Glacier and Ice-Sheet Sliding

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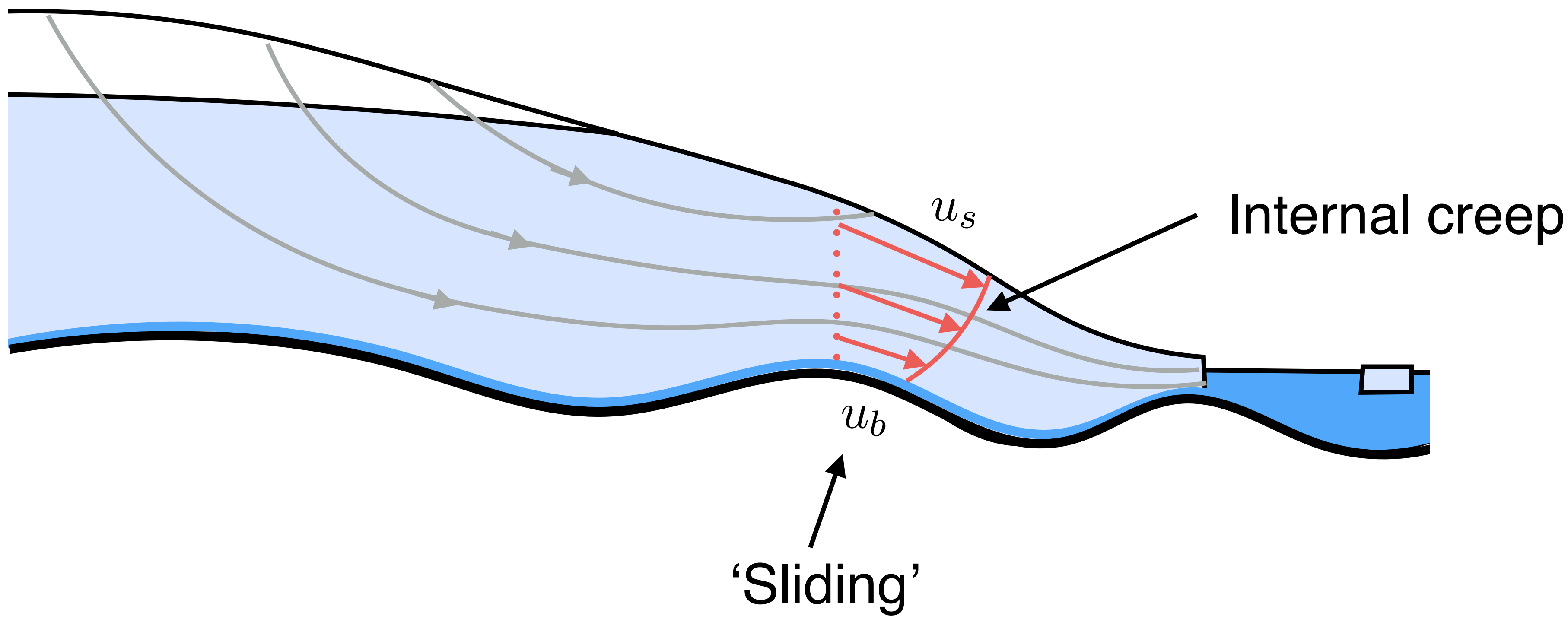
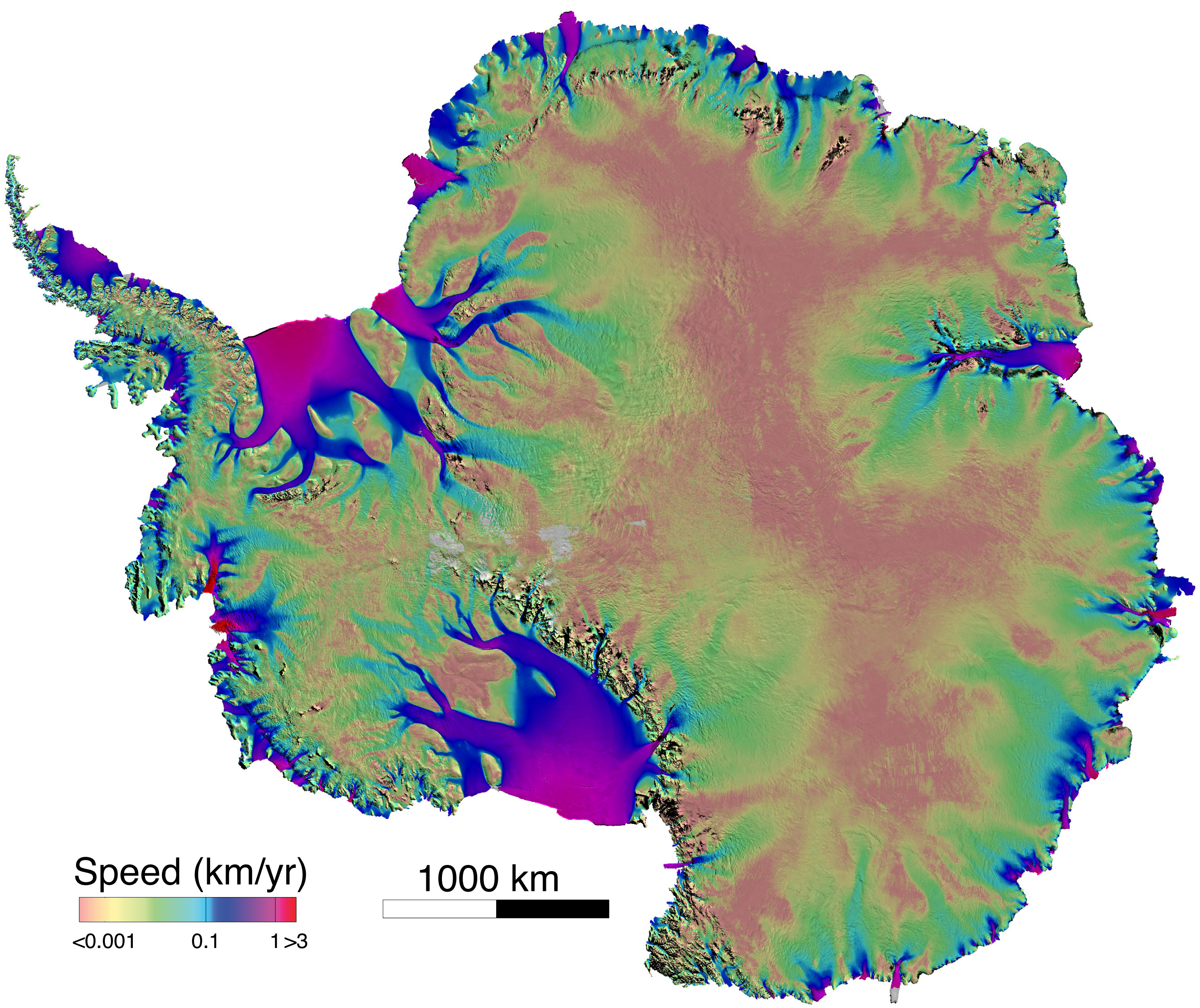
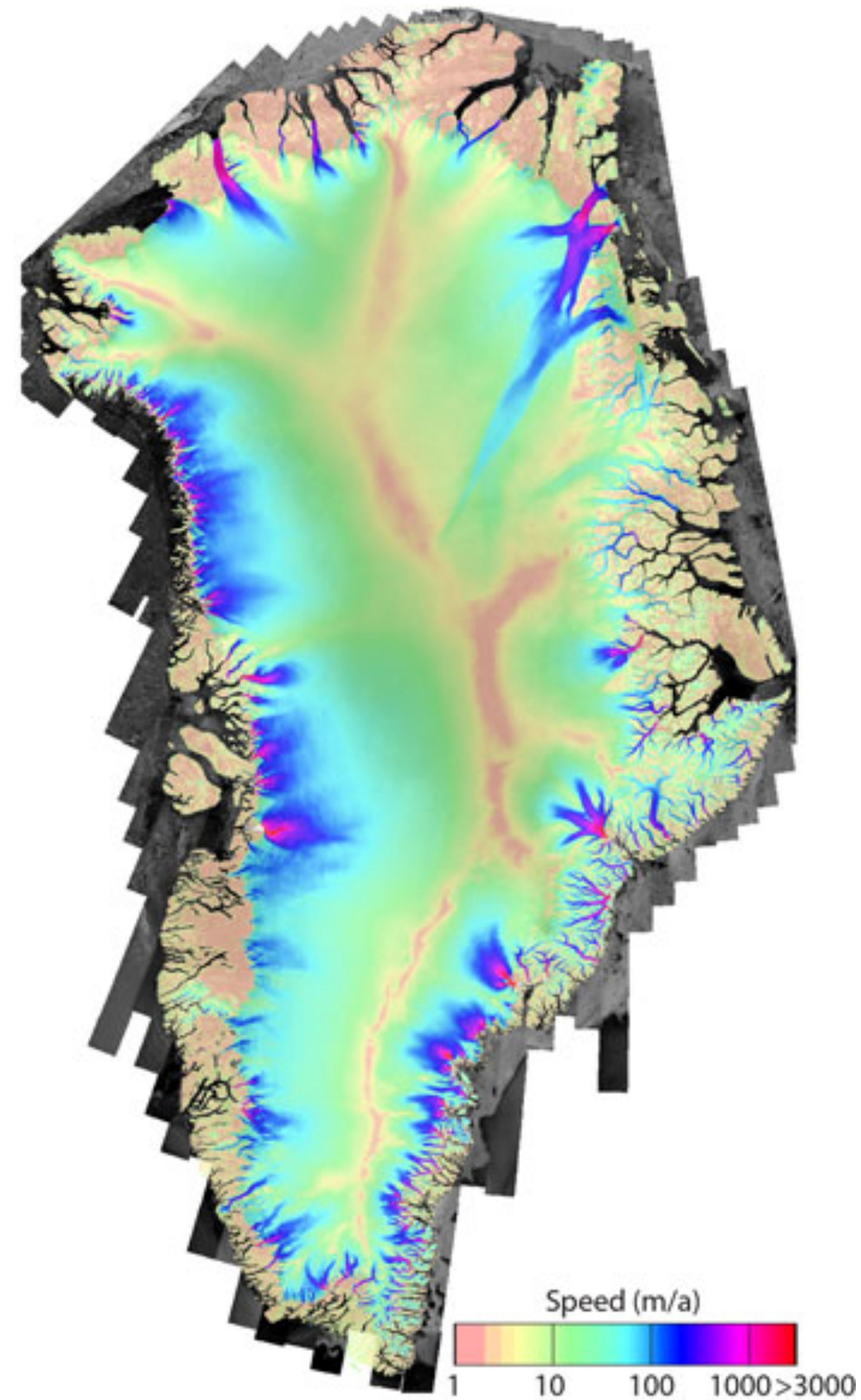


u_s
Internal creep
u_b
'Sliding'

Satellite-derived ice surface speeds

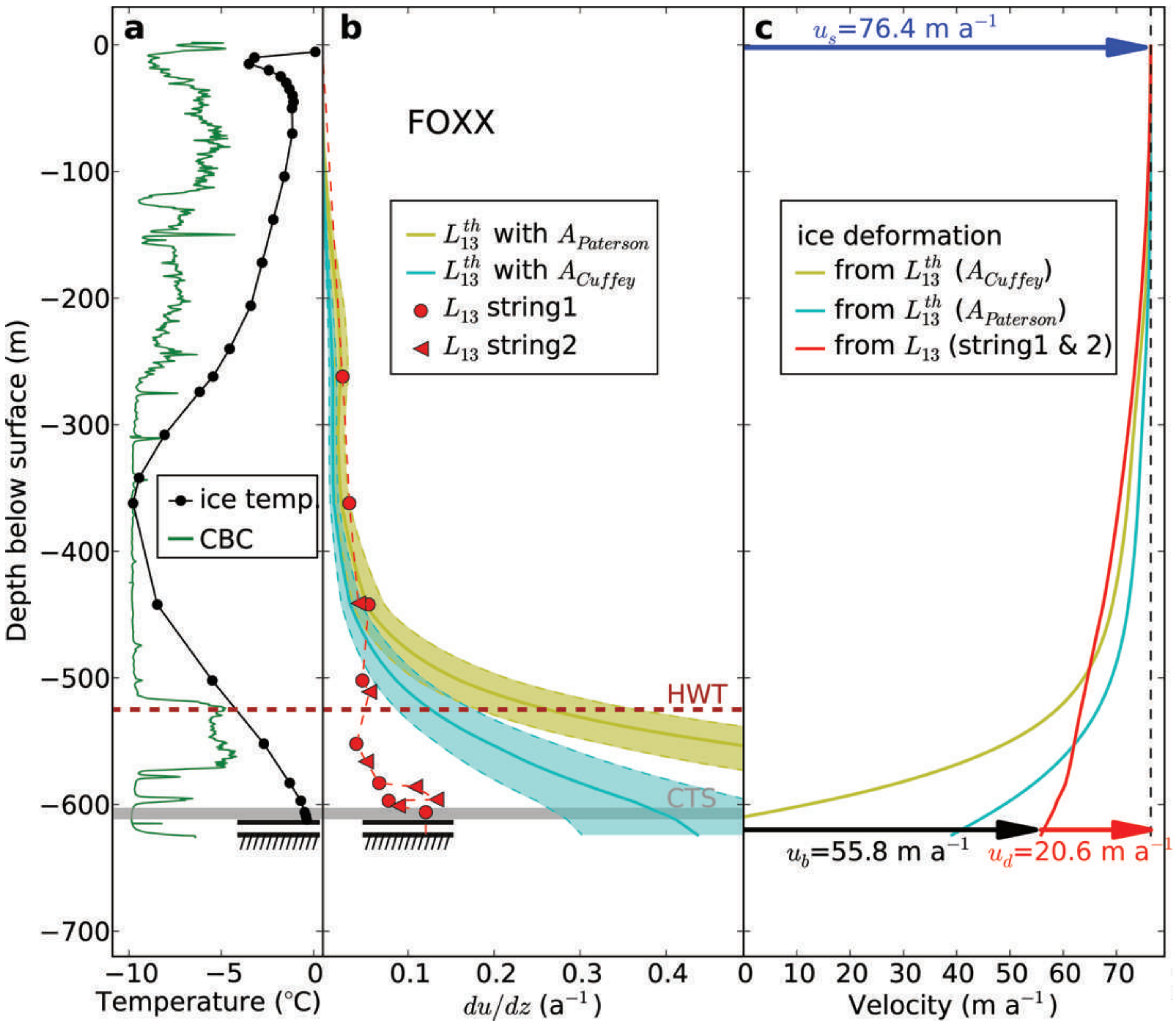


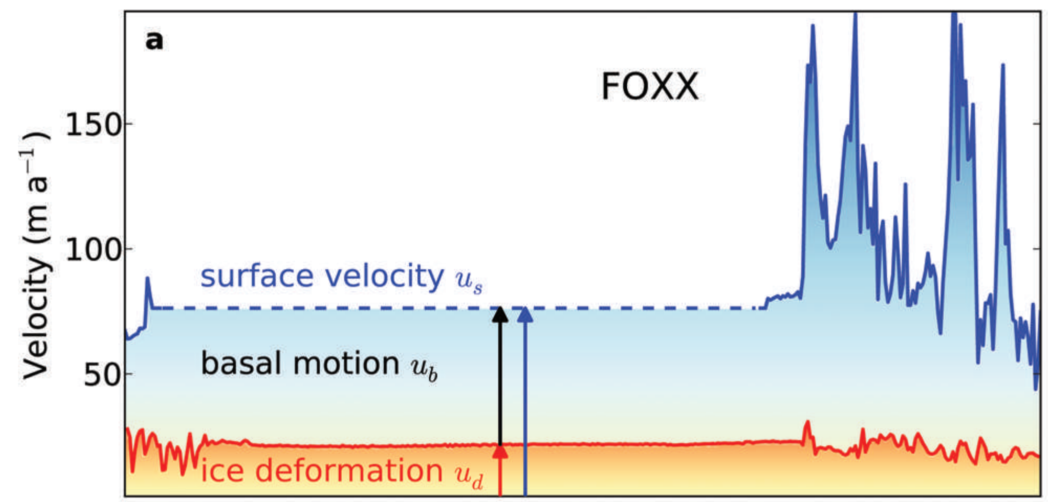
Mouginot et al 2019



Joughin et al 2018

GPS and borehole-derived ice speeds





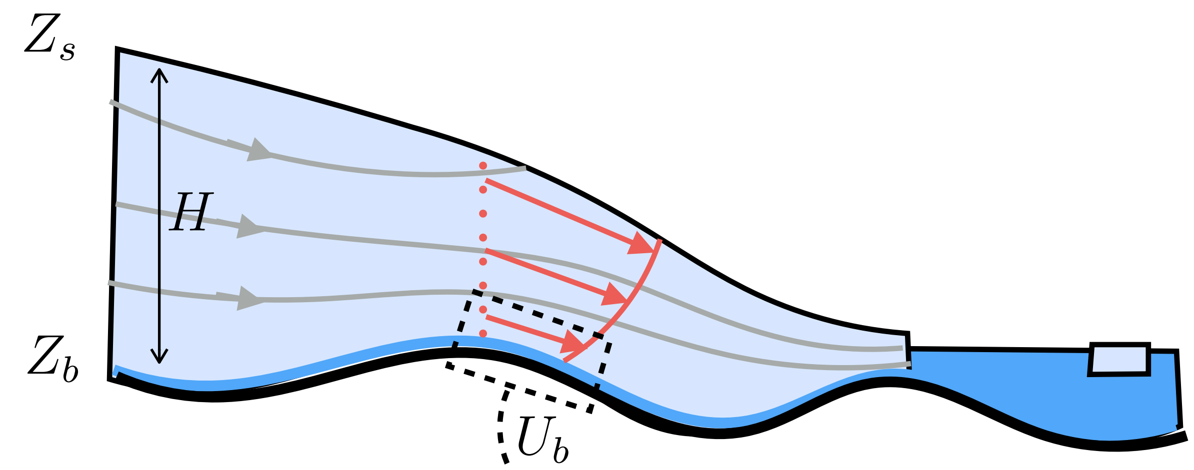
Ryser et al 2014

What controls how fast a glacier or ice sheet slides?

What physical processes enable it to slide?

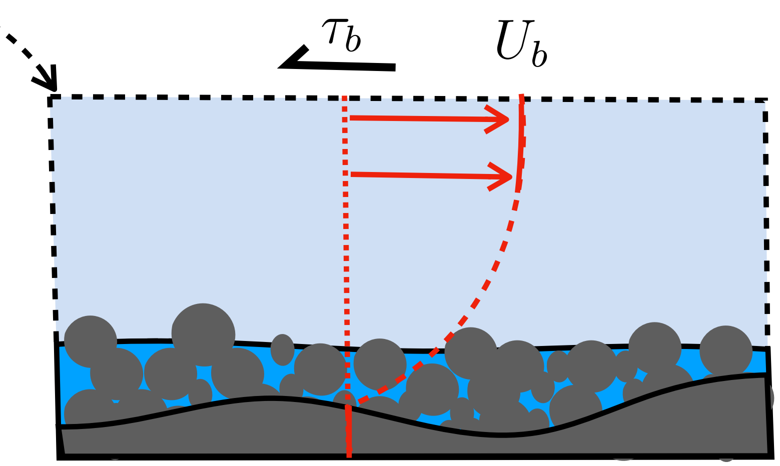
How do we describe sliding in an ice-sheet model?

Sliding law / Friction law



Stokes flow $$0 = -\nabla p + \nabla \cdot \boldsymbol{\tau} + \rho_i \mathbf{g}$$
$$\nabla \cdot \boldsymbol{u} = 0$$

To calculate ice flow we need a **basal boundary condition** which relates **basal shear stress** $\tau_b = |\boldsymbol{\tau}_b|$ and **basal speed** $U_b = |\mathbf{u}_b|$



$$\tau_b = f(U_b, \ldots)$$

This is a parameterization of **unresolved** processes close to the bed.

Historically thought of as ‘sliding’ law $U_b = F(\tau_b, \ldots)$

→ May be multi-valued

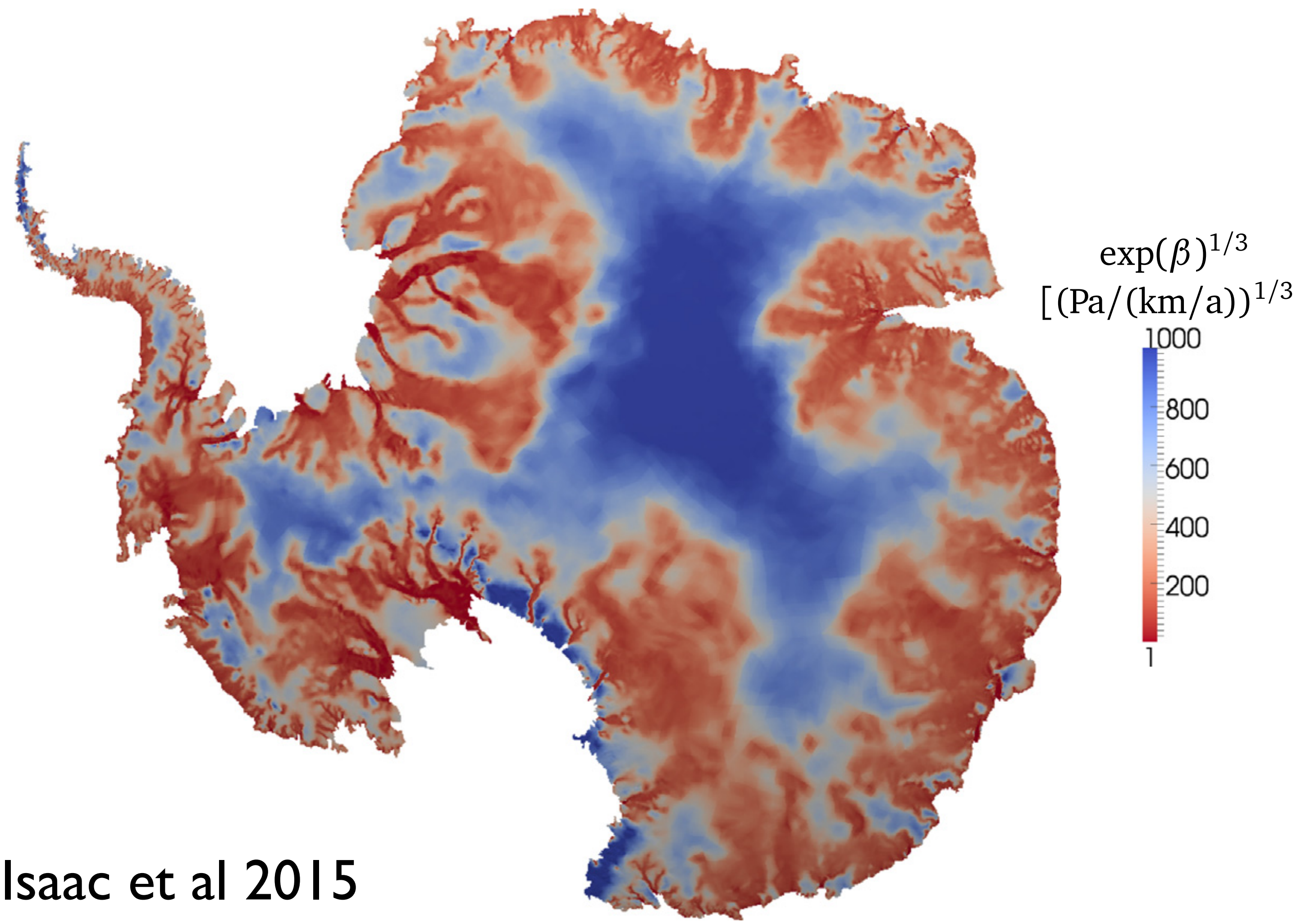
Shallow ice approximation $\boldsymbol{\tau}_b \approx -\rho_i g H \nabla Z_s$

Modern view point $\tau_b = f(U_b, \ldots)$

Numerical ice-sheet models

Many numerical models use a **friction law** of the form $\boldsymbol{\tau}_b = C|\mathbf{u}_b|^{m-1}\mathbf{u}_b$

The coefficient $C = C(x,y)$ is usually treated as a fitting parameter(s), chosen to achieve a good fit with observations of surface velocities.

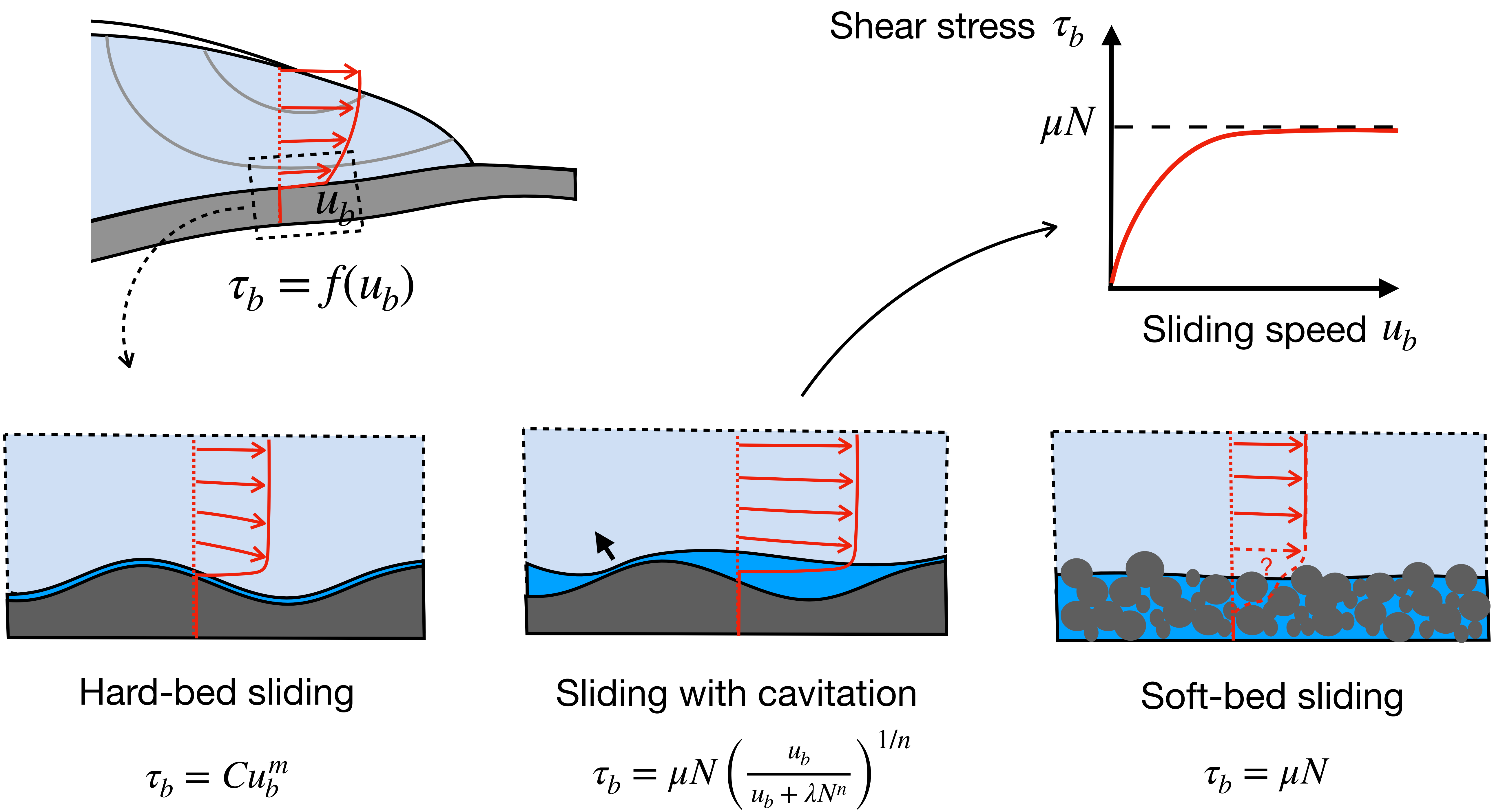


Isaac et al 2015

But the coefficient reflects properties of the bed that may vary with time.

⇨ We want to understand what **physical processes** govern the friction law.

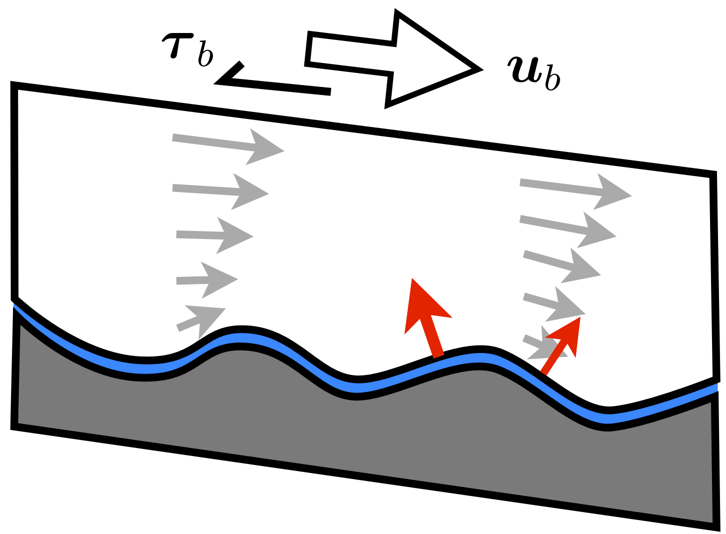
Overview



u_b
$\tau_b = f(u_b)$
Shear stress τ_b
μN
Sliding speed u_b
?
Hard-bed sliding
$\tau_b = C u_b^m$
Sliding with cavitation
$\tau_b = \mu N \left(\frac{u_b}{u_b + \lambda N^n} \right)^{1/n}$
Soft-bed sliding
$\tau_b = \mu N$

Hard-bed sliding

Hard-bed sliding Weertman 1957



A **film of water** exists between ice and the underlying bedrock (a few microns thick).

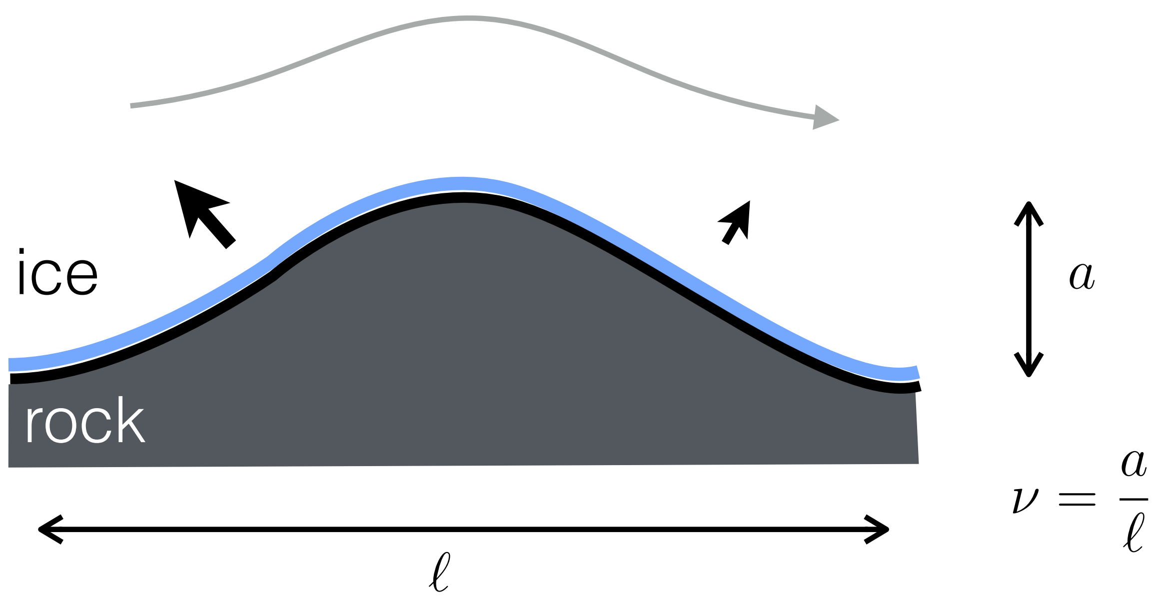
Microscopically, free slip is allowed (i.e. $\tau_{b\ \mathrm{micro}} = 0$).

Macroscopic resistance comes from the **roughness** of the bedrock ($\tau_b = f(U_b)$).

Flow over roughness occurs via **regelation** and **viscous (plastic) deformation**.

Viscous flow and regelation

The ice deforms viscously around obstacles in the bed



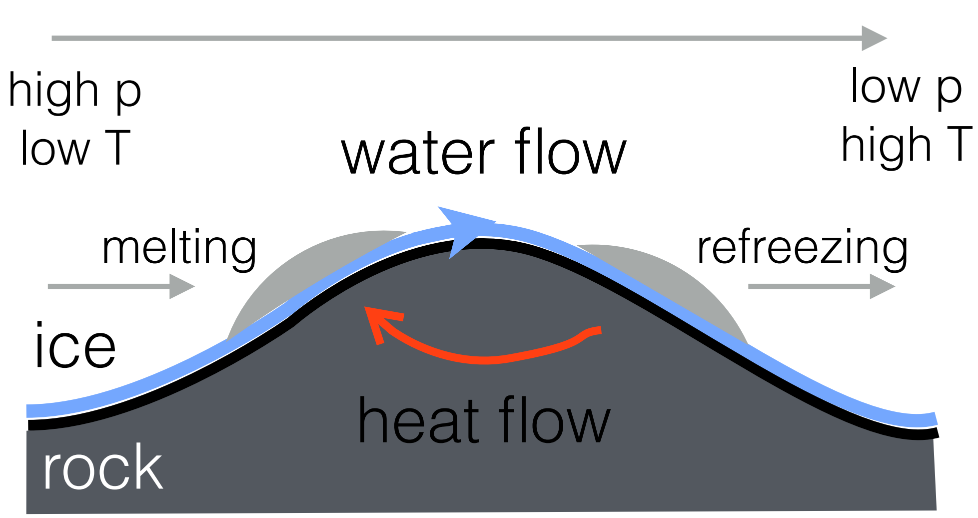
Dimensional analysis, using Glen's flow law

$$U_V \approx \left(\frac{aA}{2^n}\right)\frac{\tau_b^n}{\nu^{2n}}$$

$$\nu = \frac{a}{\ell}$$ 'roughness'

Regelation: pressure difference across obstacles causes a temperature difference

- results in upstream melting and downstream freezing

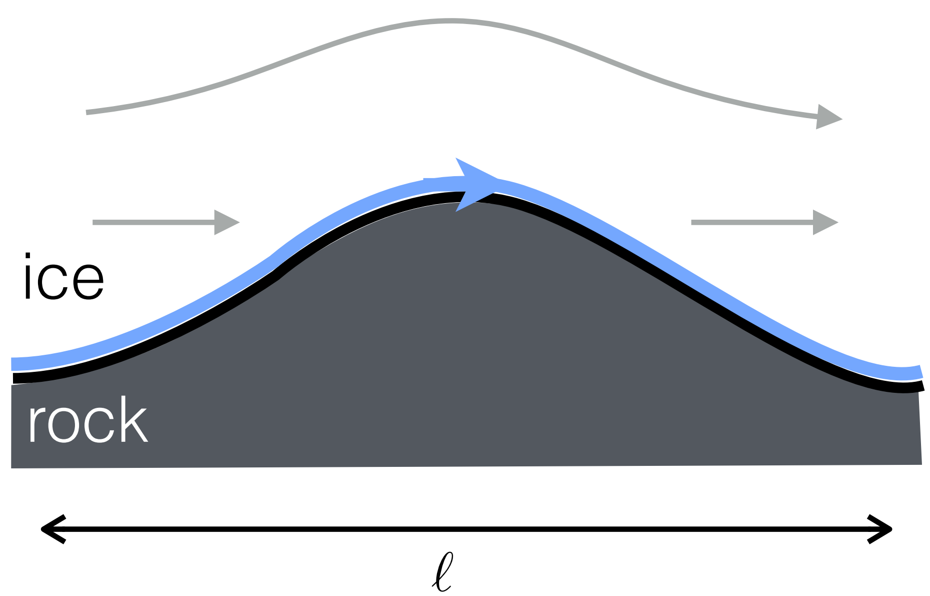


Balance of conductive / latent heat flow

$$U_R = \left(\frac{k\Gamma}{\rho_i L a}\right)\frac{\tau_b}{\nu^2}$$

Viscous flow and regelation

Combining these two mechanisms:



$$a \qquad \nu = \frac{a}{\ell} \qquad \ell$$

$$U_V \approx \left(\frac{aA}{2^n}\right)\frac{\tau_b^n}{\nu^{2n}}$$ effective for LARGE bumps

$$U_R = \left(\frac{k\Gamma}{\rho_i L a}\right)\frac{\tau_b}{\nu^2}$$ effective for SMALL bumps

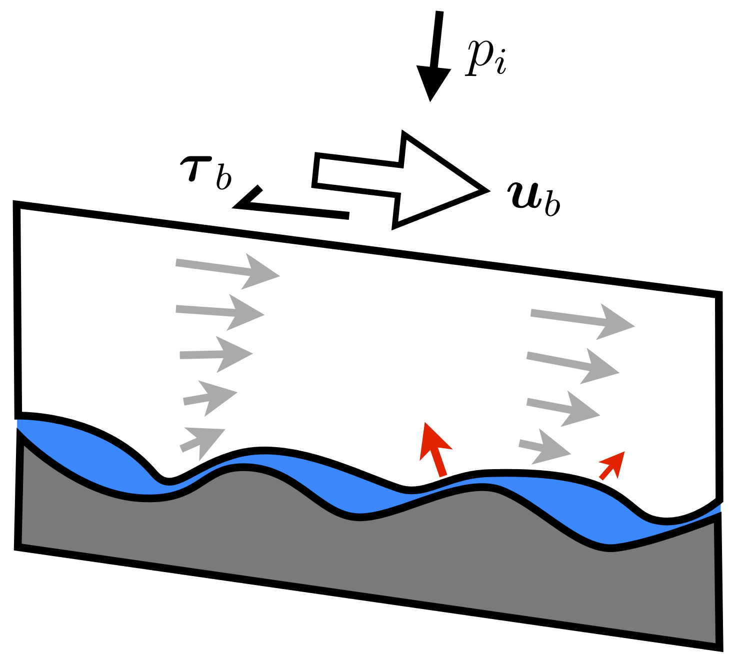
There is a '**controlling obstacle size**' for which stress / speed cross over: $a \propto U_b^{-(n-1)/(n+1)}$

⇨ Weertman sliding law

$$\tau_b = \nu^2 \; R \; U_b^{2/(n+1)}$$

$$R = \left(\frac{\rho_i L}{2k\Gamma A}\right)^{1/(n+1)}$$

Sliding with cavitation Lliboutry 1968, Iken 1981, 1983



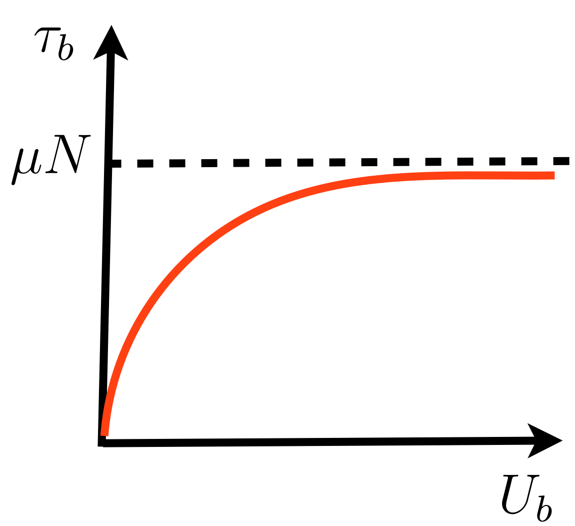
Cavitation occurs when pressure on downstream face of bumps reduces to critical level p_c

For steady-state cavities, friction law becomes dependent on **effective pressure** $N = p_i - p_c$

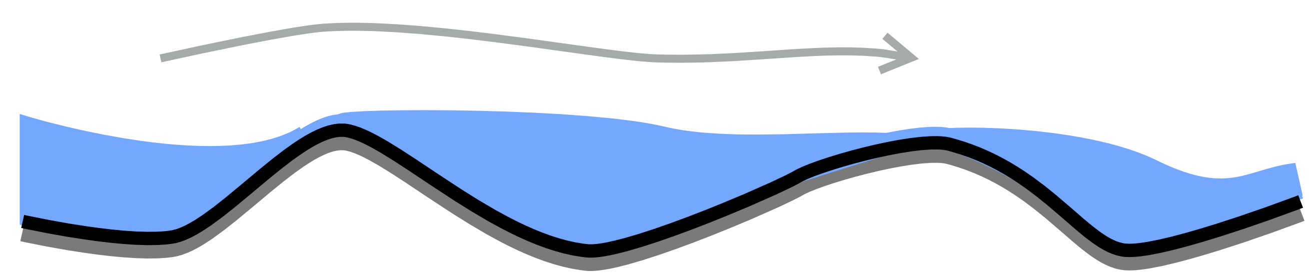
⇨ $$\tau_b = f(U_b, N)$$

p_i (macroscopic) ice normal stress

Iken suggested there should be a **maximum shear stress**



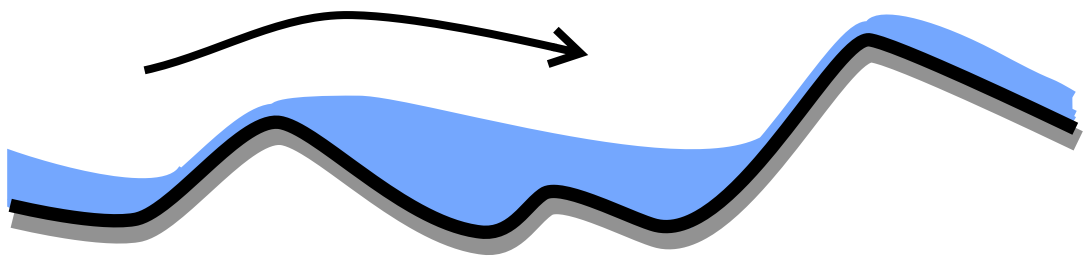
associated with cavities 'drowning' the bed roughness.



Sliding with cavitation

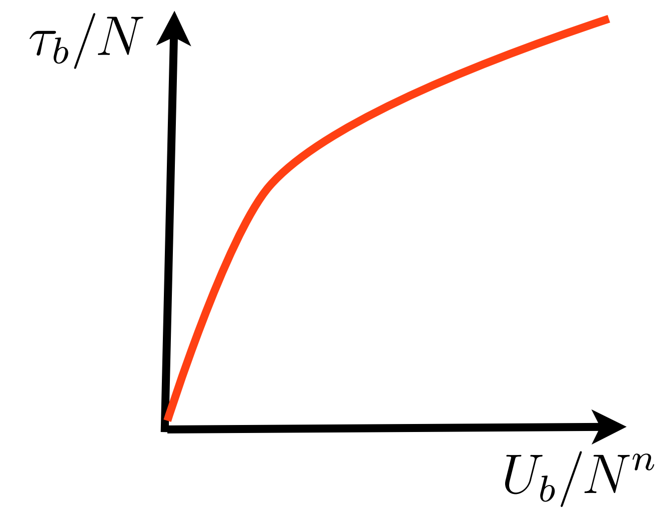
Budd et al 1979, Fowler 1986, Schoof 2005, Gagliardini et al 2007, Helanow et al 2019

Fowler suggested cavities never really 'drown' bed - stress is just transferred to larger bumps

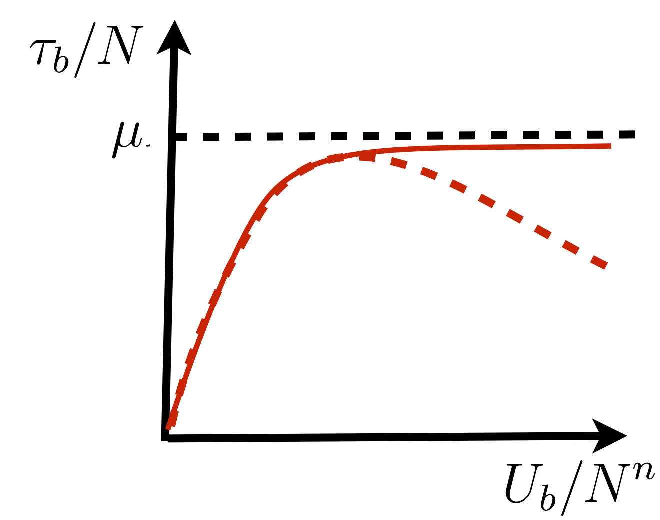


⇨ 'Generalized' Weertman law / Budd law

$$\tau_b = C U_b^p N^q$$



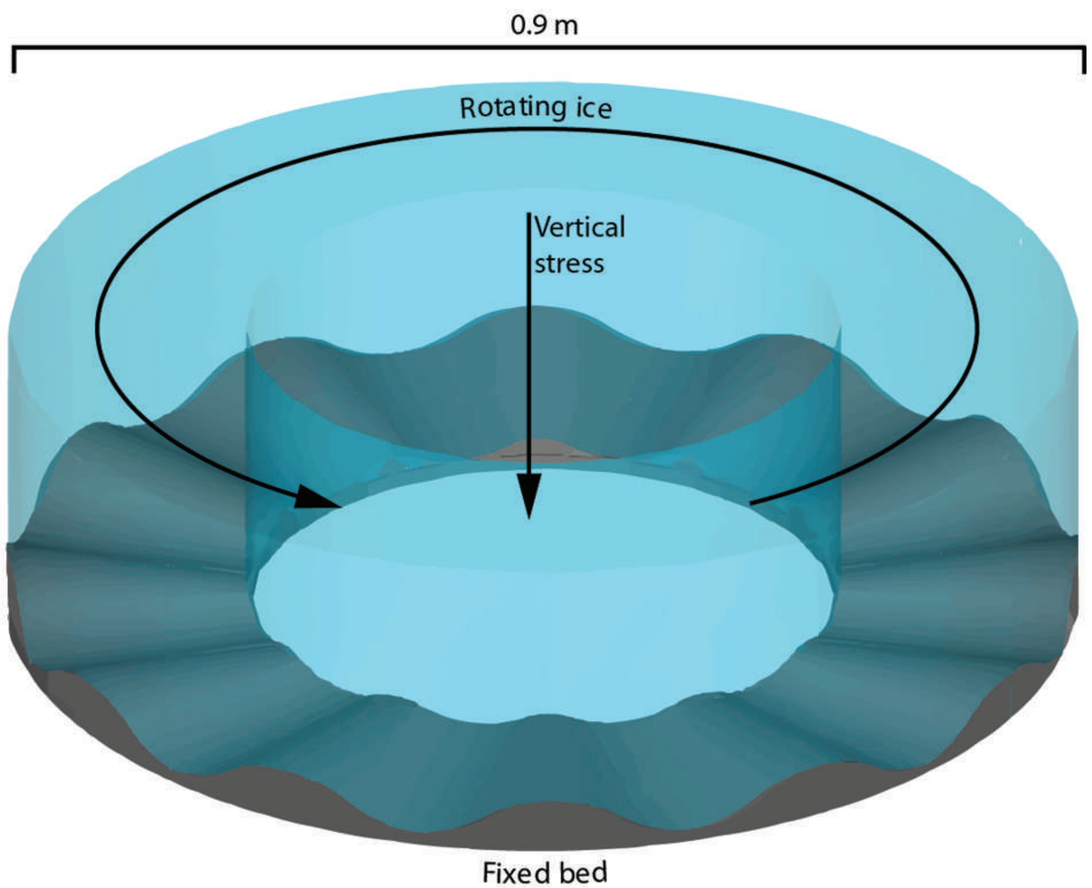
Schoof suggested an alternative with a maximum shear stress

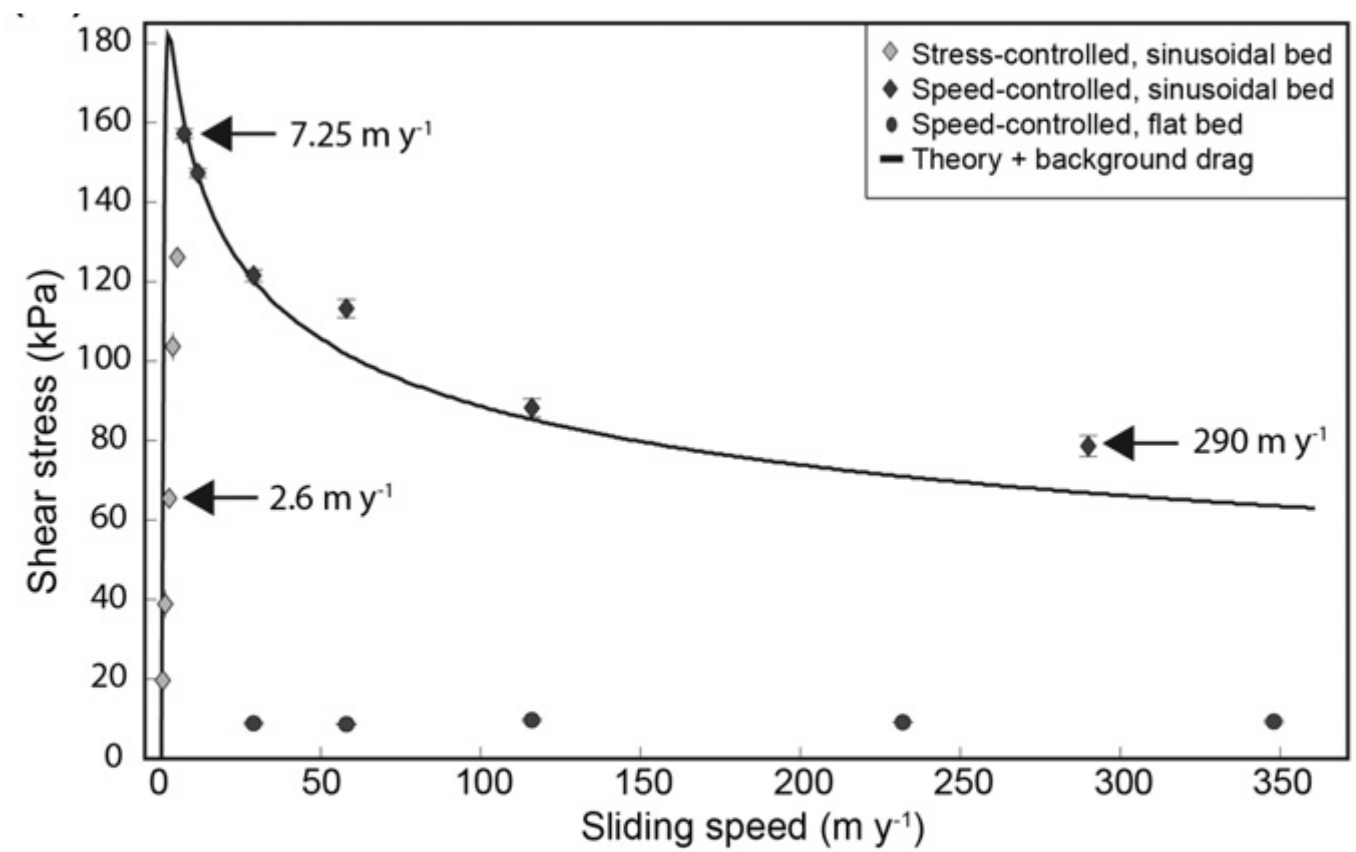


⇨ Regularised 'Coulomb' law

$$\frac{\tau_b}{N} = \mu \left(\frac{U_b}{U_b + \lambda A N^n} \right)^{1/n}$$

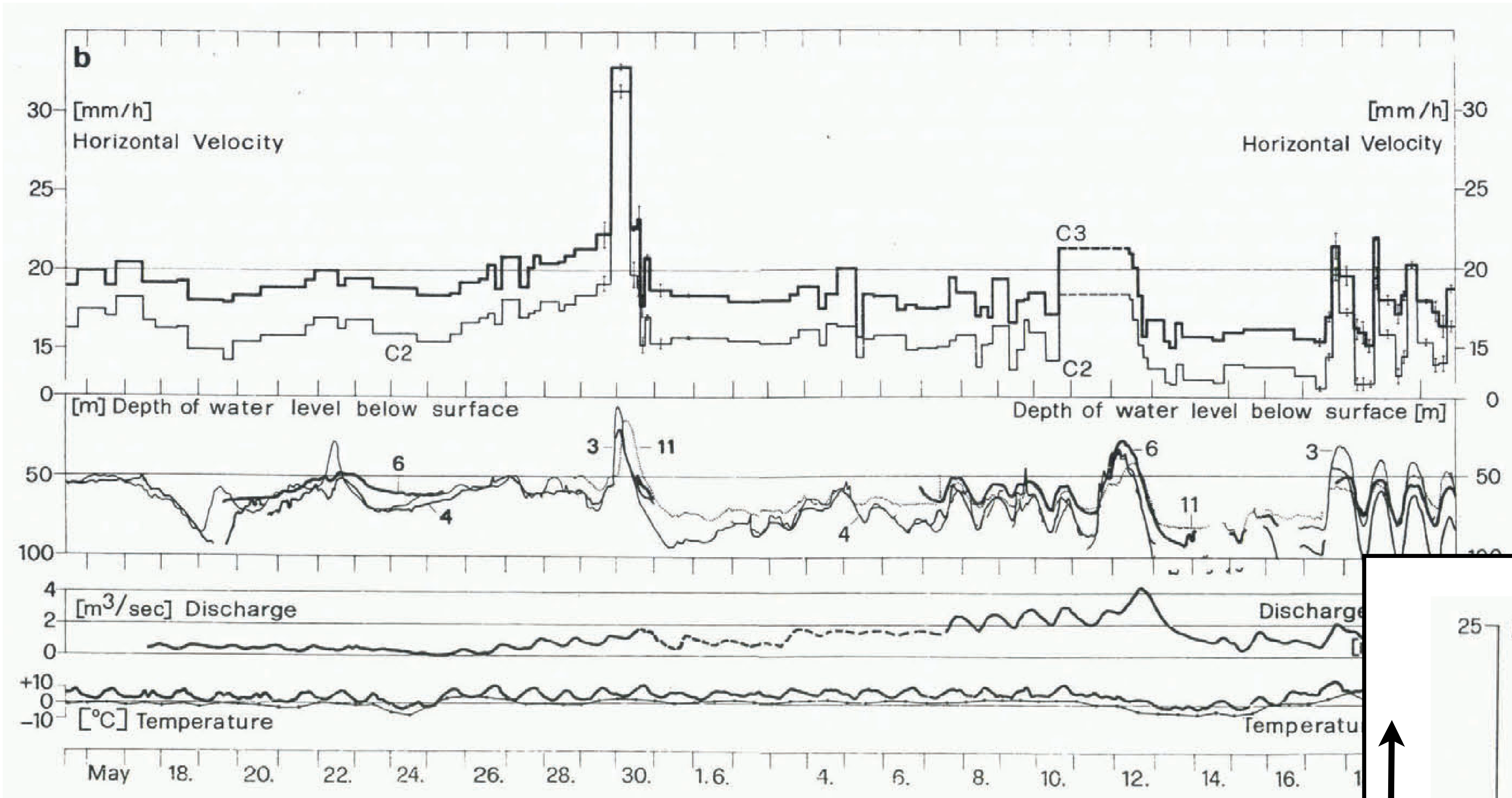
Laboratory experiments Iverson & Zoet 2015





Zoet & Iverson 2015

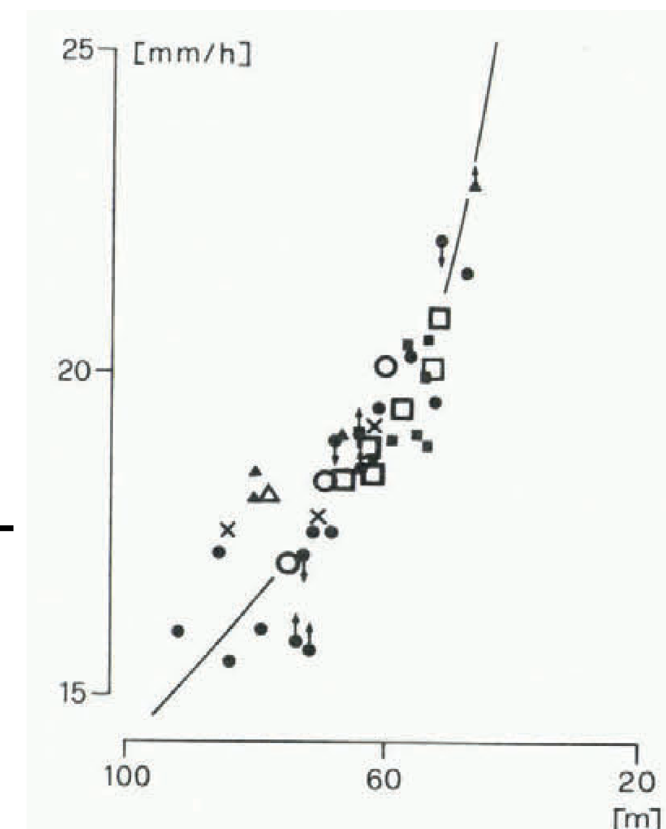
Field measurements



Iken & Bindschadler 1986

Speed

Water pressure



Speed

Water pressure

Iken & Bindschadler 1986

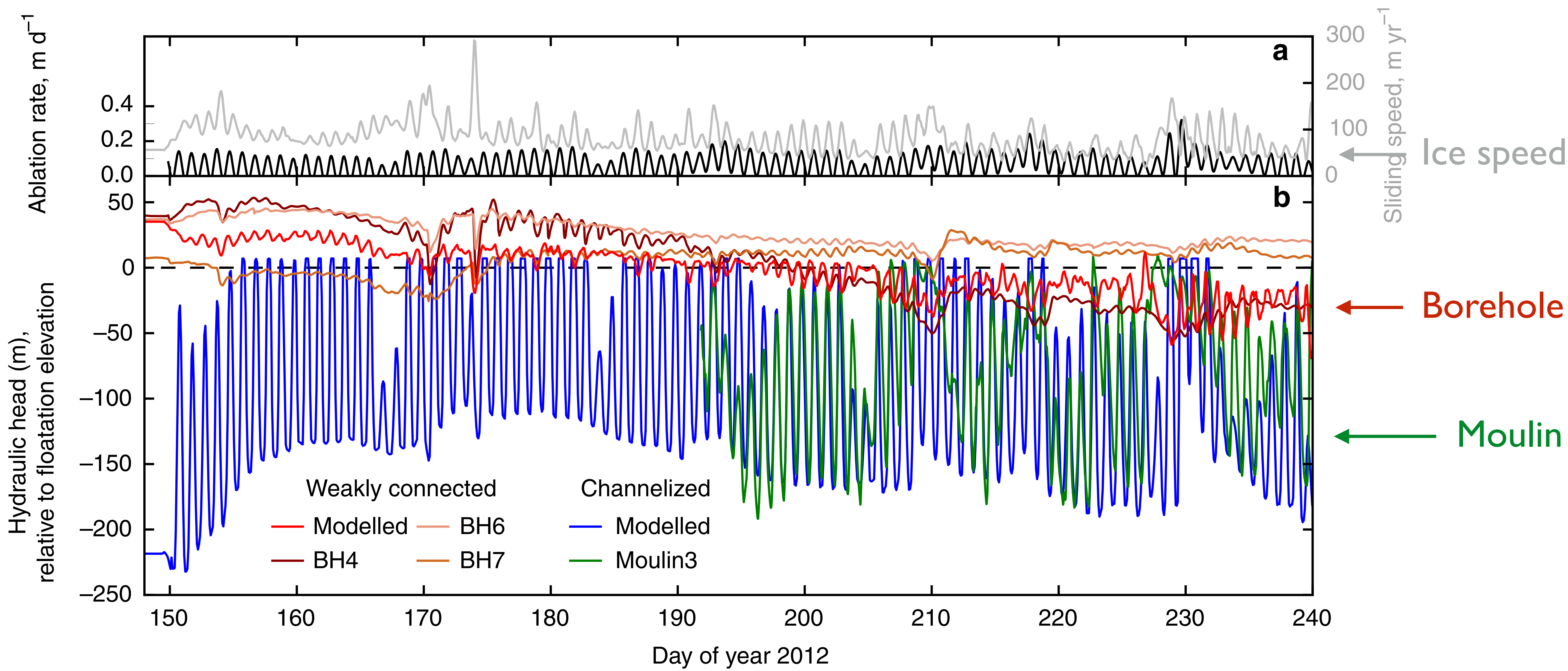
Some field work shows a definite relationship between ice speed and borehole water pressure eg. Bindschadler 1983, Iken & Bindschadler 1986

However, a consistent relationship is **not** always observed

eg. Sugiyama & Gudmundsson 2004, Harper et al 2007, Howat et al 2008, Fudge et al 2009

Field measurements

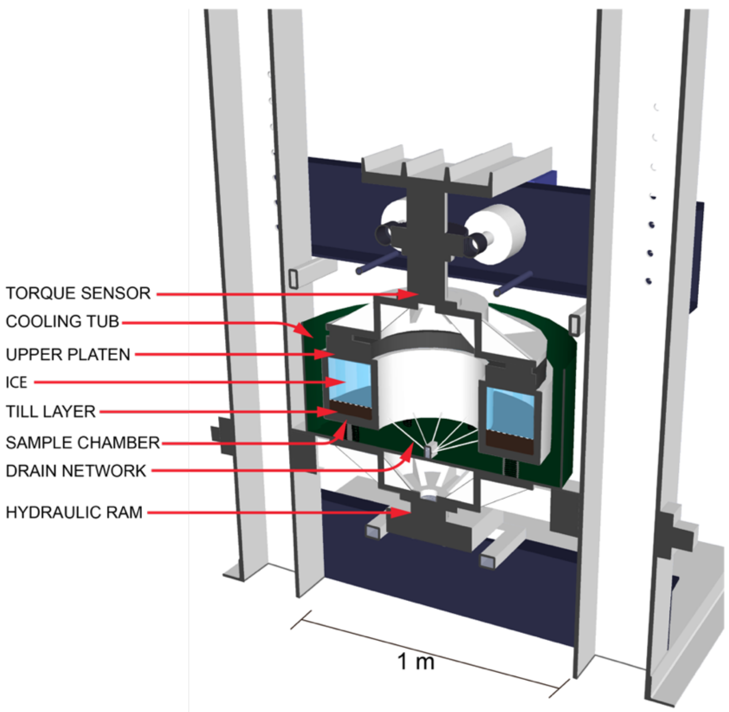
Measurements from **west Greenland** suggest diurnal variations in ice velocity correlate with water pressure in moulins, but are out of phase with pressure in boreholes.



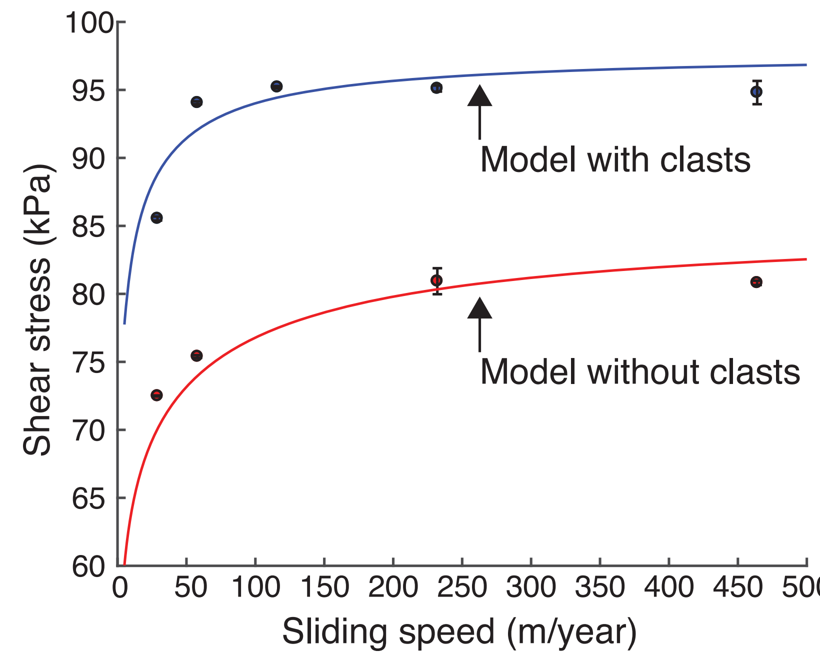
Hoffman et al 2016

Soft-bed sliding

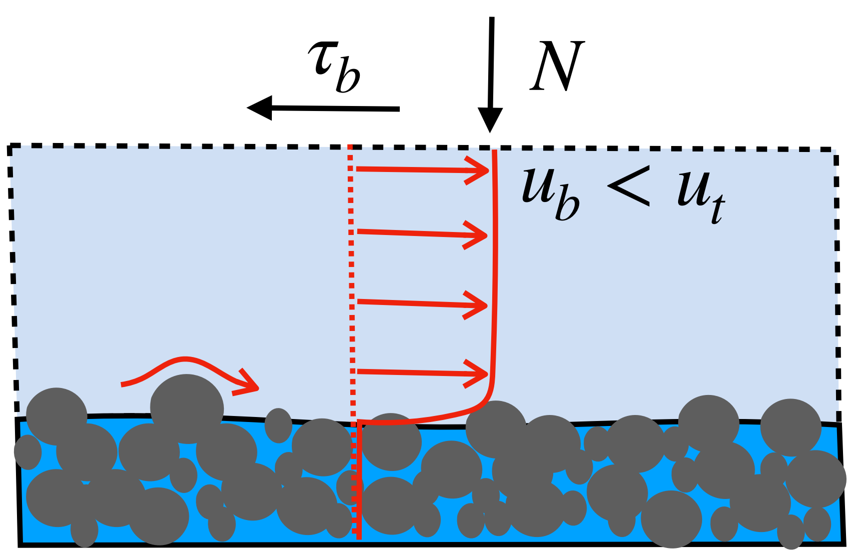
Laboratory experiments Zoet & Iverson 2020



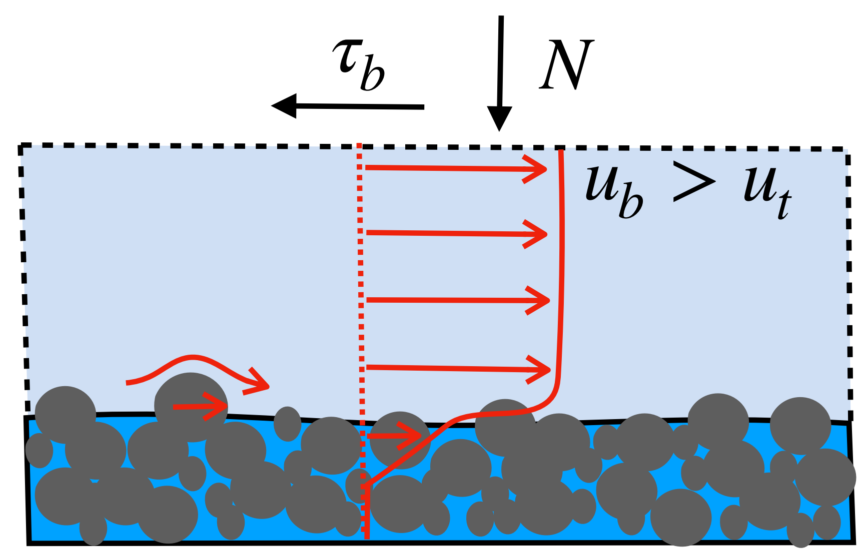
Shear stress τ_b



Sliding speed u_b

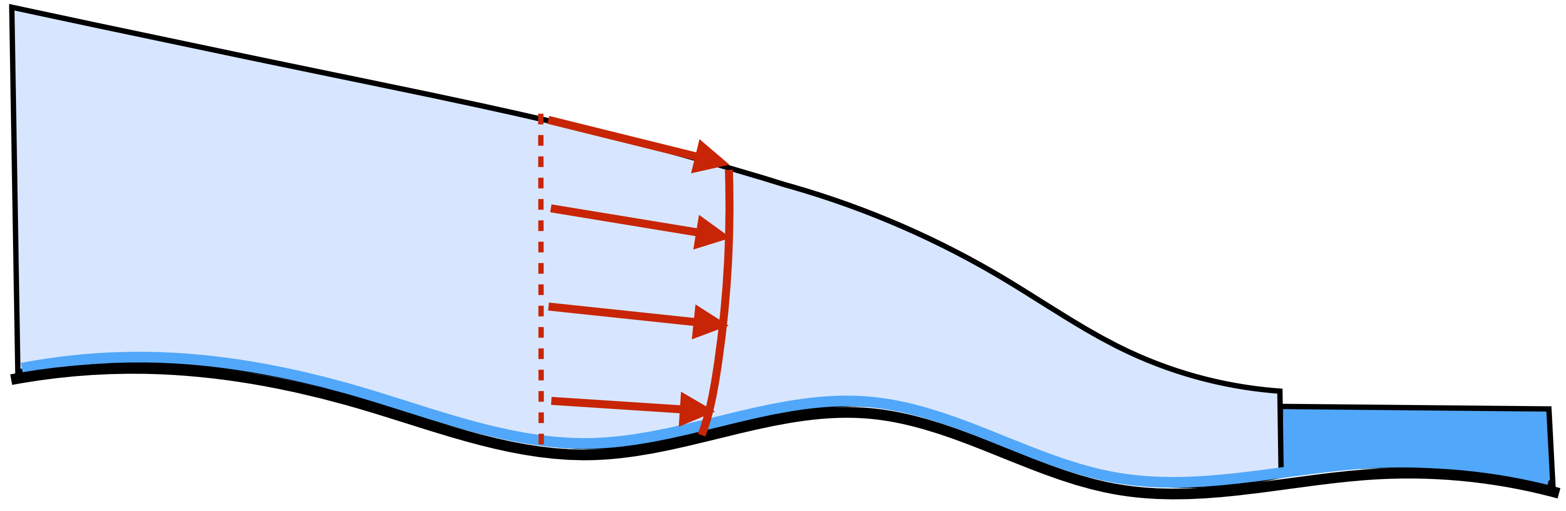


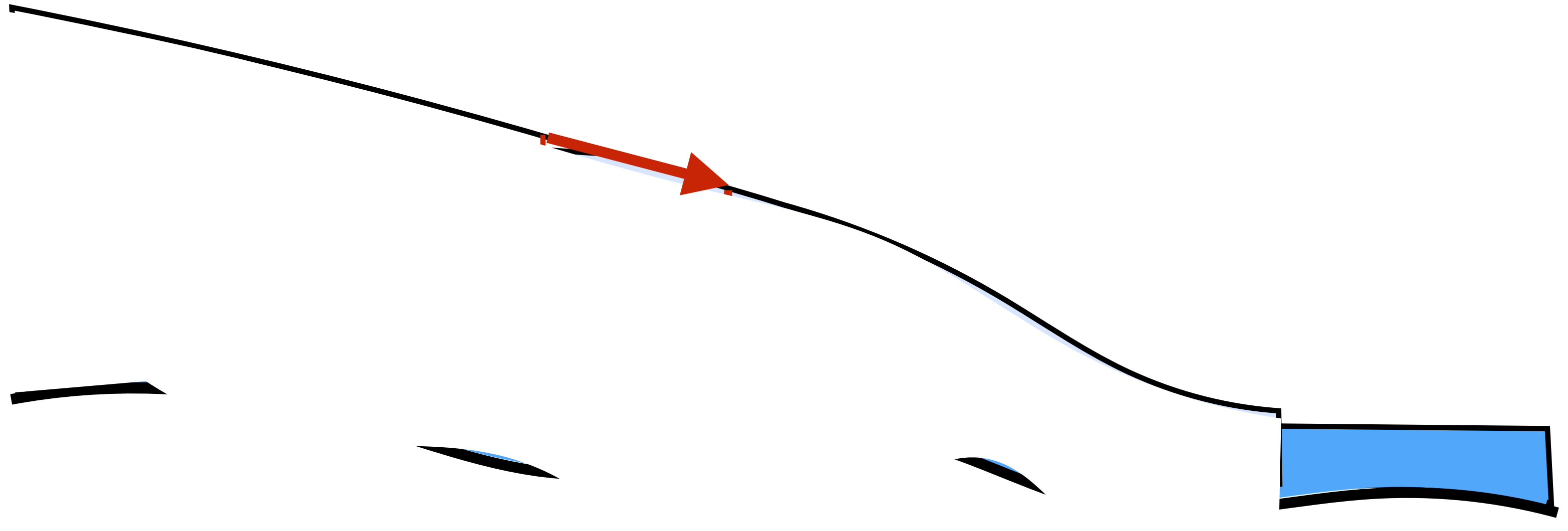
Unyielded till - slip at interface



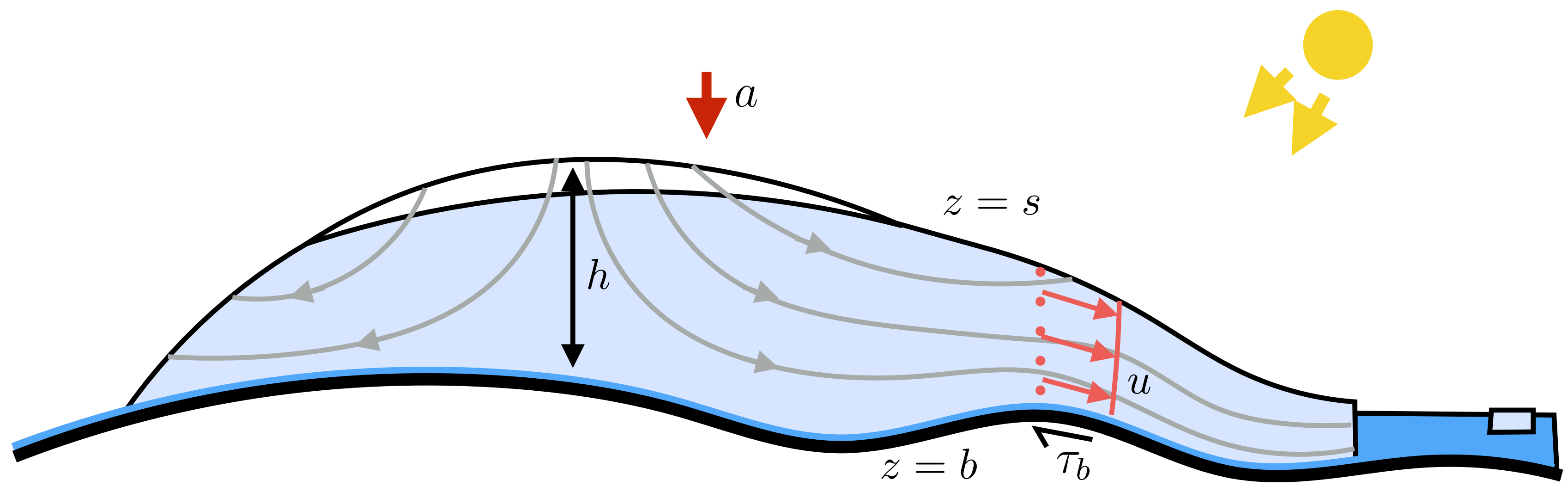
Yielded till - larger clasts plough through deforming till

Ice-sheet modelling and basal inversions





Inverse methods



Forward model eg SSA

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{u}) = a$$

$$\mathbf{0} = -\rho g h \nabla s - C|\mathbf{u}|^{m-1}\mathbf{u} + \nabla \cdot (h\boldsymbol{T}) \qquad \boldsymbol{T} = \eta \begin{pmatrix} 4\frac{\partial u}{\partial x} + 2\frac{\partial v}{\partial y} & \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} & 2\frac{\partial u}{\partial x} + 4\frac{\partial v}{\partial y} \end{pmatrix}$$

Maps input parameters to outputs $\mathcal{F} : P \to Y$

Running the model gives $\mathbf{y} = \mathcal{F}(\mathbf{p})$ which we can compare with observations $\mathbf{y}_{\text{obs}}$

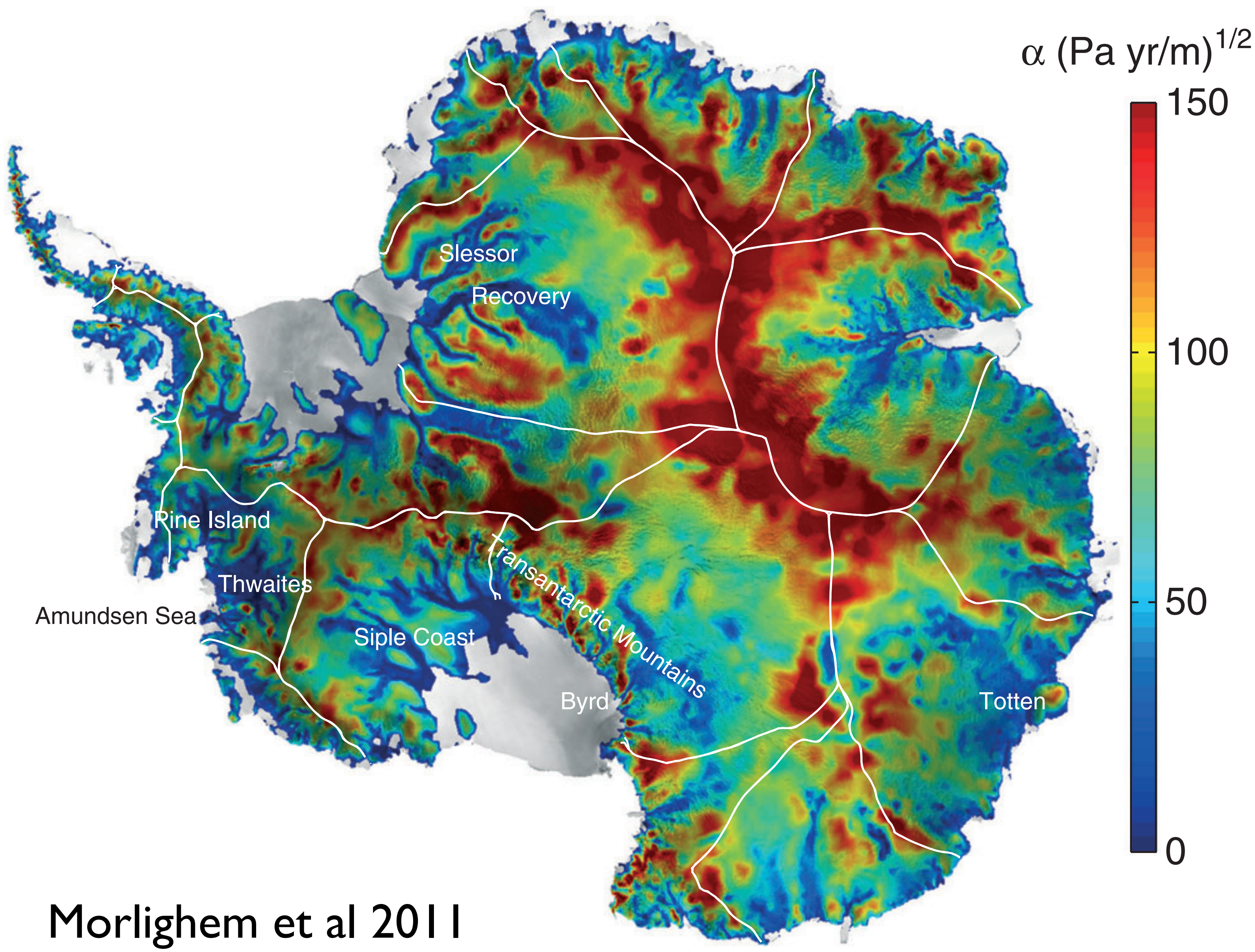
Inverse methods used to find input parameters that best fit observations

(or to find a ‘posterior’ probability distribution)

Minimise a cost function

$$\mathcal{J}(\mathbf{p}) = \frac{1}{2}\int_{\Omega} |\mathbf{y} - \mathbf{y}_{\text{obs}}|^2 \, \mathrm{d}S + \mathcal{R}(\mathbf{p})$$

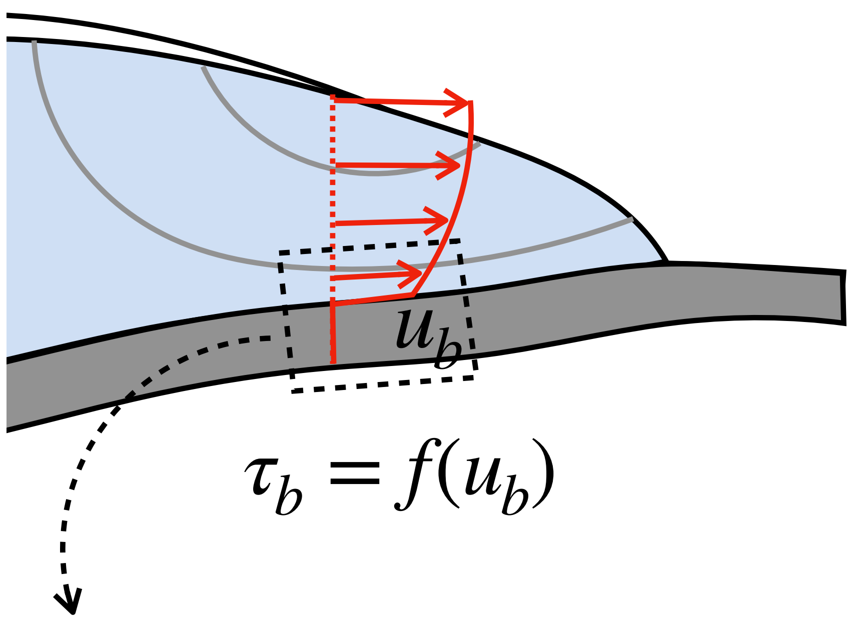
Inferred basal friction coefficient

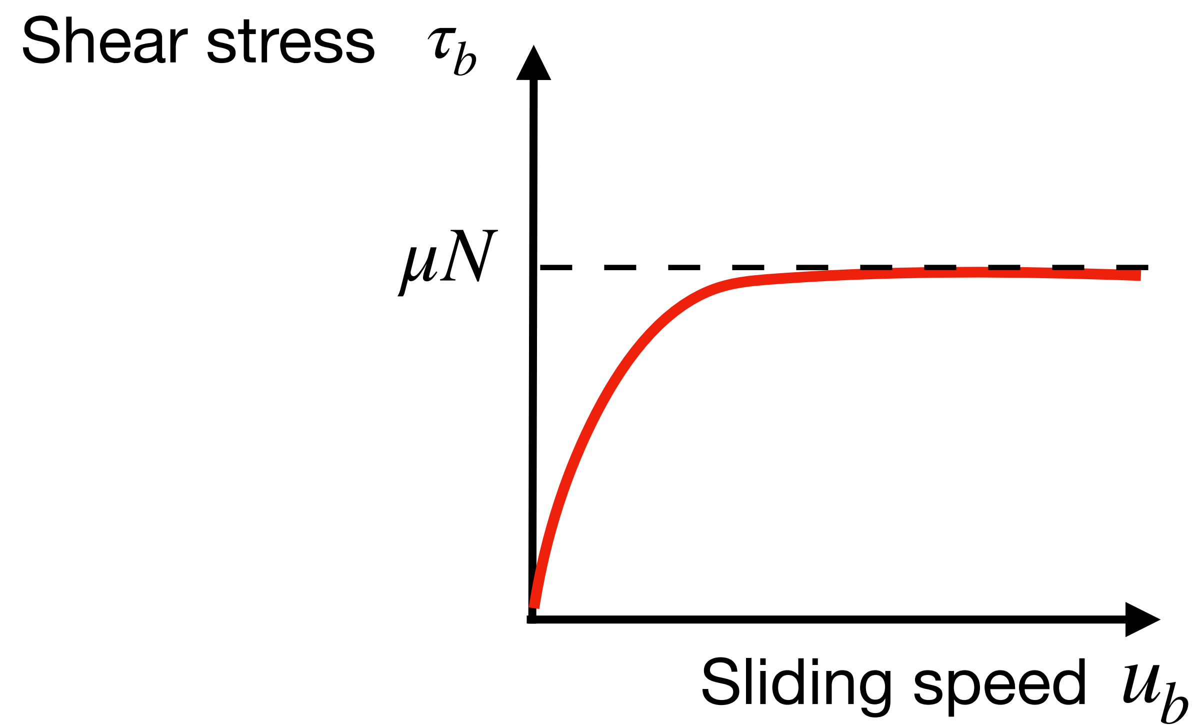


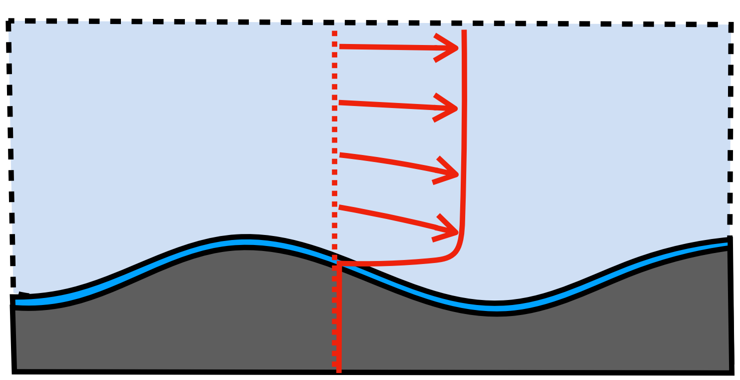
Morlighem et al 2011

Note: the 'correct' friction law and value of coefficients depend on the **resolution** of your model (the friction law is to describe unresolved processes!)

Summary

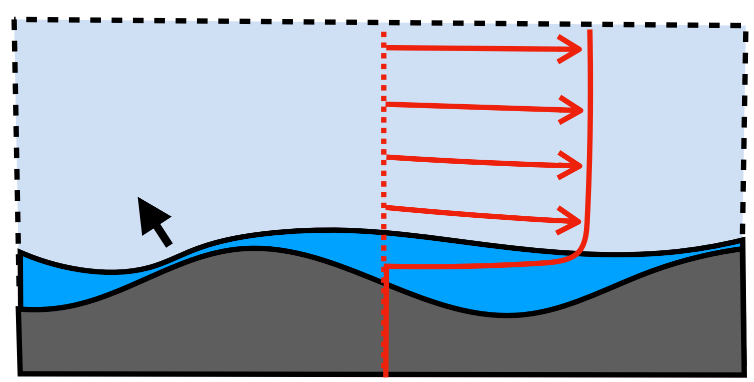






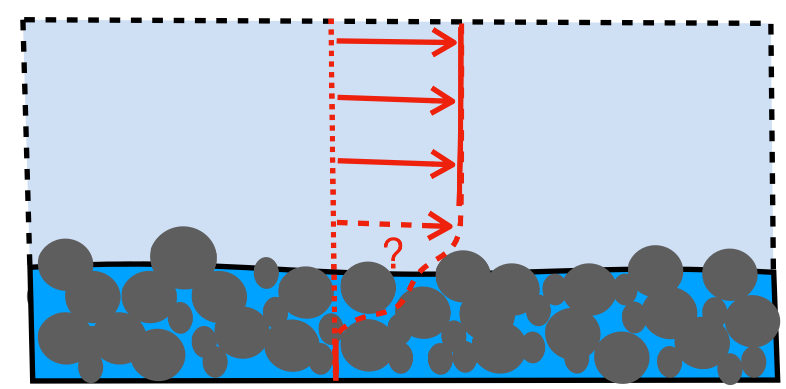
Hard-bed sliding

$$\tau_b = C u_b^m$$



Sliding with cavitation

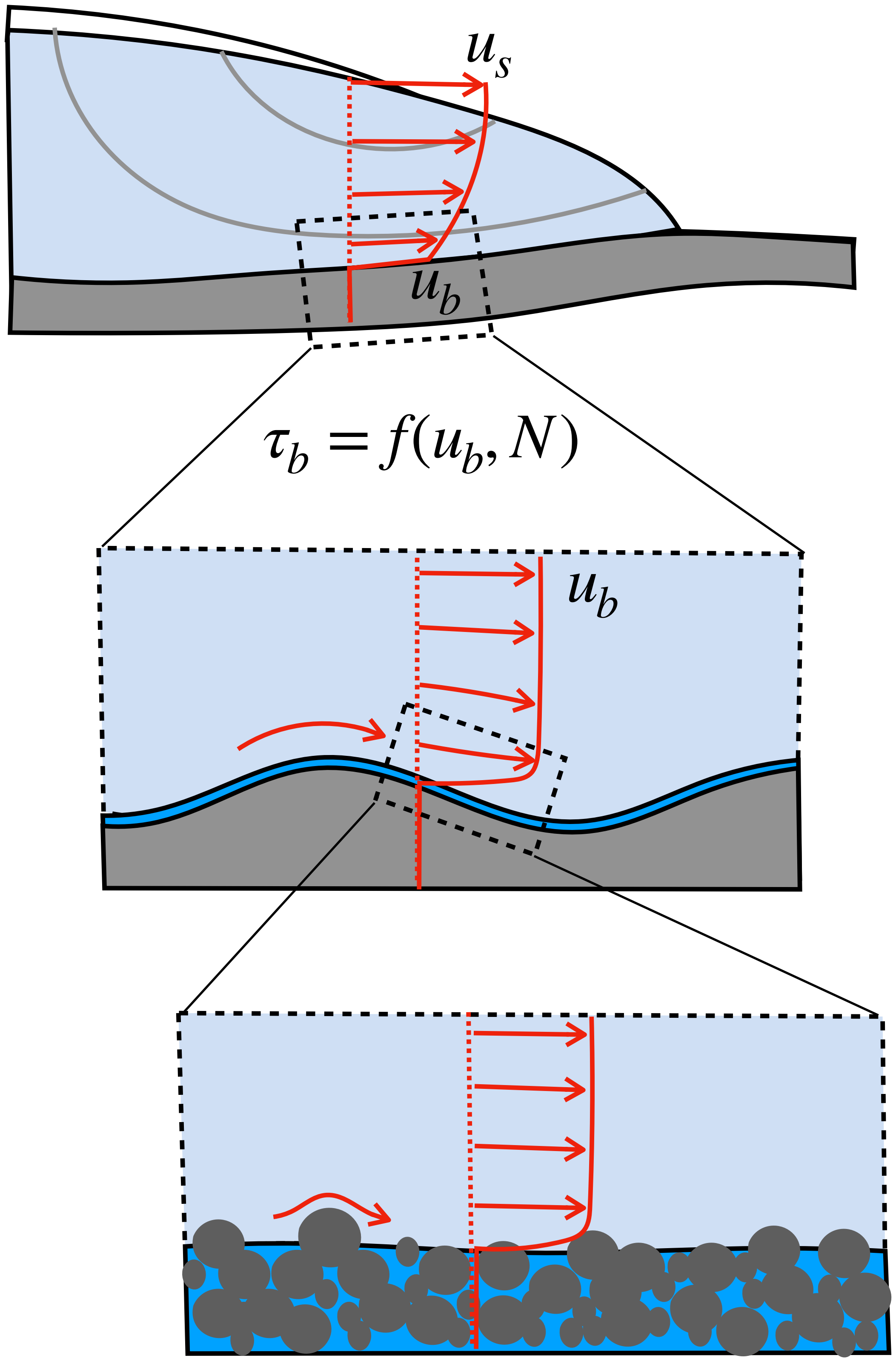
$$\tau_b = \mu N \left(\frac{u_b}{u_b + \lambda N^n} \right)^{1/n}$$



Soft-bed sliding

$$\tau_b = \mu N$$

The importance of 'form drag'



The sliding law needs to account for **all sub-grid scale 'roughness'**.

That often includes larger scales than those for which cavitation / bed deformation are relevant.

$$\tau_b = \mu N \left(\frac{u_b}{u_b + \lambda N^n} \right)^{1/n} + C u_b^{1/n}$$