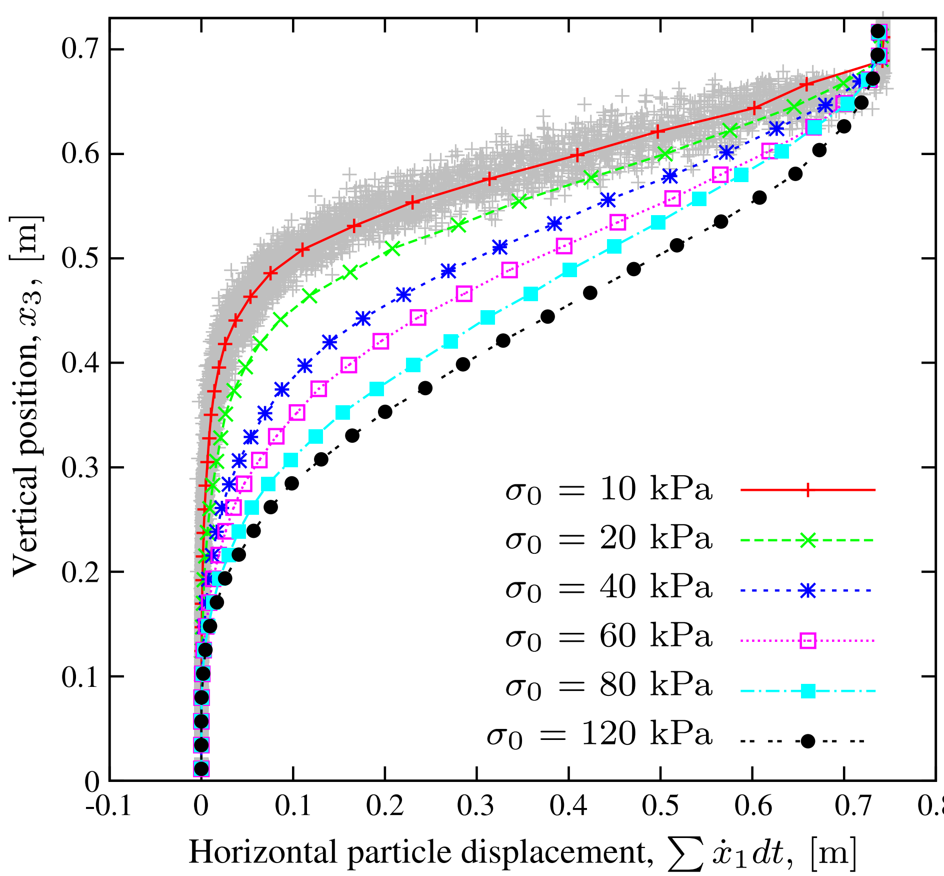
A similar law applies to describe ice flow over sediments with topography

$$\tau_b = \mu N + R U_b^{1/n}$$

Computational experiments

Discrete particle (DEM) experiments under imposed shearing velocity

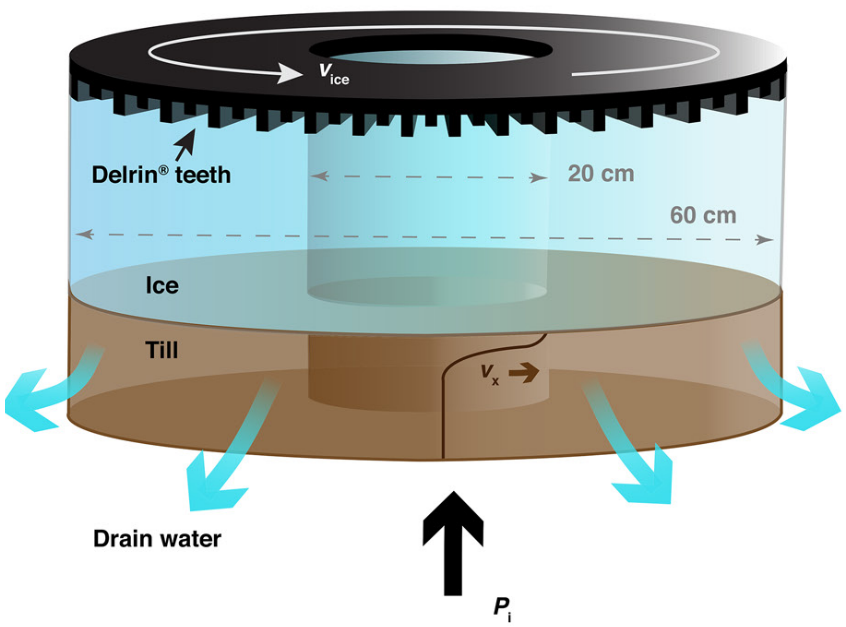


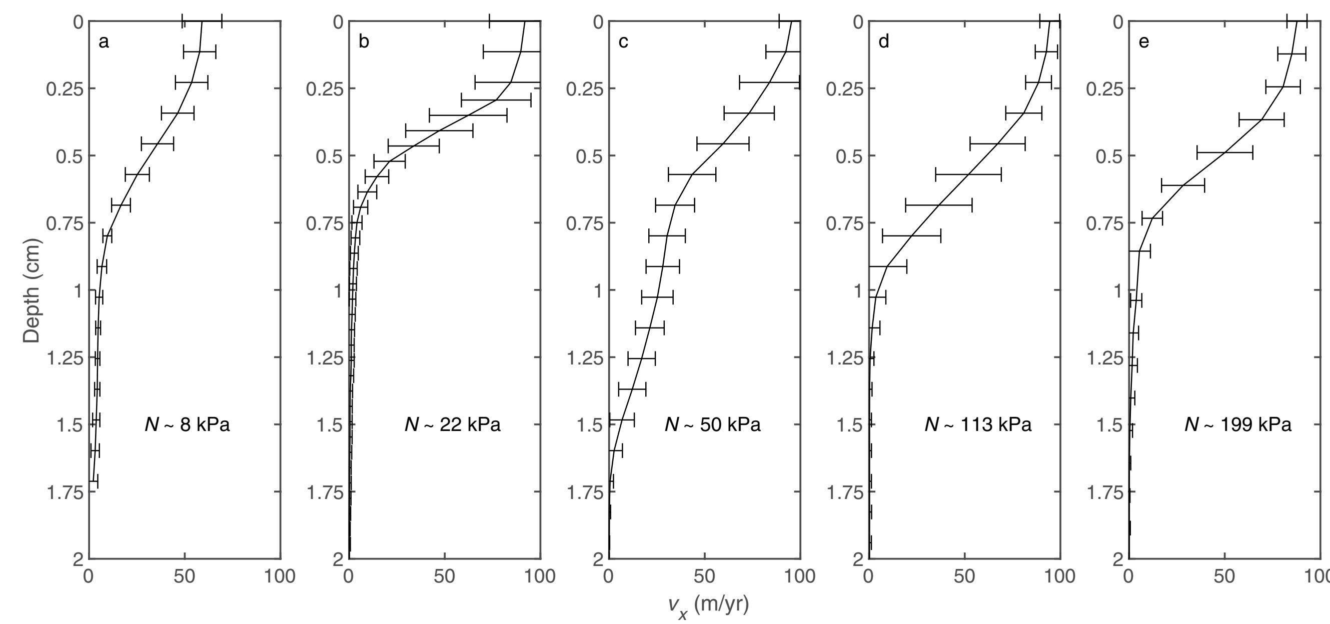


Damsgaard et al 2013

Laboratory experiments

Laboratory ring shear experiment visualise till deformation, sediment flux, and ice-till slip



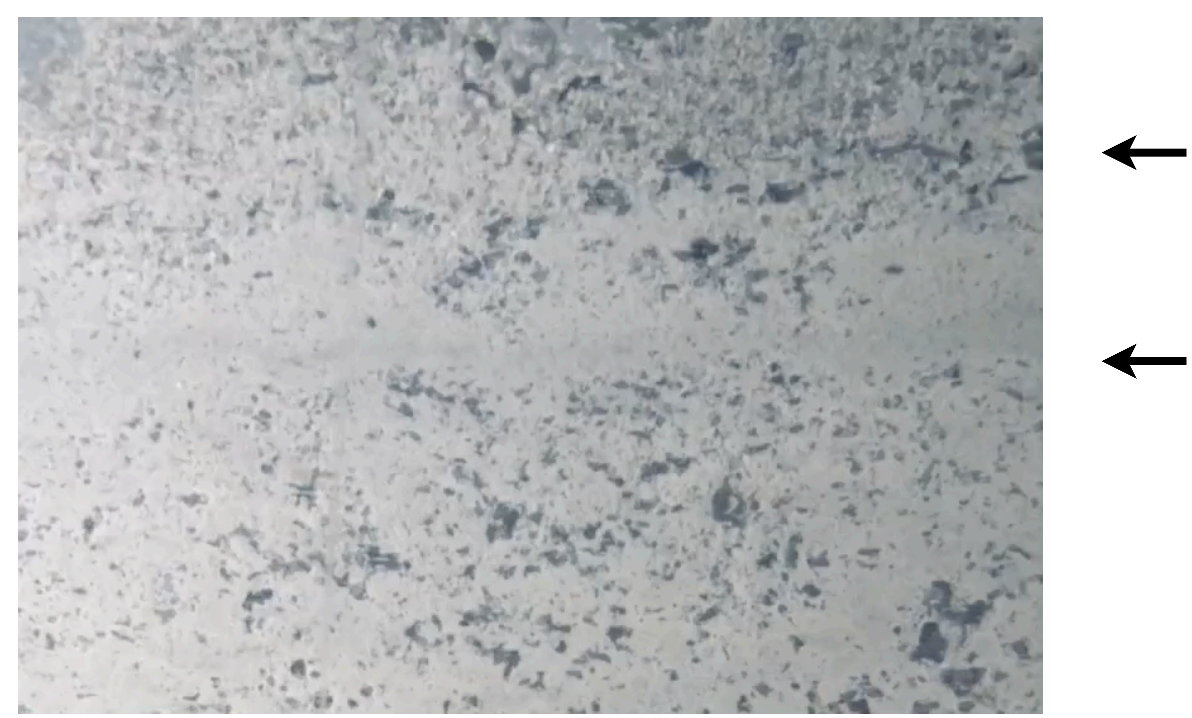


Hansen & Zoet 2022

Ice-till slip occurs at low effective pressure / low sliding speeds.

Depth of deformation increases then decreases with effective pressure.

Sediment flux scales approximately **linearly with sliding speed**, and **non-monotonically** with effective pressure.



Drainage in sediments

Drainage through till

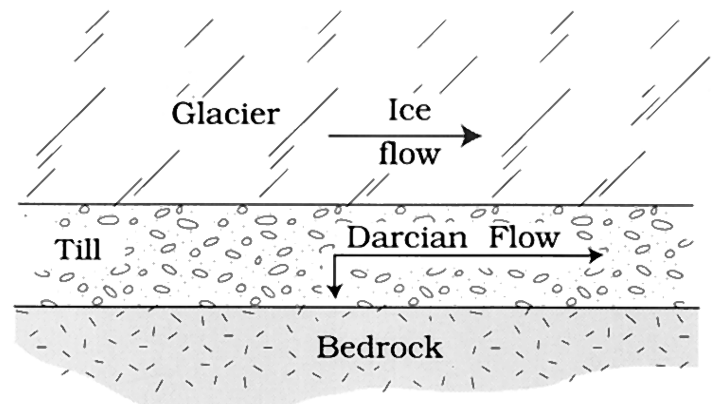
Estimates of **hydraulic conductivity** vary, but it is generally thought to be low.

Although water seeps **vertically** into the till, **horizontal** transport **through** the till is most likely **insufficient** to evacuate the water produced from melting.

eg. in Antarctica

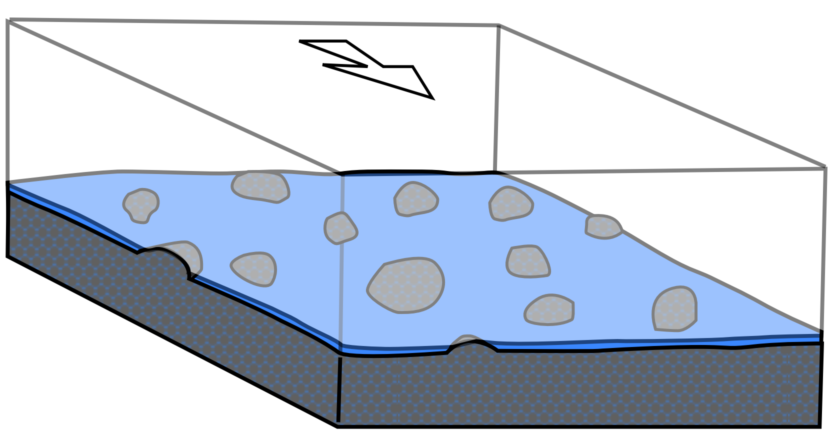
$$q = \frac{K h_T}{\rho_w g}\nabla\phi \approx \frac{10^{-7}\cdot\ 10}{10^3\cdot\ 10}\cdot\ 10\ \ \frac{\mathrm{m\,s^{-1}\ m}}{\mathrm{kg\,m^{-3}\ m\,s^{-2}}}\frac{\mathrm{Pa}}{\mathrm{m}} \approx 10^{-10}\ \mathrm{m^2\,s^{-1}}$$

$$\int m\,\mathrm{d}x \approx 5\cdot\ 10^3\ \ \mathrm{mm\ \ y^{-1}\ km} \approx 1.6\times 10^{-4}\ \mathrm{m^2\,s^{-1}}$$



Fountain & Walder 1998

⇨ Water flows in a **patchy film** at the ice-till interface, or in some form of **channels or canals**.

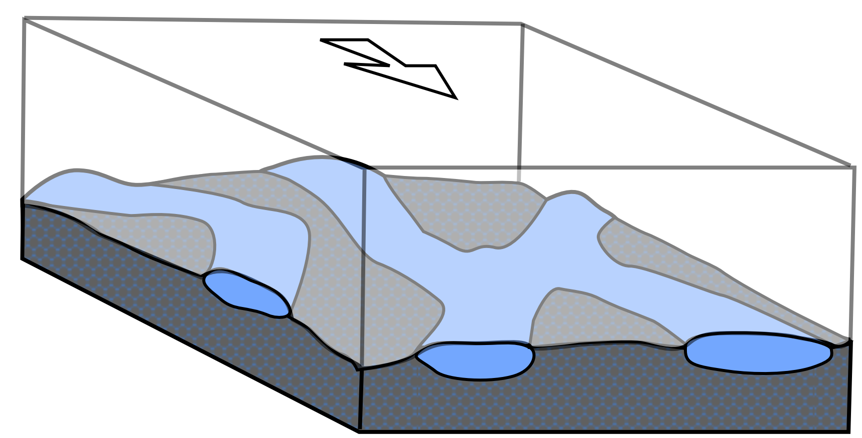


Patchy sheet

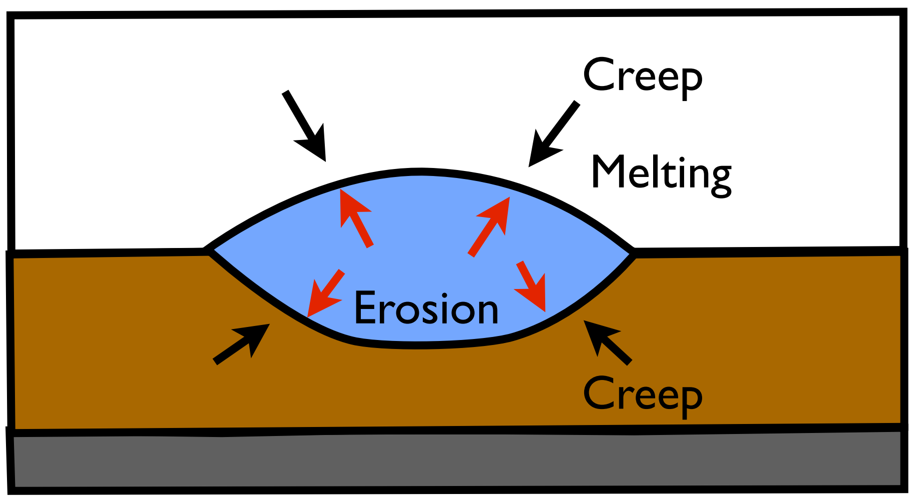
Alley 1989, Creyts & Schoof 2009

Canals

Walder & Fowler 1994



Canals
Walder & Fowler 1994, Ng 2000



Gravitational potential gradient

$$\Psi = \rho_i g \tan\alpha + (\rho_w - \rho_i) g \tan\theta$$

Walder & Fowler suggested two possibilities for steady states:

Channels - mostly melted into ice	$N \propto \Psi^{7/15} Q^{1/15}$	$N > \tilde{N}$
Canals - mostly eroded into sediment	$N \propto \Psi^{-1/3} Q^{-1/3}$	$N < \tilde{N}$

$\tilde{N} \approx 0.8\ \mathrm{MPa}$

⇨ Effective pressure in canals DECREASES with increasing discharge Q (so might expect distributed system)

The crucial difference seems to be that erosion tends to produce a **wide cross-section**.

Canals are favoured when the potential gradient is small (e.g. interior of ice sheets).

Interaction of sliding and drainage

A consequence of $\frac{\partial N}{\partial Q} < 0$ is the potential for a **positive feedback**

Initiation of sliding $\uparrow U_b$

→ Increased melting $\uparrow m \propto \tau_b U_b / L$

→ Increased discharge $\uparrow Q$

→ Lower effective pressure $\downarrow N$

Positive feedback

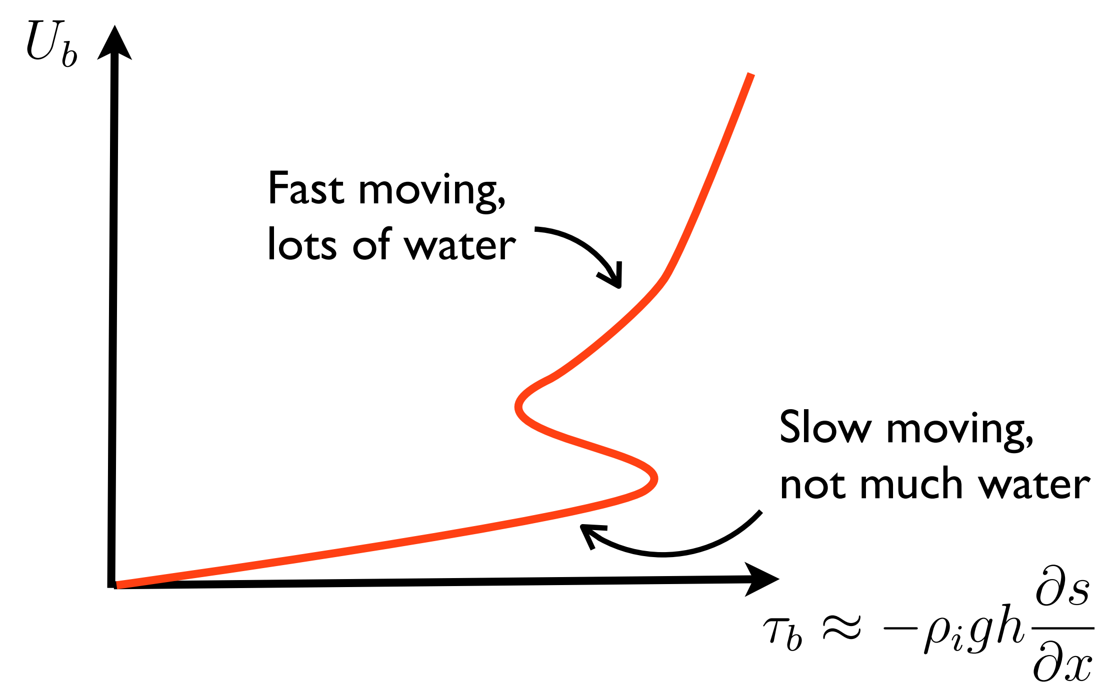
Model

$$Q = \frac{G + \tau_b U_b - k U_b^{1/2}}{\rho_w L} A$$

$$N = c / Q^{1/3}$$

$$\tau_b = C U_b^p N^q$$

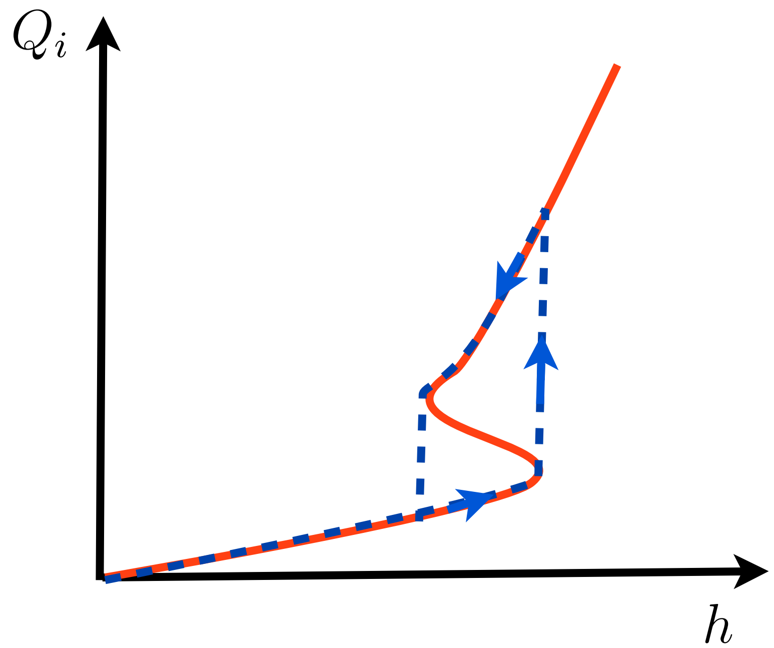
⇨



The relationship between ice thickness and speed can become **multivalued**

Surges and ice streaming

Temporal variability



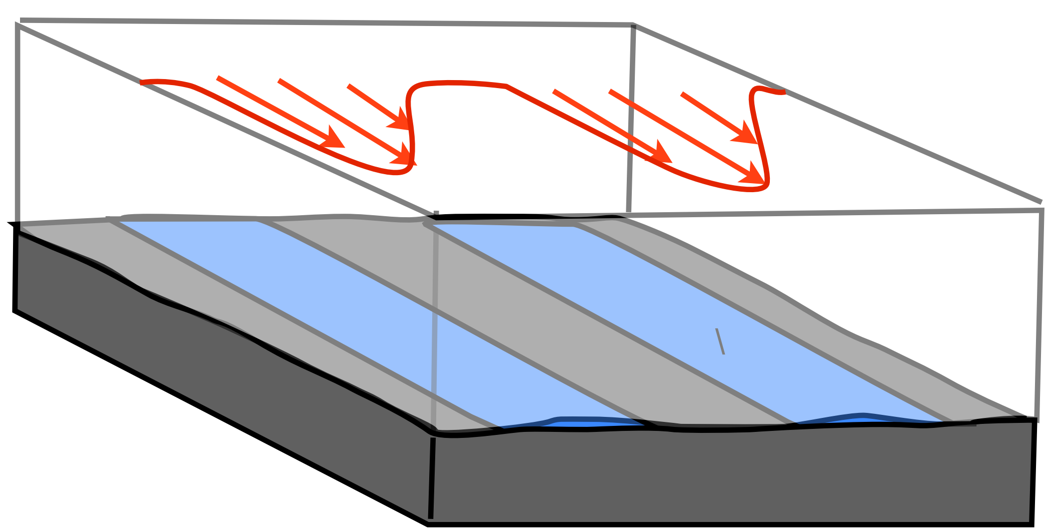
accumulation

$$\frac{\mathrm{d}h}{\mathrm{d}t} \propto A - Q_i$$

ice flux

⇨ Surges

Spatial variability



⇨ Ice streams

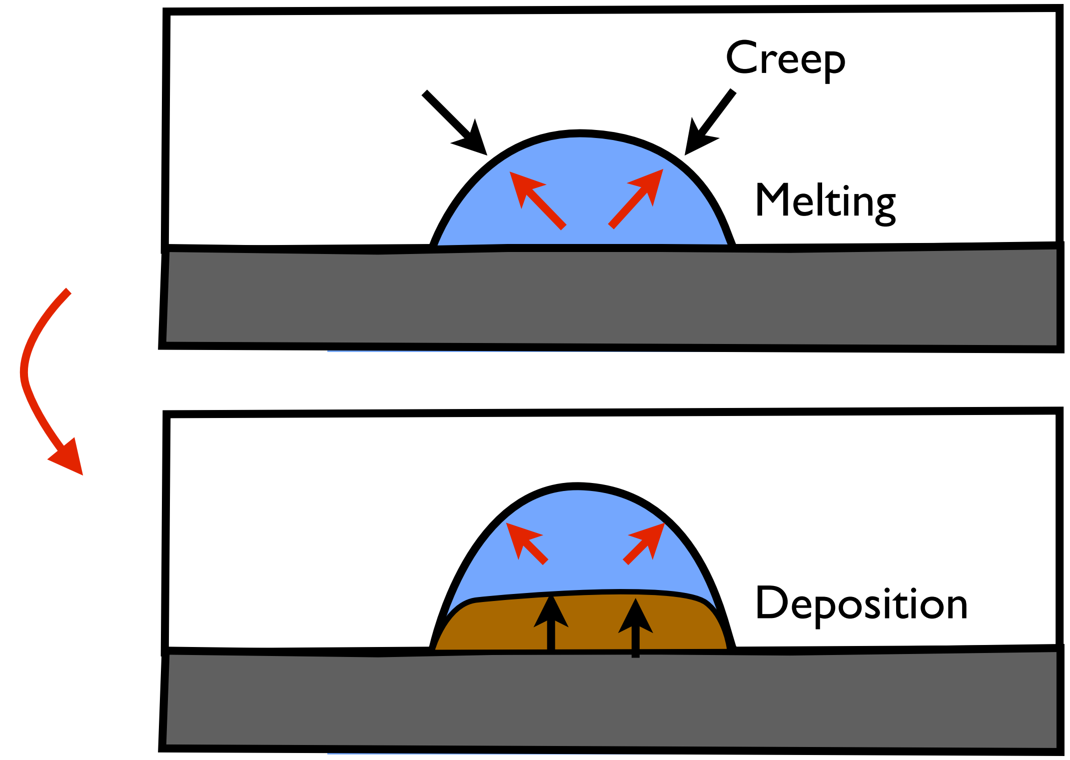
Meltwater deposits

Meltwater deposits

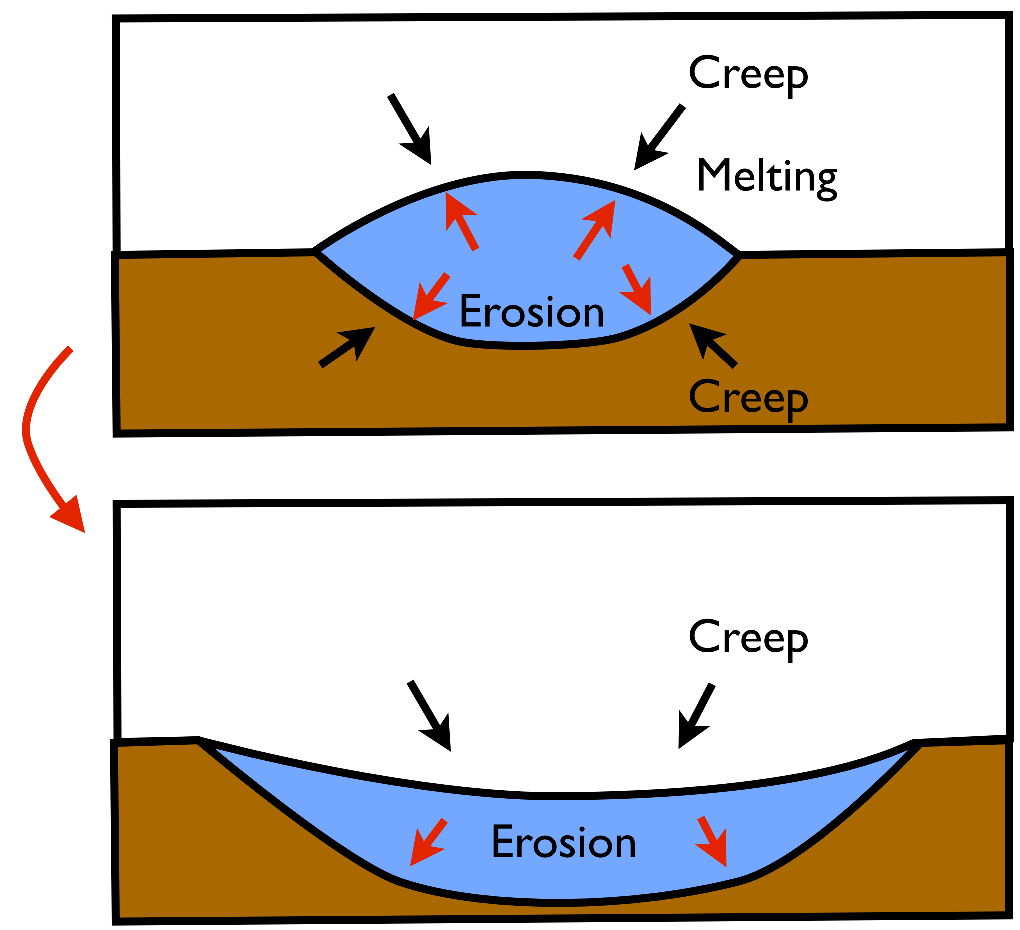
Deposition of sediments in Röthlisberger channels can build **eskers**

- Most likely under falling water speed, near margin

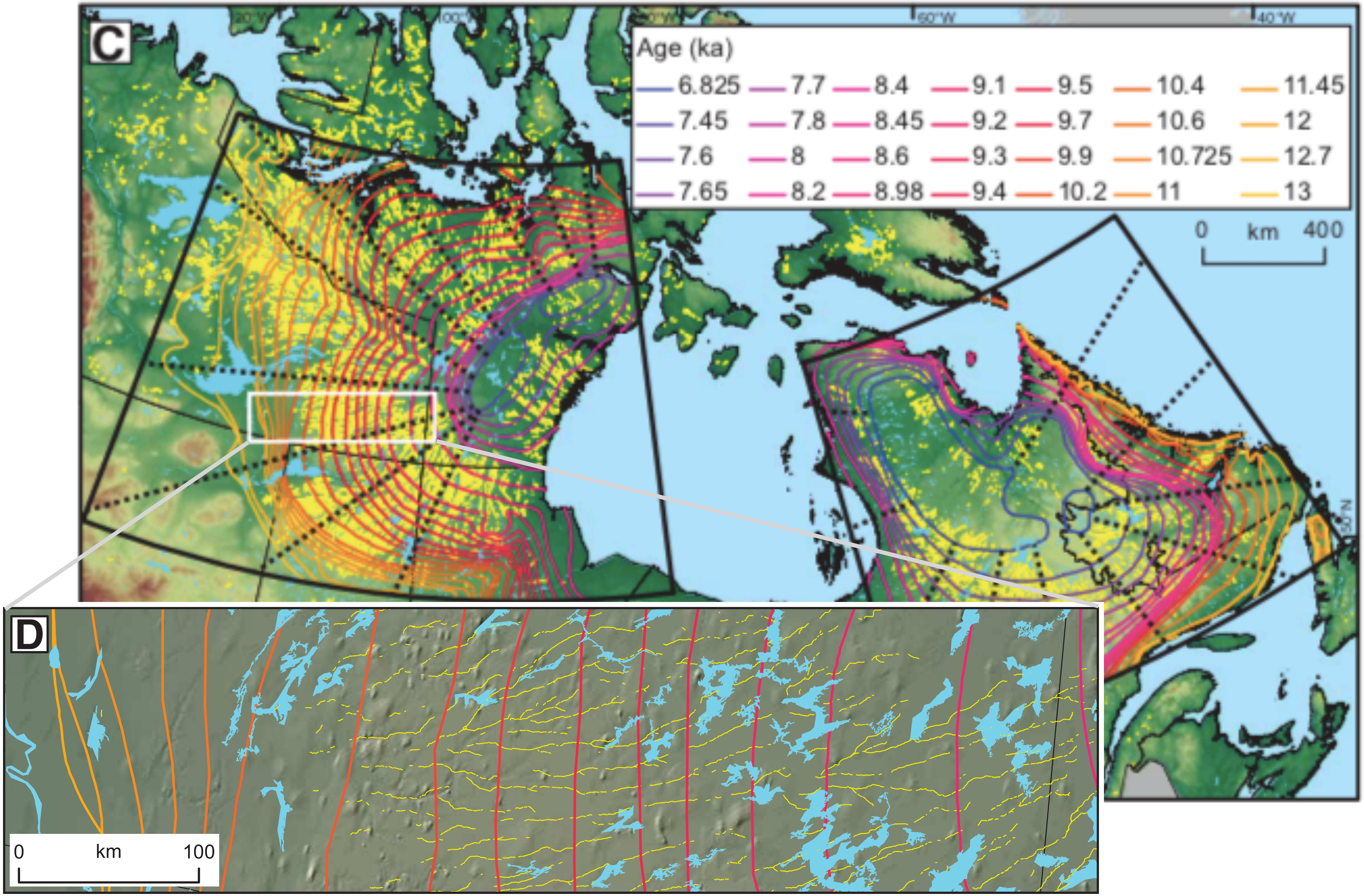
- Sediment is flushed from the surrounding bed



Erosion of sediments from canals can create **tunnel valleys**

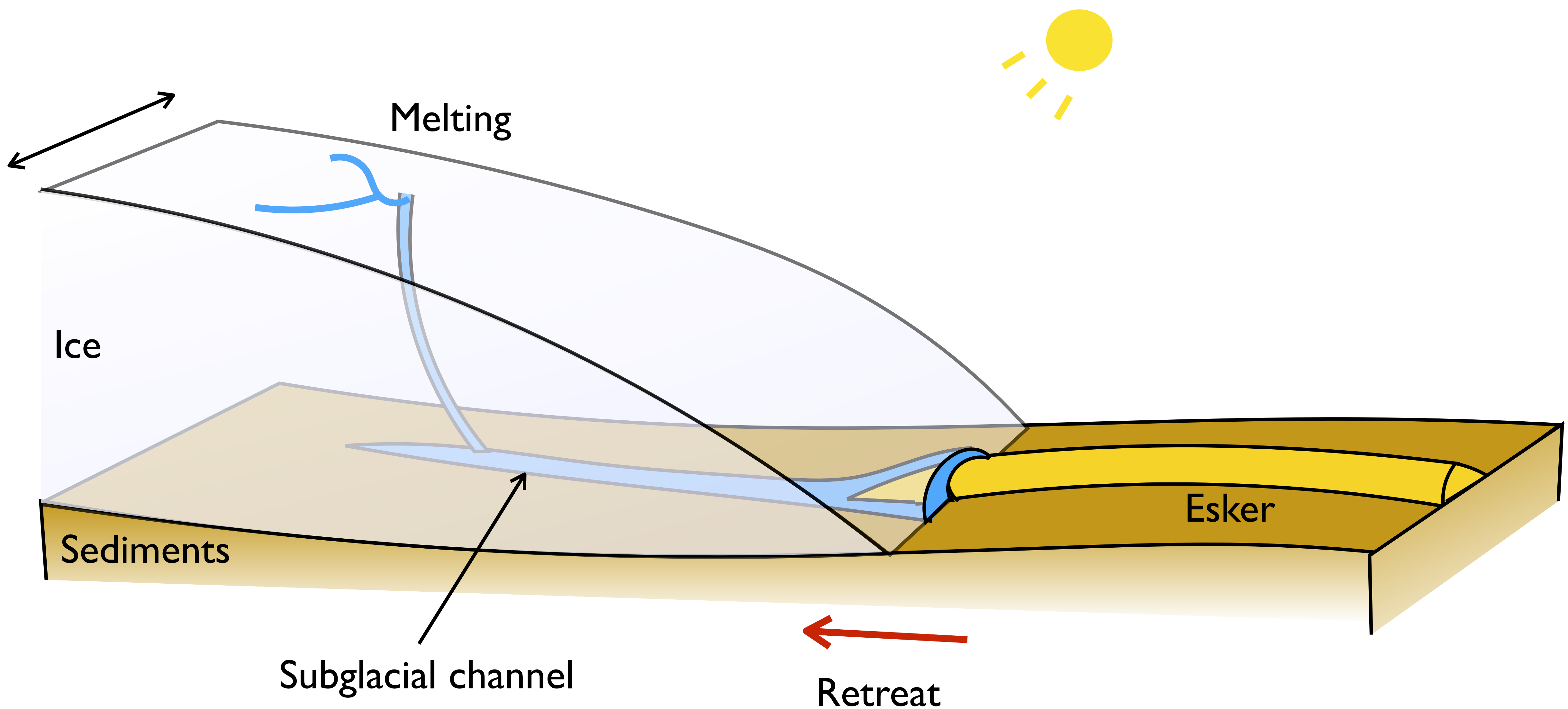


Eskers



Storrar et al 2014 *Geology*

Eskers

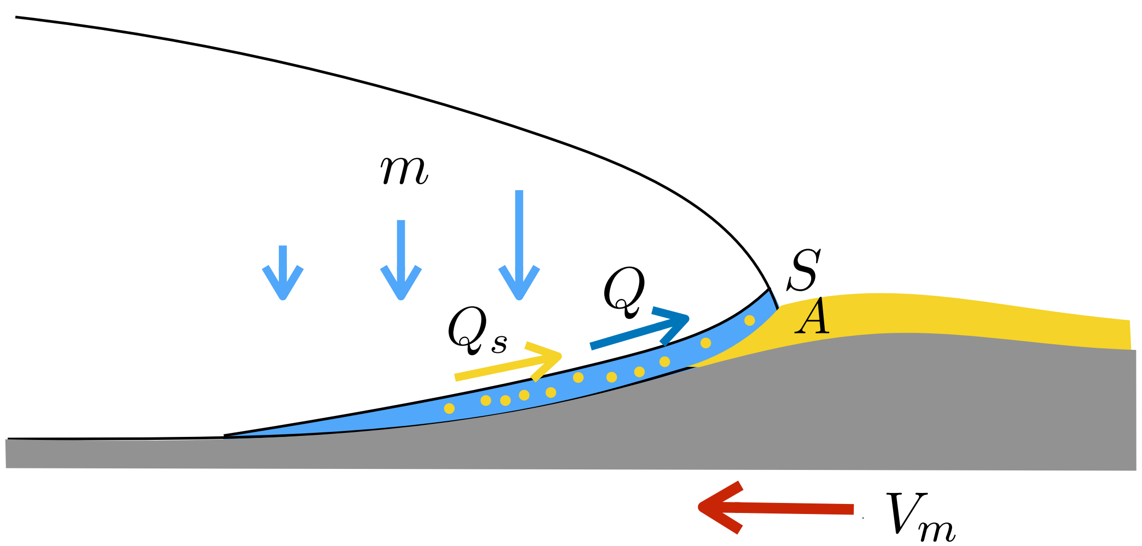


Model

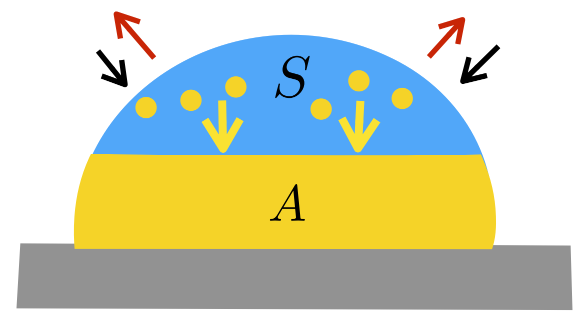
An extended version of the Röthlisberger channel model that incorporates sediment transport.

Eskers

Sediment **deposition** acts to clog the channel:



Hewitt & Creyts 2019



$$\frac{\partial S}{\partial t} = \frac{Q\Psi}{\rho_i L} - \tilde{A} S N^n - D$$

$$\frac{\partial A}{\partial t} = D$$

Boundary-layer analysis leads to an approximation of the cross-sectional area:

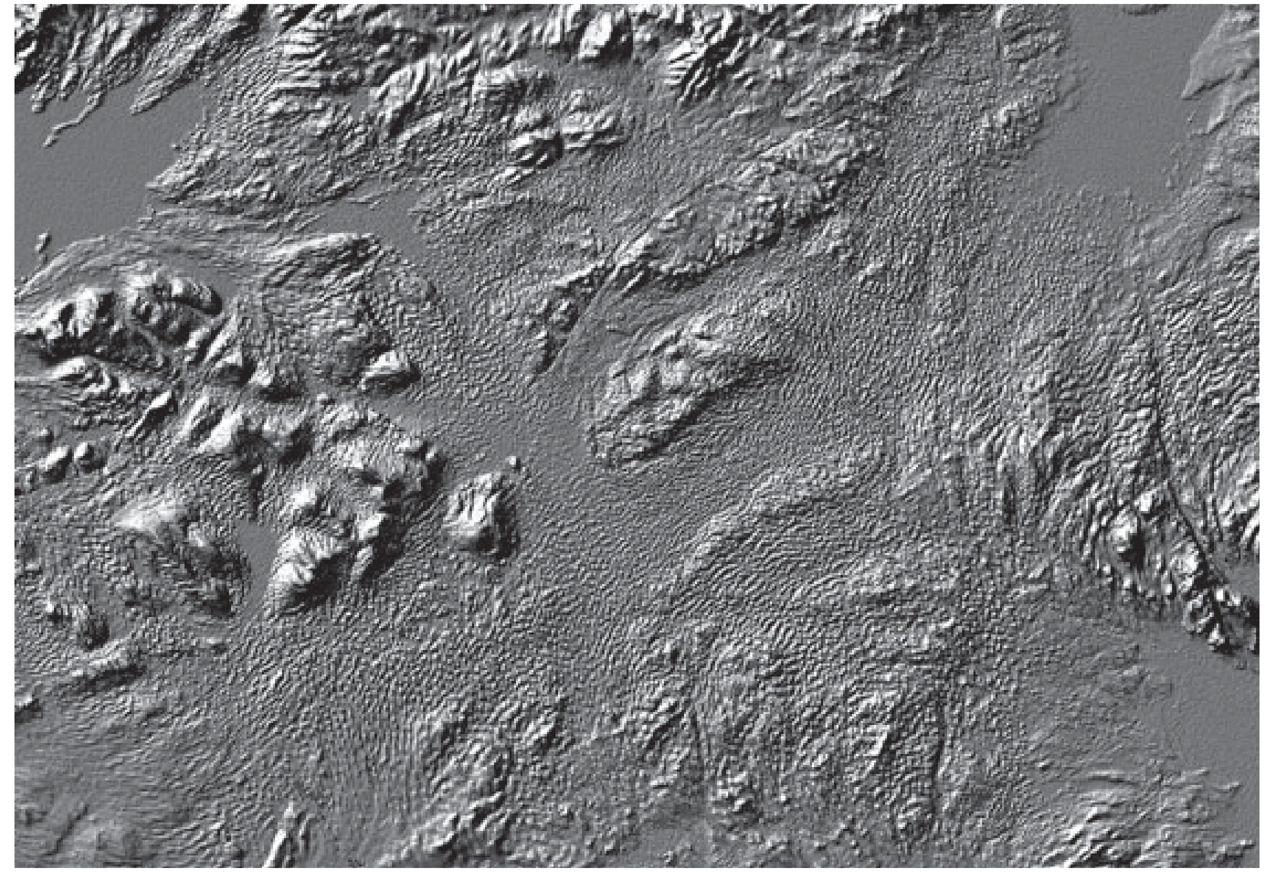
$$A = \mathcal{C}\, Q_m^{-4/5} Q_{s\,m}^{29/15} V_m^{-1}$$

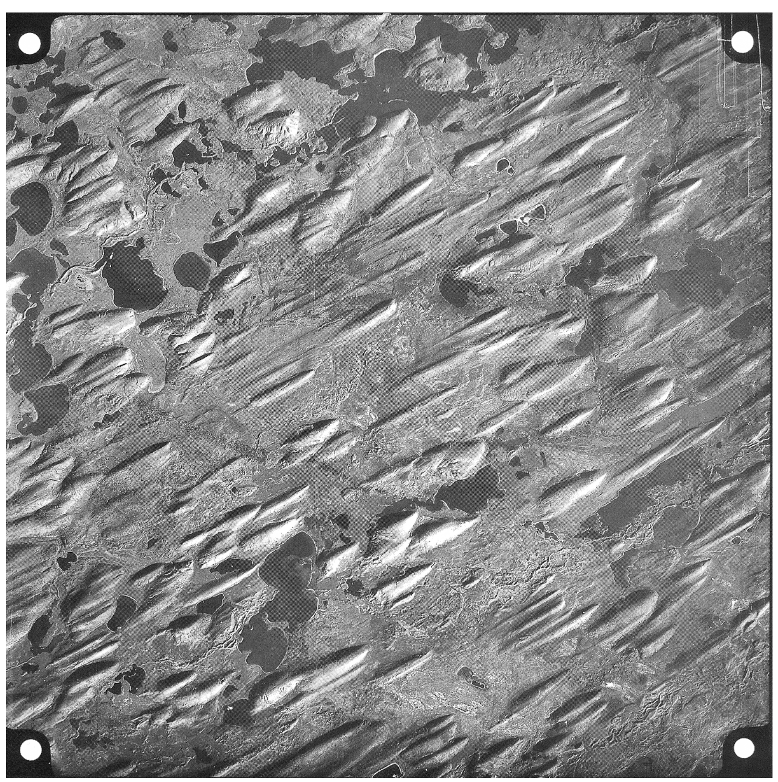
Meltwater flux — Sediment flux — Retreat rate

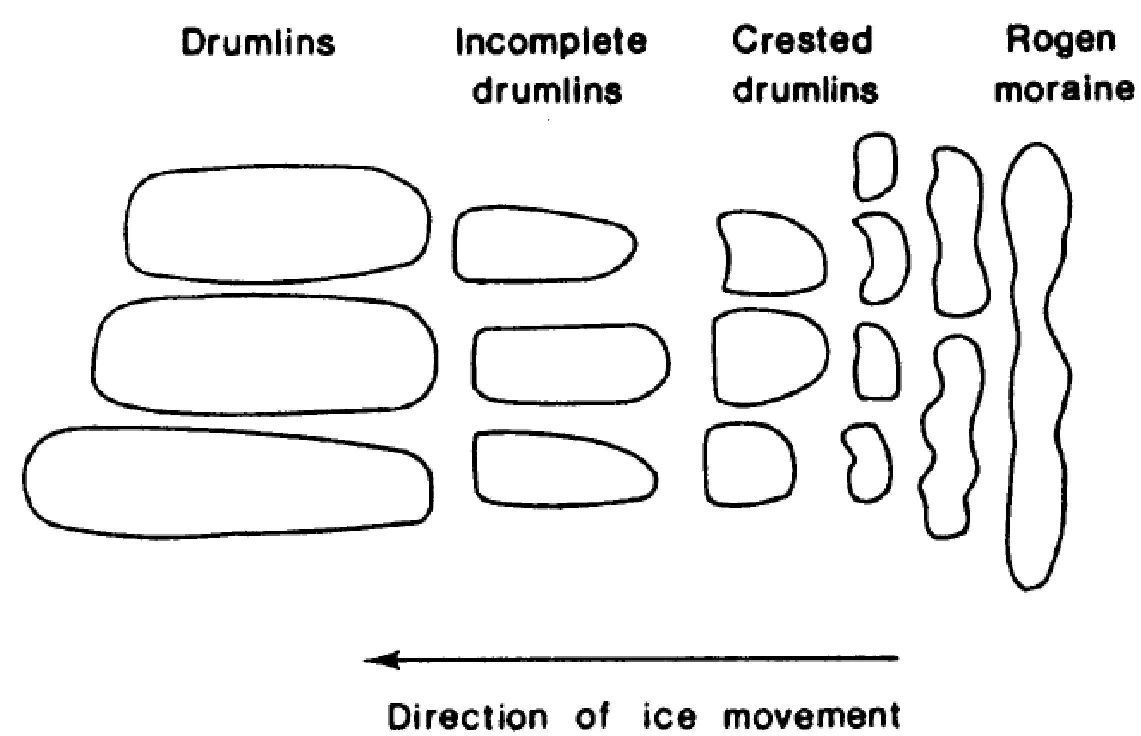
Esker size **increases** roughly quadratically with **sediment flux**, **decreases** roughly linearly with **meltwater flux**, and **decreases** linearly with **ice-sheet retreat rate**.

Deformational deposits

Subglacial bedforms

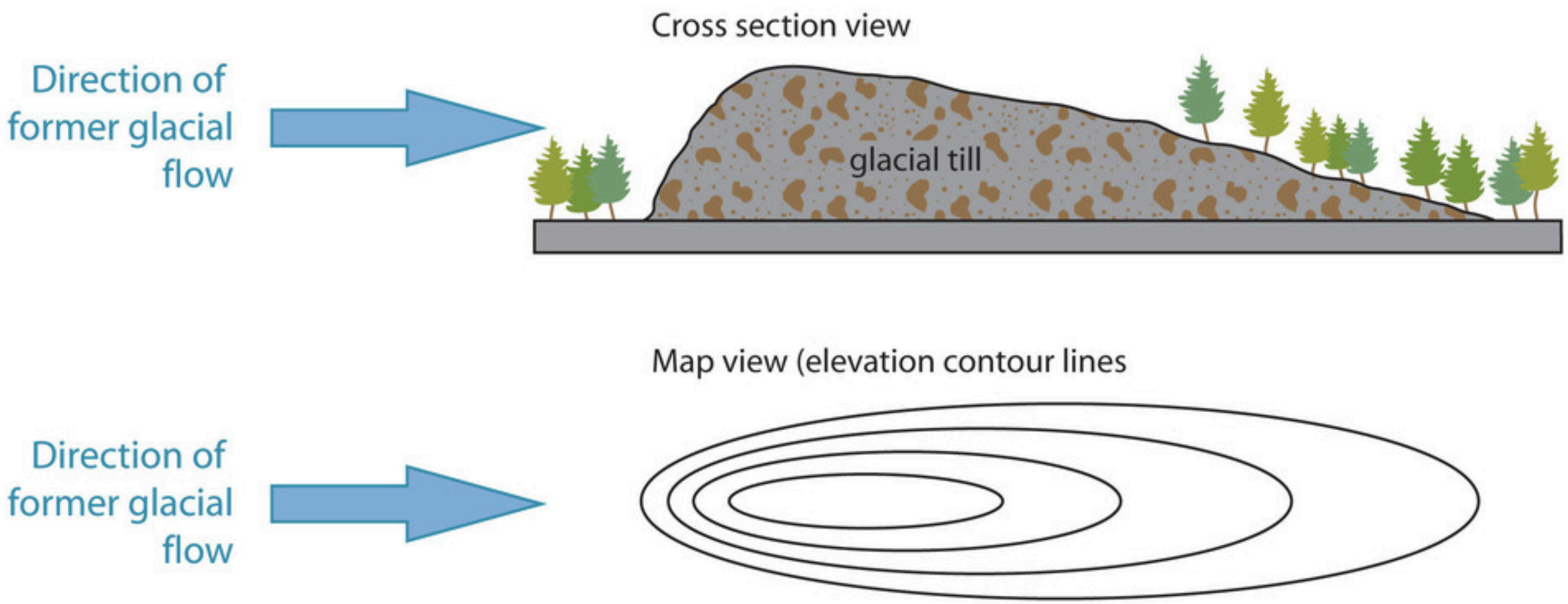






Sugden & John 1976

Drumlins

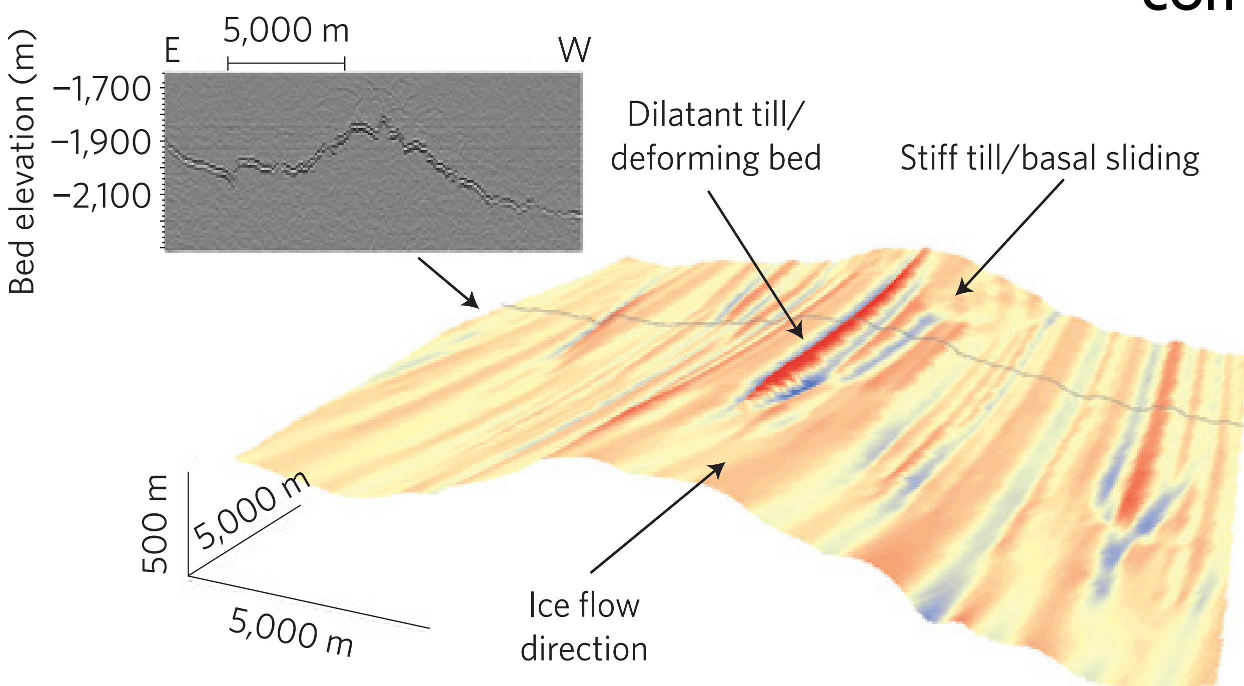


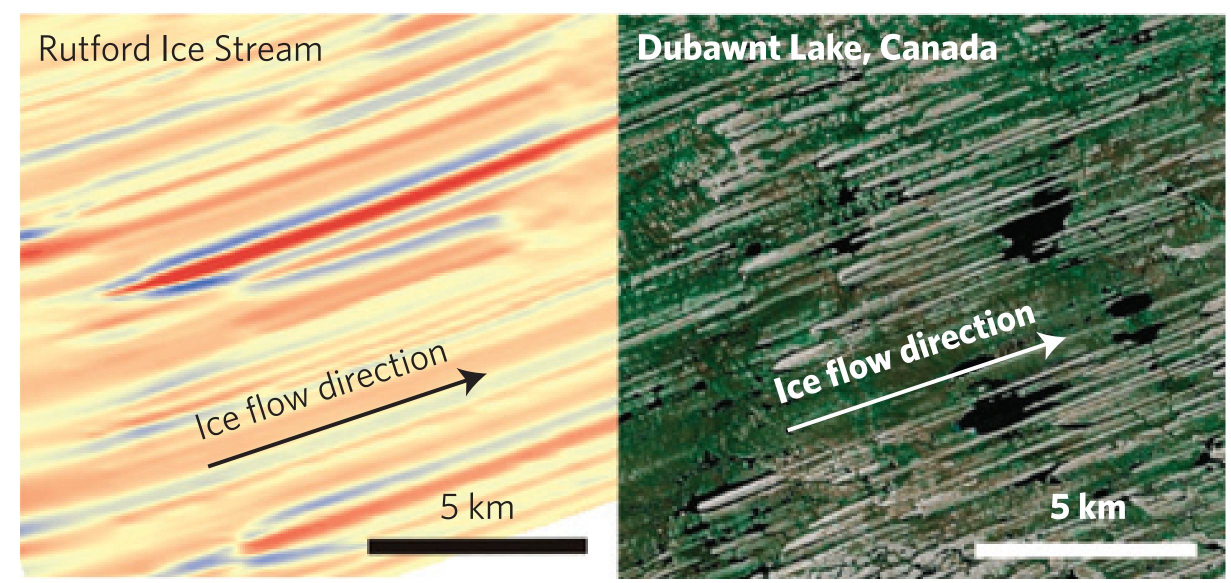
Subglacial waveforms formed by deformation/erosion by ice flow

Mega-scale glacial lineations

Radar profile of bed beneath Rutford ice stream (West Antarctica)

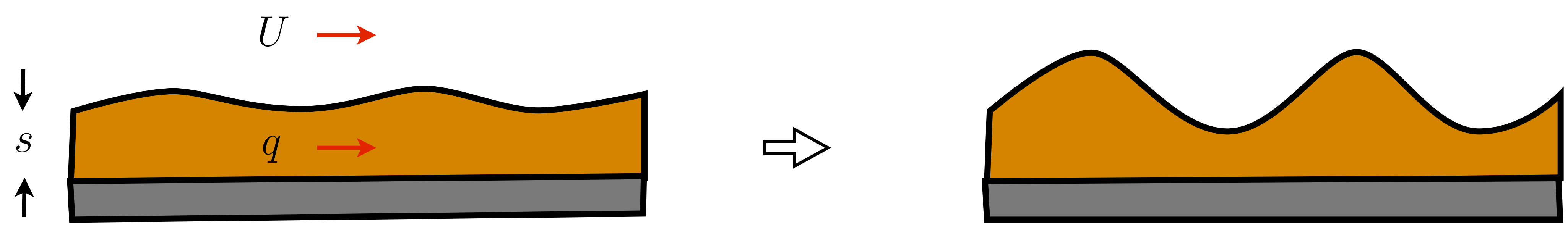
compared with de-glaciated bed of former ice stream.





King et al 2009

Instability theory Hindmarsh 1998, Schoof 2007, Fowler 2000, 2001, 2002, 2009, 2010, 2010

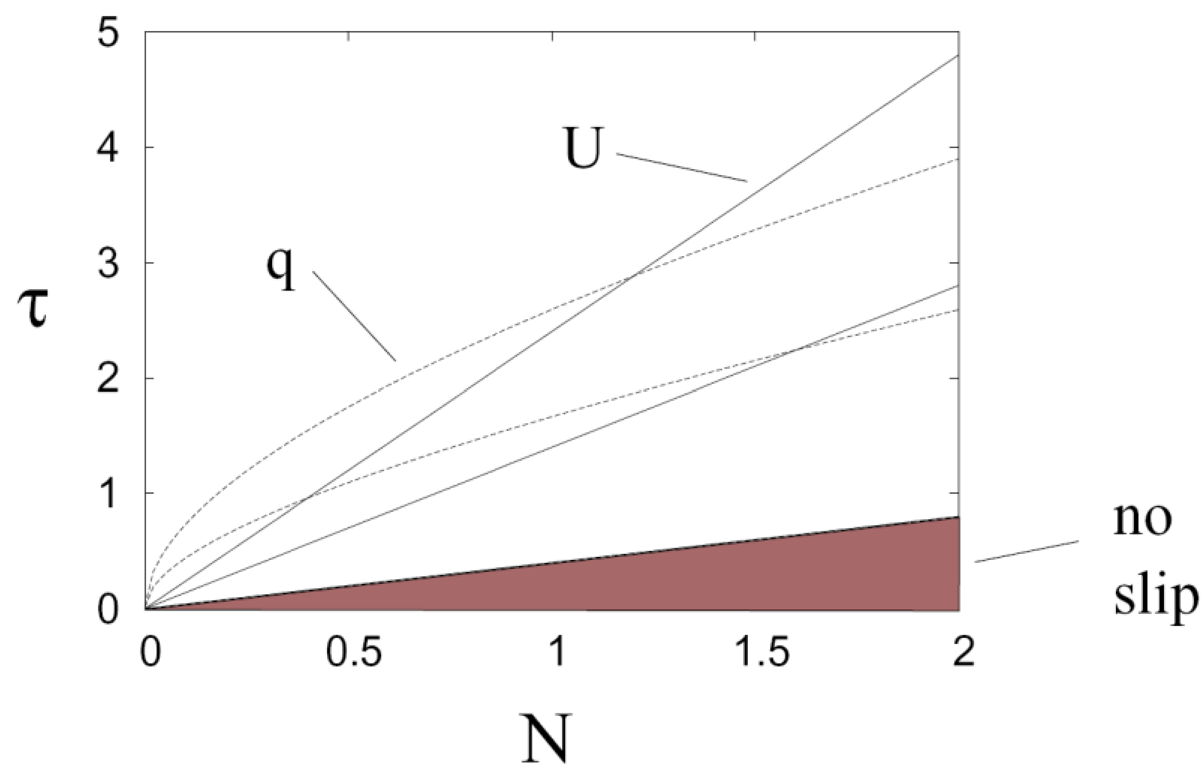


Conservation of sediment $\frac{\partial s}{\partial t} + \frac{\partial q}{\partial x} = 0$

Sediment flux $q = q(\tau, N)$

Sliding law $\tau = \tau(U, N)$

Instability if $\left.\frac{\partial \tau}{\partial N}\right|_U > \left.\frac{\partial \tau}{\partial N}\right|_q$



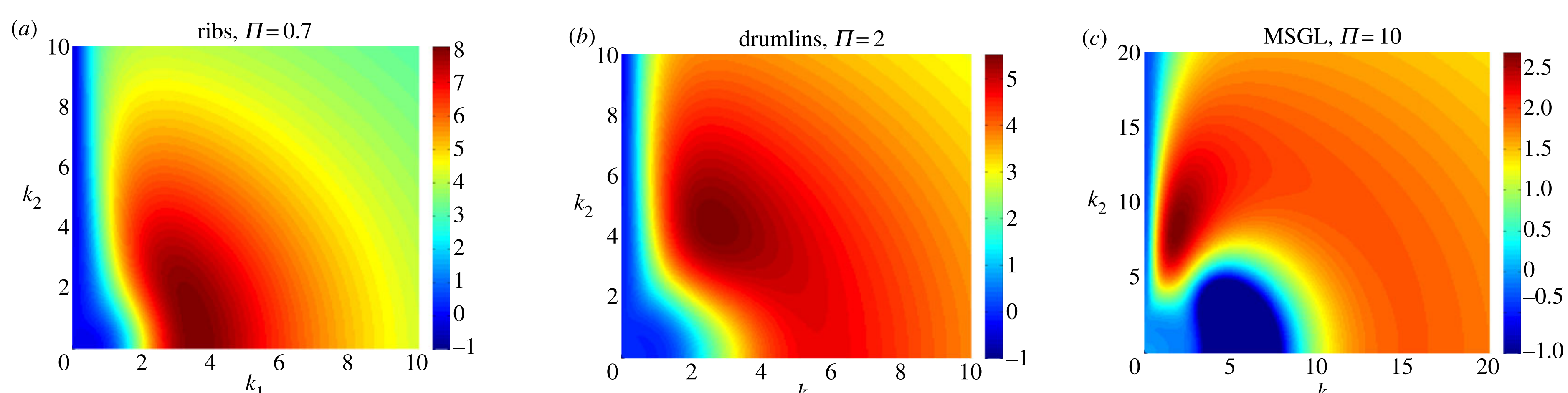
Instability theory Fowler & Chapwanya 2014

Modified theory may explain evolution of ribbed moraine, drumlins, and mega-scale lineations

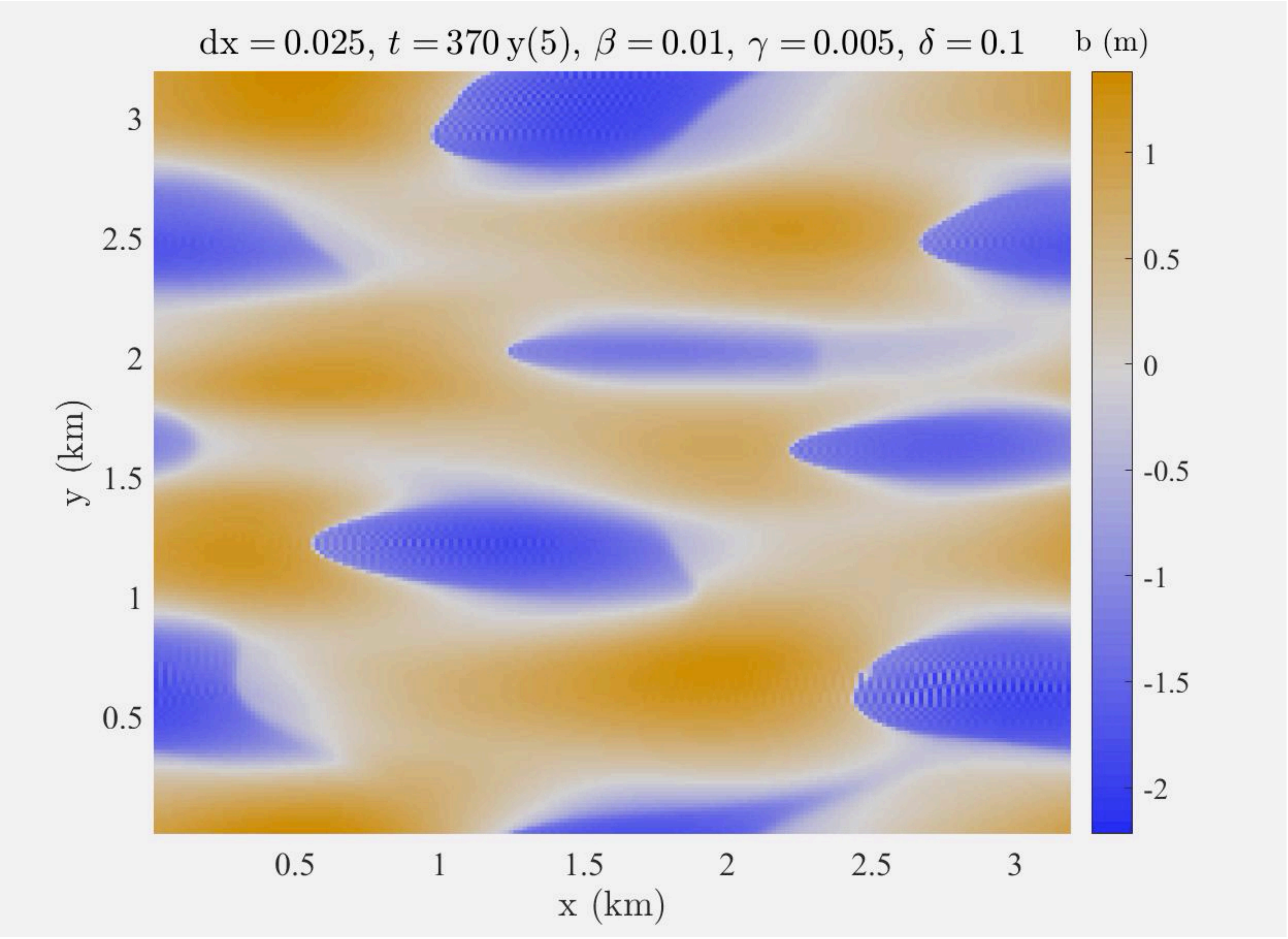
$$\left.\begin{aligned}
&\sigma(h^3)_x = \boldsymbol{\nabla}.[h^3 \boldsymbol{\nabla}\psi],\\
&\Psi = s - N + \Phi,\\
&A = \frac{1}{2}\left[\frac{f(\bar{u},N)}{\mu} - N\right]_+,\\
&\varepsilon r h_t = 1 - \Pi h N,\\
&b = s - \delta h,\\
&b_t + \boldsymbol{\nabla}.[A\bar{u}\mathbf{i}] + \sigma\gamma[B(\tau_e)h]_x = \beta\boldsymbol{\nabla}.[A^3\boldsymbol{\nabla}N] + \gamma\boldsymbol{\nabla}.[B(\tau_e)\{h\boldsymbol{\nabla}\Psi + \theta\boldsymbol{\nabla}b\}],\\
&\boldsymbol{\tau}_e = \sigma h\mathbf{i} - \{h\boldsymbol{\nabla}\Psi + \theta\boldsymbol{\nabla}b\}\\
&\alpha s_t + \bar{u}s_x = w(\Phi, N).
\end{aligned}\right\} \qquad (2.59)$$

- Ice flow over sediments causes **transverse dune-like** instability.
- Water flow carrying sediments causes **longitudinal rill-like** instability.

Growth rates

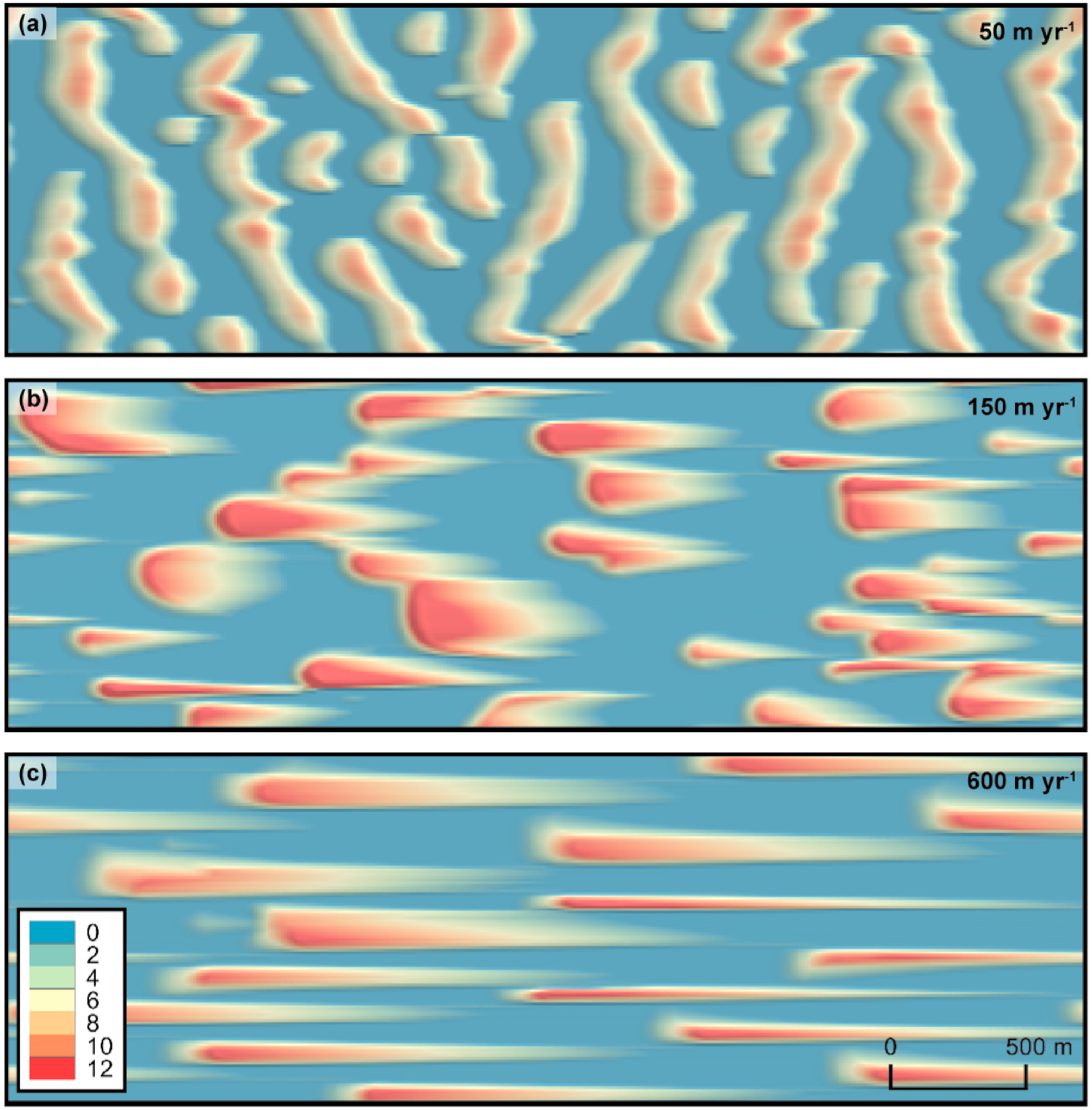


Instability theory



Fannon et al 2017

Instability theory



Barchyn et al 2016

Summary

Friction laws for soft beds are **similar to hard-bed sliding laws** (even though the local slip / deformation mechanism may be different).

Drainage over till may occur through films, cavities and canals.

Eskers form through deposition in Röthlisberger channels.

Ice flow over **deforming till** can be unstable and produce **ribbed moraine**, **drumlins**, and **mega-scale glacial lineations**.

An important open question is how the development of these bed-forms controls/affects ice dynamics.