# EXAM III PRACTICE

(A) For 1–4, you are given $f(x)$ on $[a,b]$. Find (but don't solve) integrals which compute

a) Arclength of graph of $f$ above $[a,b]$

b) Surface area when that graph is rotated about $x$-axis

1. $f(x) = \sqrt{3-x^2}$, on $[0,\sqrt{3}]$
2. $f(x) = \ln(x)$, on $[1,e^2]$
3. $f(x) = x\cos^2(x)$ on $[0,\pi]$
4. $f(x) = (x/2)^4 + (1/2x^2)$ on $[1,4]$ ← (These integrals you can really solve!)

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(B) 5. Calculate the work done to build a pyramid with square base of side length 2 and height 4, assuming your material has constant density $\rho$.

6. Same as above, but with a cone of base-radius 1 and height 4.

7. Find the present-value of the income-stream

$$I(t) = \begin{cases} t & \text{for } 0 \le t \le 10 \\ t^2 & \text{for } 10 \le t < \infty \end{cases}$$

If the interest rate is 10%.

8. Let $R$ be the region between $y = 1-x$ and $y = 1-x^2$ in the positive quadrant $(x \ge 0, y \ge 0)$

Find the moment of inertia of R when rotating about the y-axis, assume density $\rho(x) = \frac{1}{1-x}$.

9. A hemispheric cup (i.e, ) of radius R is filled with (constant-density) water to full capacity. How much work is done against gravity?

10. An isosceles triangle T sits with its base on the x-axis from $x=1$ to $x=2$. If it has a height of 1, find the moment of inertia when rotating about the y-axis. Assume density $\rho(x) = 1/2x$ throughout.

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(C) For each $f(x)$ on the interval $[a,b]$ given below, find

a) $c$ so that $cf(x)$ is a prob. density on $[a,b]$
b) $\mathbb{E}[X]$, the expectation.
c) $\mathbb{V}[X]$, the variance.

Setting up the integrals is more important than solving...

11. $f(x) = e^{-2x}$ on $[1,\infty)$

12. $f(x) = \ln(1+x)$ on $[e-1, e^2-1]$

13. $f(x) = \frac{1}{1+x^2}$ on $[-\infty,\infty]$

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(D) Given $f(x)$ and $g(x)$ on $[a,b]$, find centroids of the region between graphs of $f$ and $g$ on $[a,b]$ when density is $\rho(x)$

14. $f(x) = x^2$, $g(x) = \sqrt{x}$ on $[0,1]$, $\rho = 1$

15. $f(x) = e^x$, $g(x) = 1$ on $[0,1]$, $\rho(x) = x$

16. $f(x) = \ln x$, $g(x) = x-1$ on $[1,3]$, $\rho = 1$.