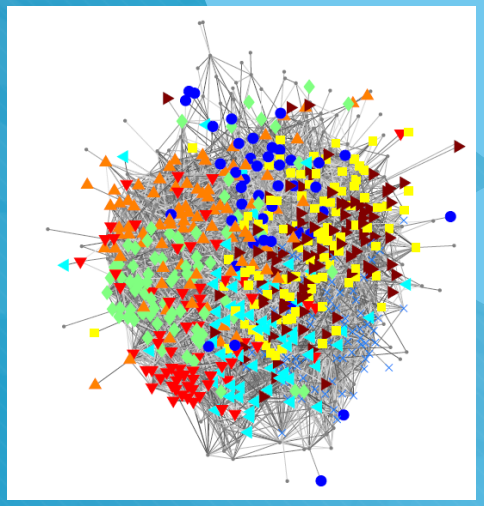
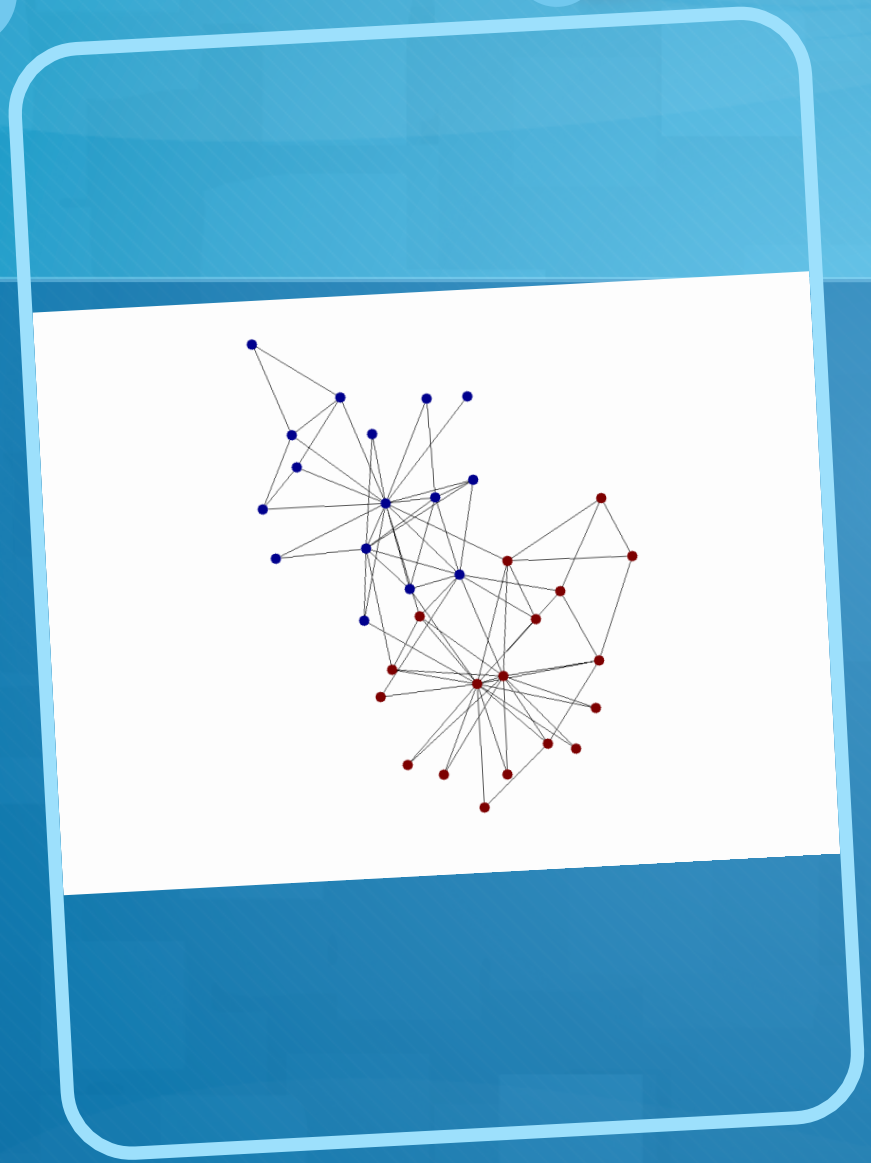


Mathematics and Social Networks

Mason A. Porter (applied mathematics tutor)

Somerville College
University of Oxford



What is a network?

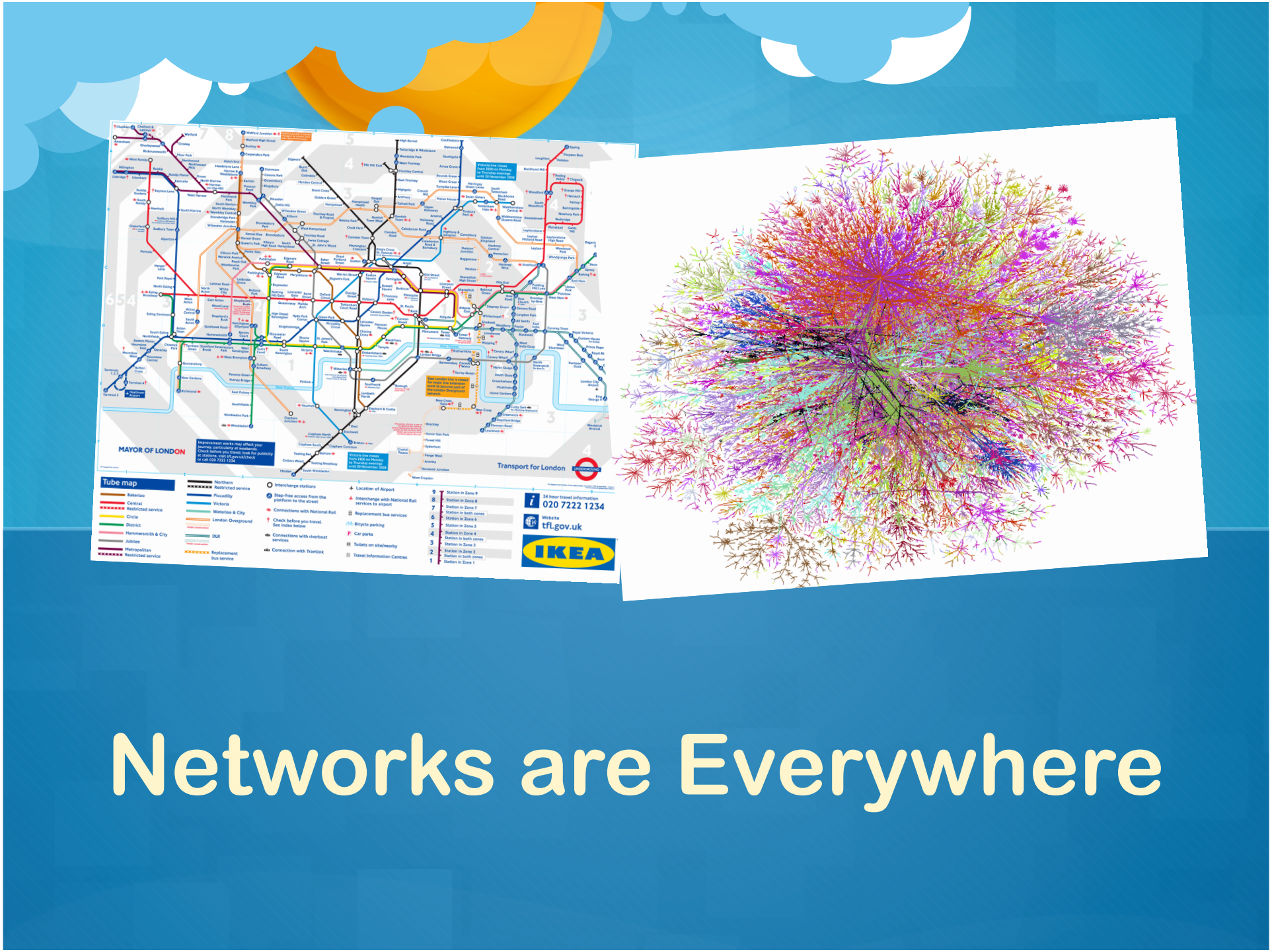
A network consists of *nodes* representing entities. Nodes are connected by *edges* representing ties between the entities.

Examples:

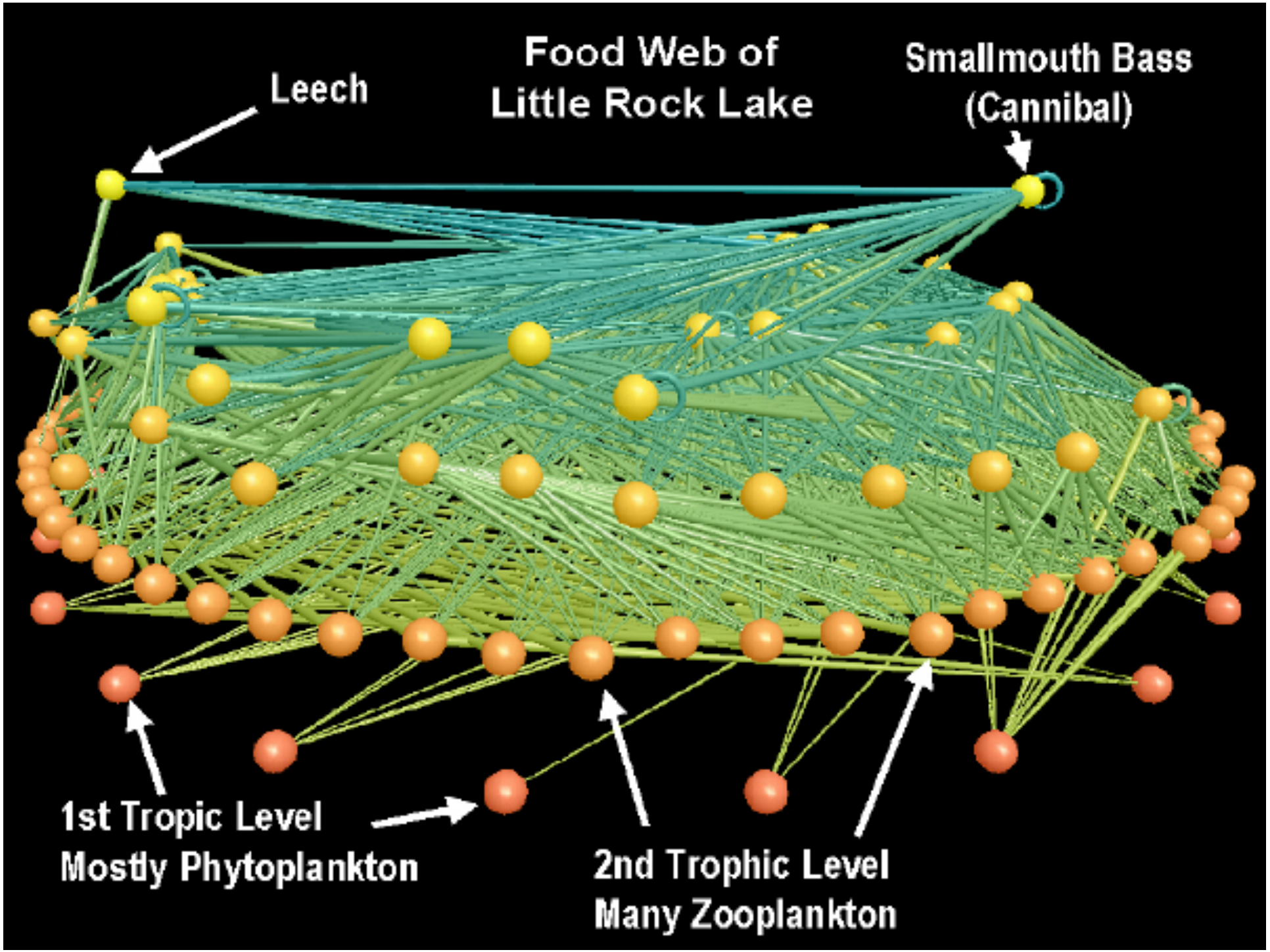
Individuals connected by Facebook “friendships”.

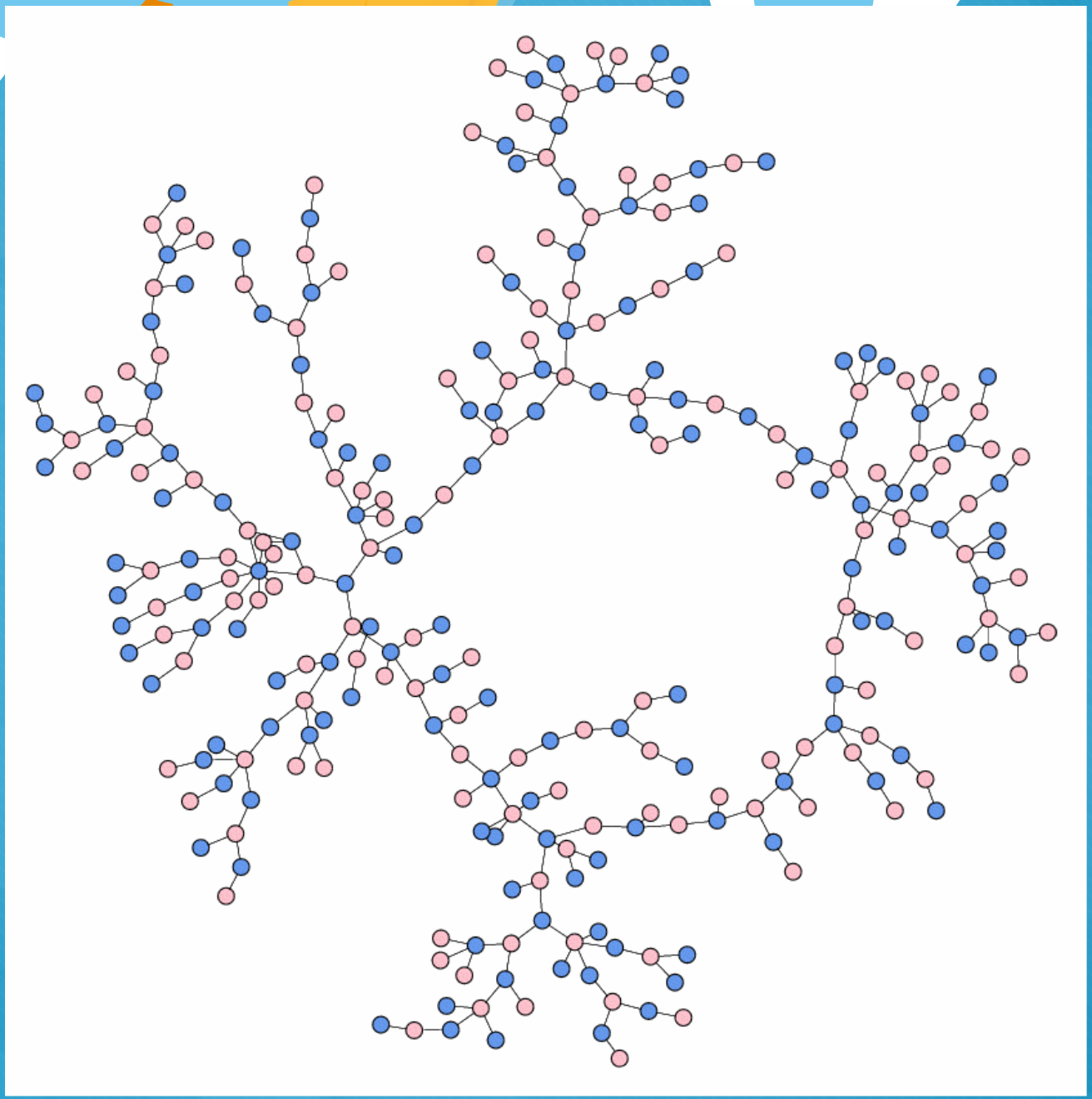
Web pages connected by hyperlinks.

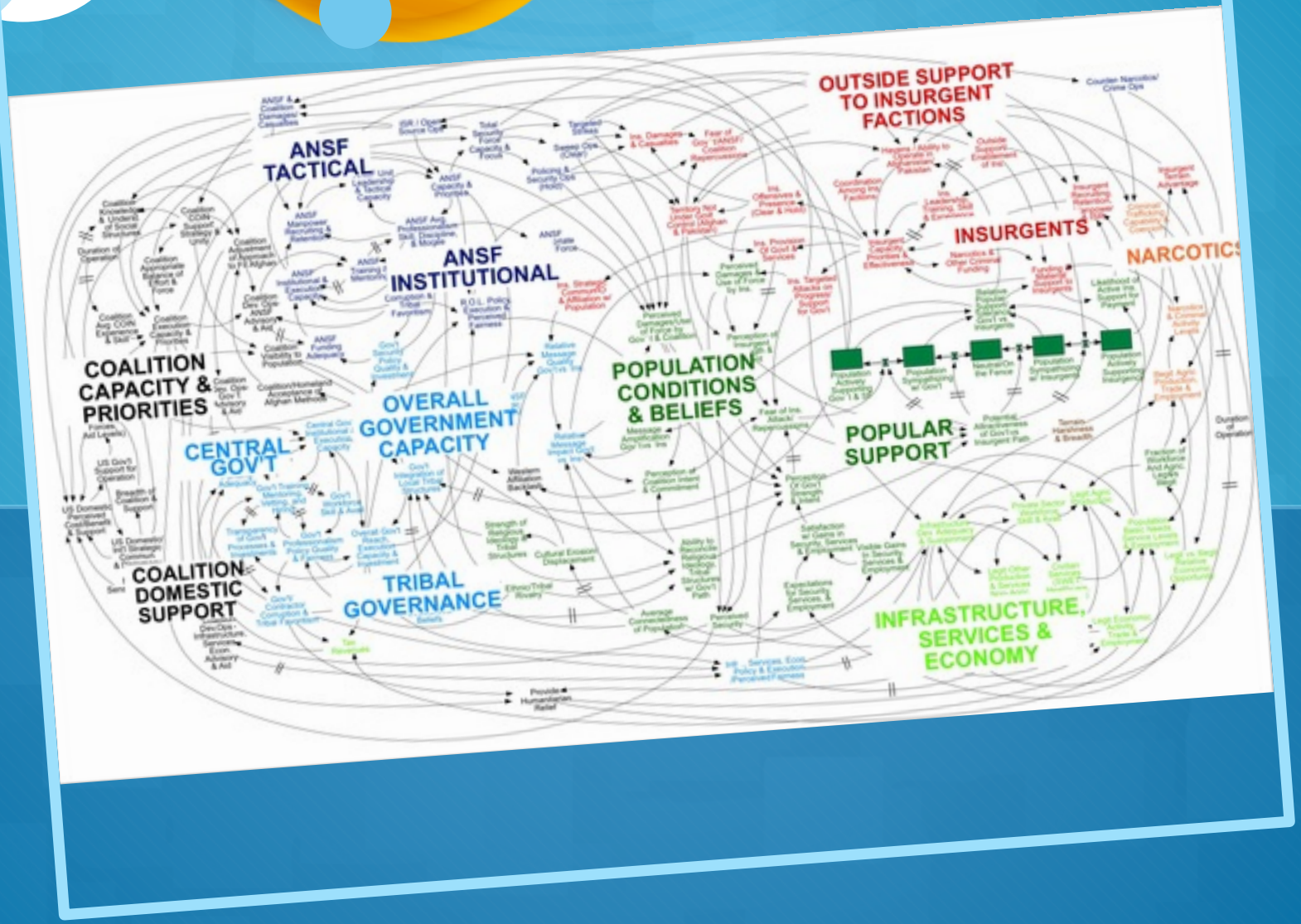
Contiguous cities on a train route.



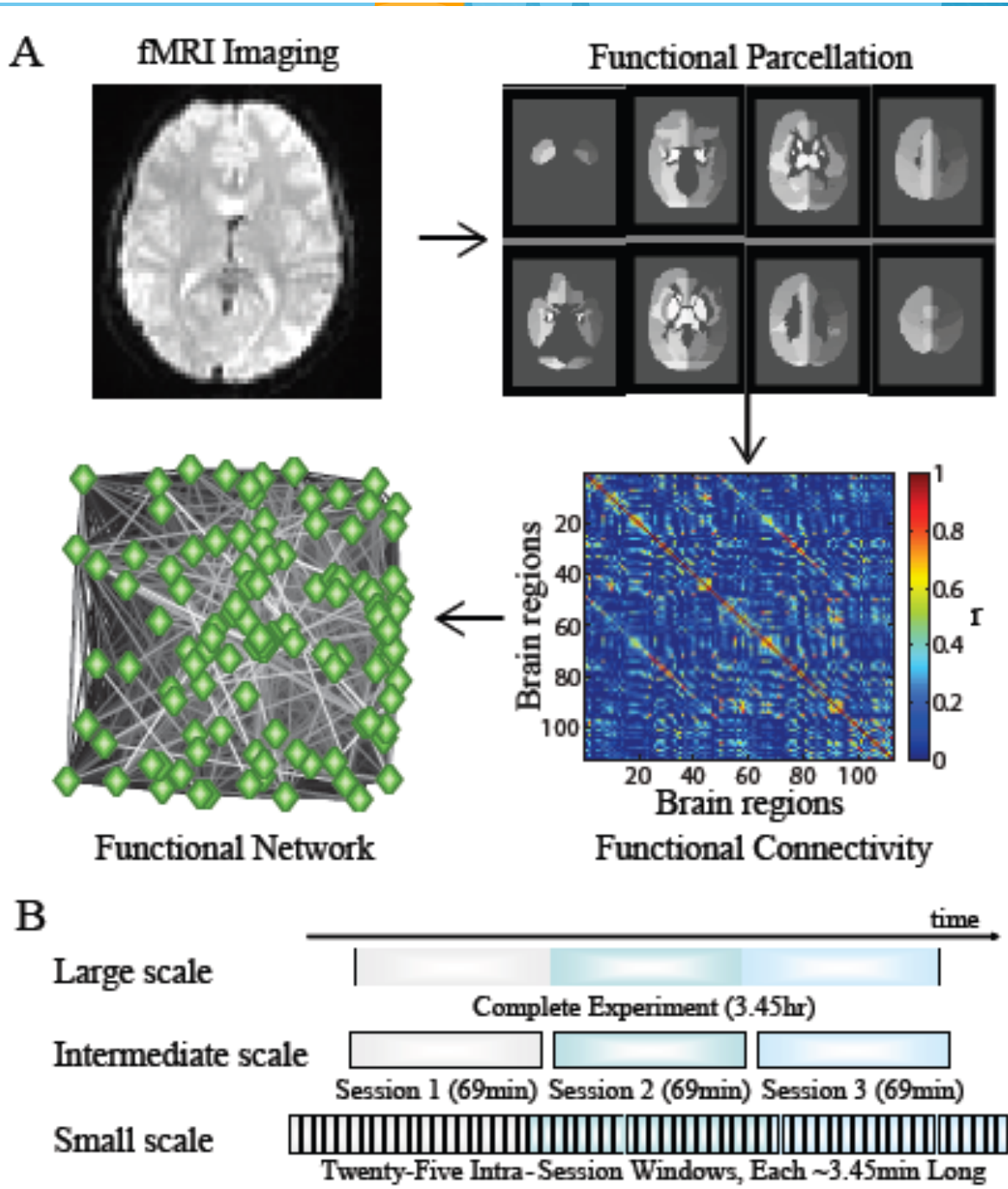
Networks are Everywhere


Food Web of
Little Rock Lake
Leech
Smallmouth Bass
(Cannibal)
1st Tropic Level
Mostly Phytoplankton
2nd Trophic Level
Many Zooplankton





"When we understand this slide, we'll have won the war."


A
fMRI Imaging
Functional Parcellation
Functional Network
Functional Connectivity
Brain regions
Brain regions
r
B
time
Large scale
Complete Experiment (3.45hr)
Intermediate scale
Session 1 (69min)
Session 2 (69min)
Session 3 (69min)
Small scale
Twenty-Five Intra-Session Windows, Each ~3.45min Long

Types of Networks

- **Binary networks:** 1 if there is a connection and 0 if there isn't
- **Weighted networks:** Some value if there is a connection (representing strength of connection) and otherwise 0
- **Directed networks**
- **"Bipartite" networks:** only nodes of different types are connected to each other (e.g., an actor connected to a movie in which he/she appeared)
- **More …**

Representing a Network

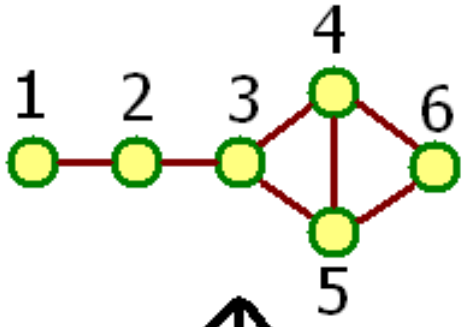
Adjacency matrix A

This example: binary ("unweighted")

A_{ij} = 1 if there is a connection between nodes I and j

A_{ij} = 0 if no connection

Question: How do we generalize the mathematical representation to weighted, directed, and bipartite examples?



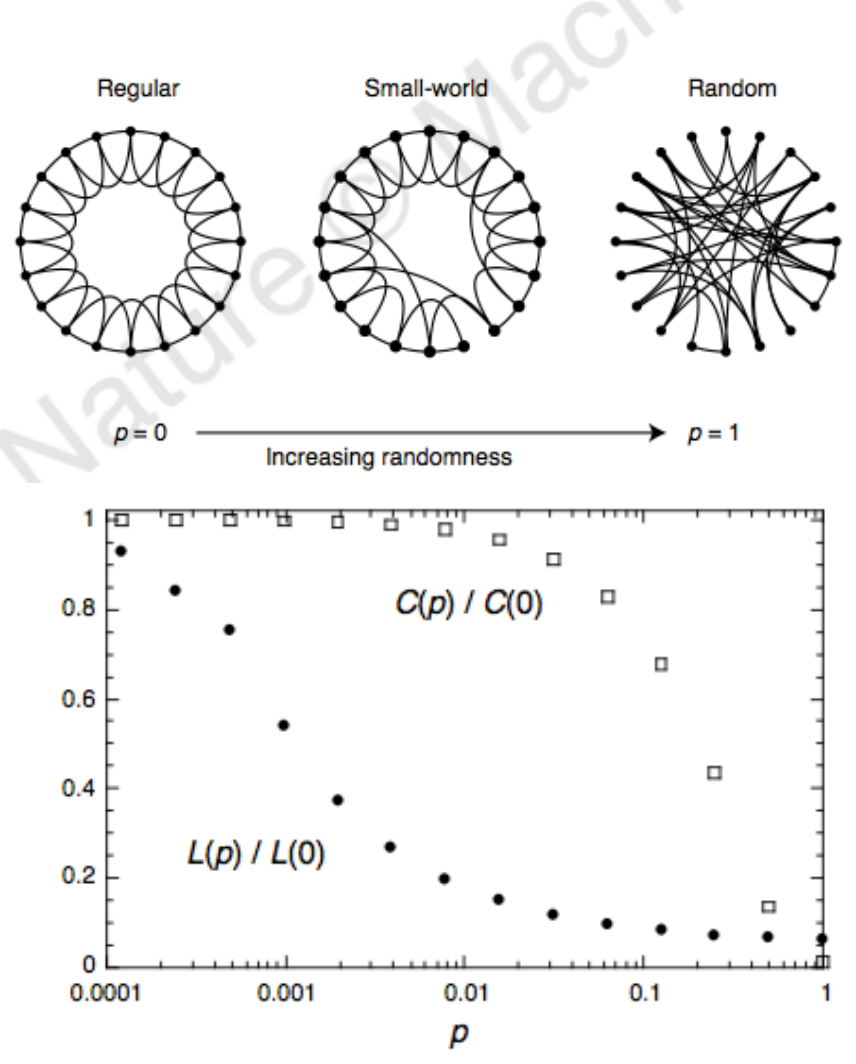
$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Goals of “Network Science”

- 1. Basic principles
  - Microscopic, mesoscopic, and macroscopic structures
- 2. Function = structure + dynamics
  - Dynamics *on* networks, dynamics *of* networks, interactions between the two
    - Time-evolution, robustness, etc.
- 3. Application
  - Inference/prediction of structure, demographics, etc.
  - Manipulation and design

Basic Principles

- Microscopic structure
  - Properties and roles of individual nodes and edges
  - E.g., Local clustering properties, node roles
- Mesoscopic and mascroscopic structure
  - Summary statistics
    - E.g., degree distribution, numerous types of centrality, assortativity, clustering coefficients, rich-club coefficient, etc.
    - caveats: sensitive to noise and perturbations, etc.
  - Modules and hierarchies
    - *Community structure*
    - Caveats: myriad ways to compute them, choice of "null model", etc.



Small Worlds

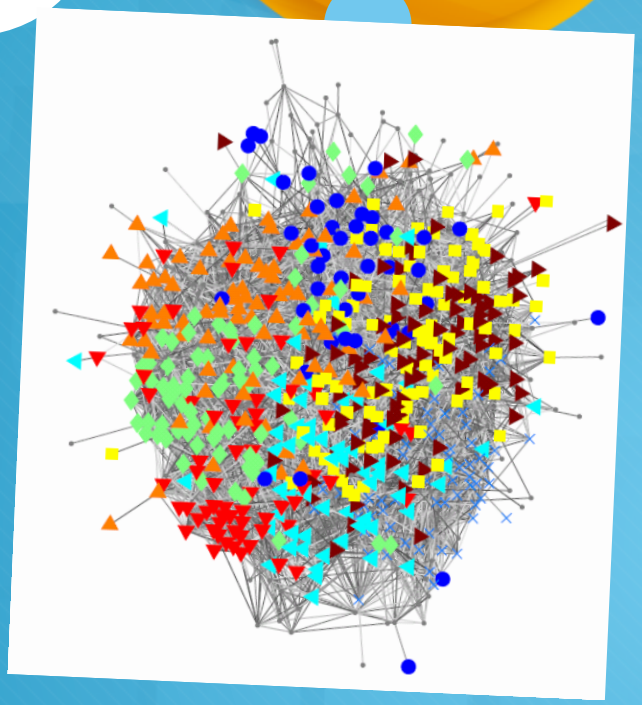
6 degrees of separation (psychologist Stanley Milgram)

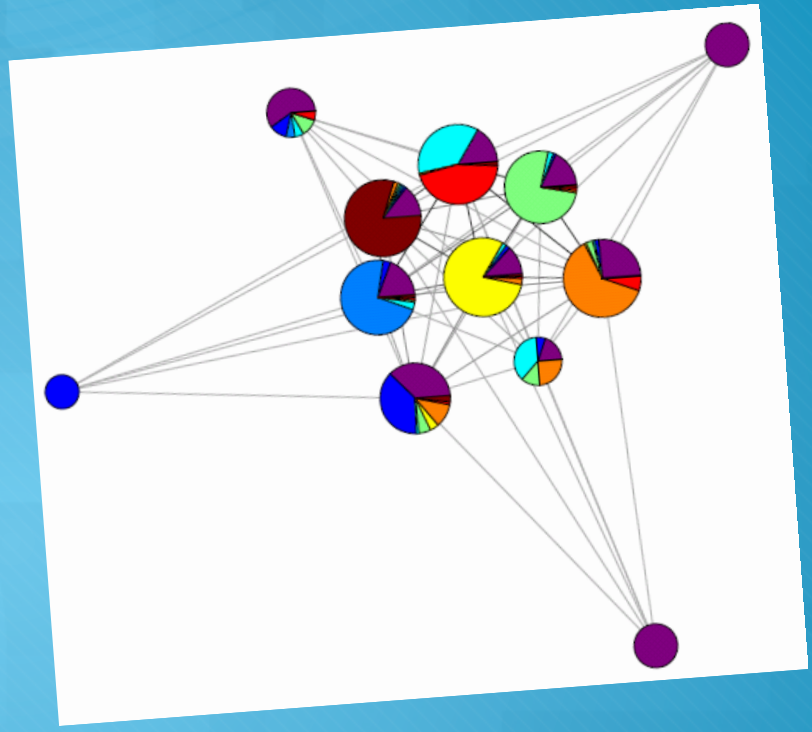
6 degrees of Kevin Bacon

Erdös numbers

Mathematical models developed starting in late 1990s to study this (starting with Watts & Strogatz, 1998)

How to navigate small worlds?

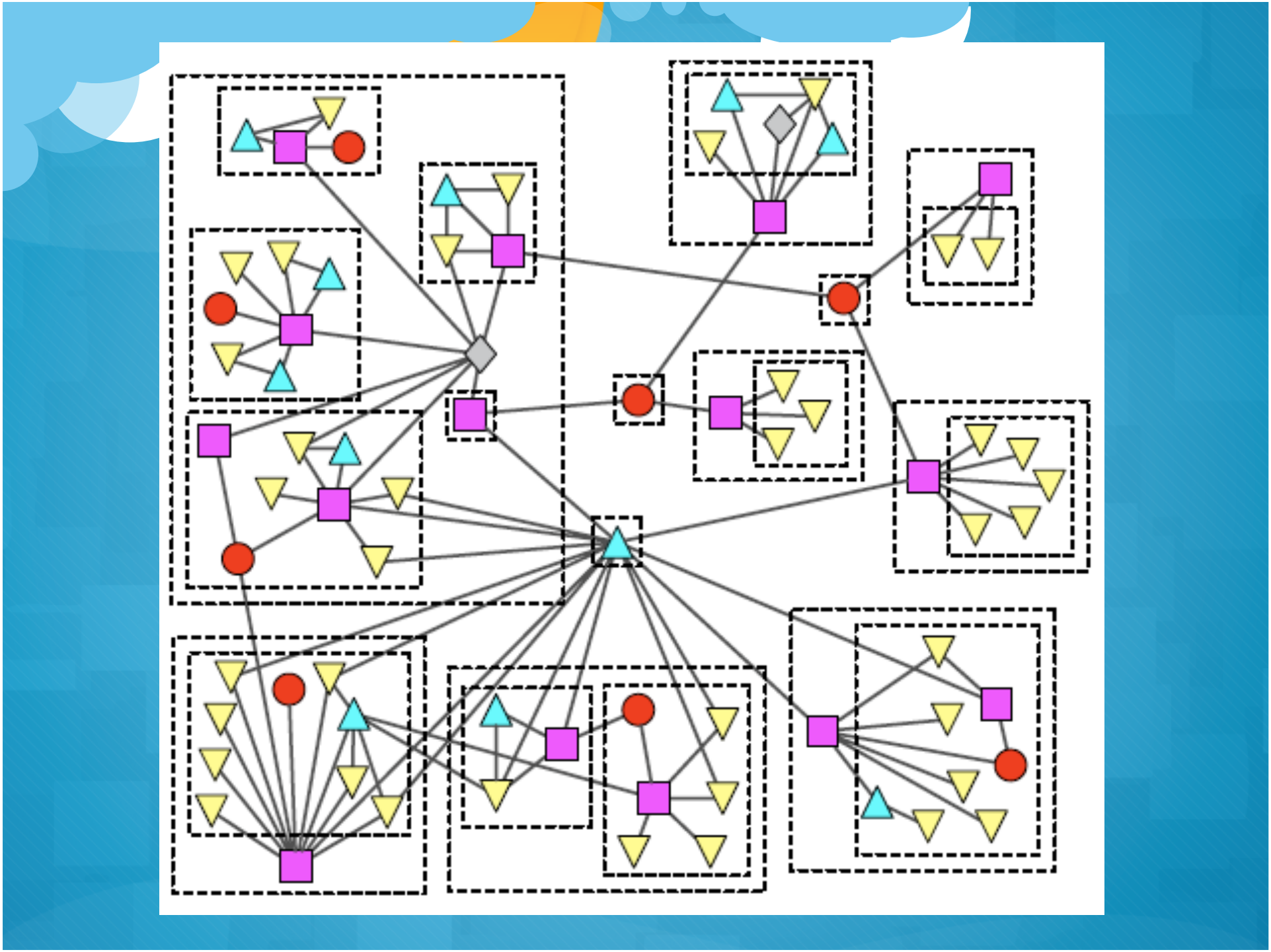


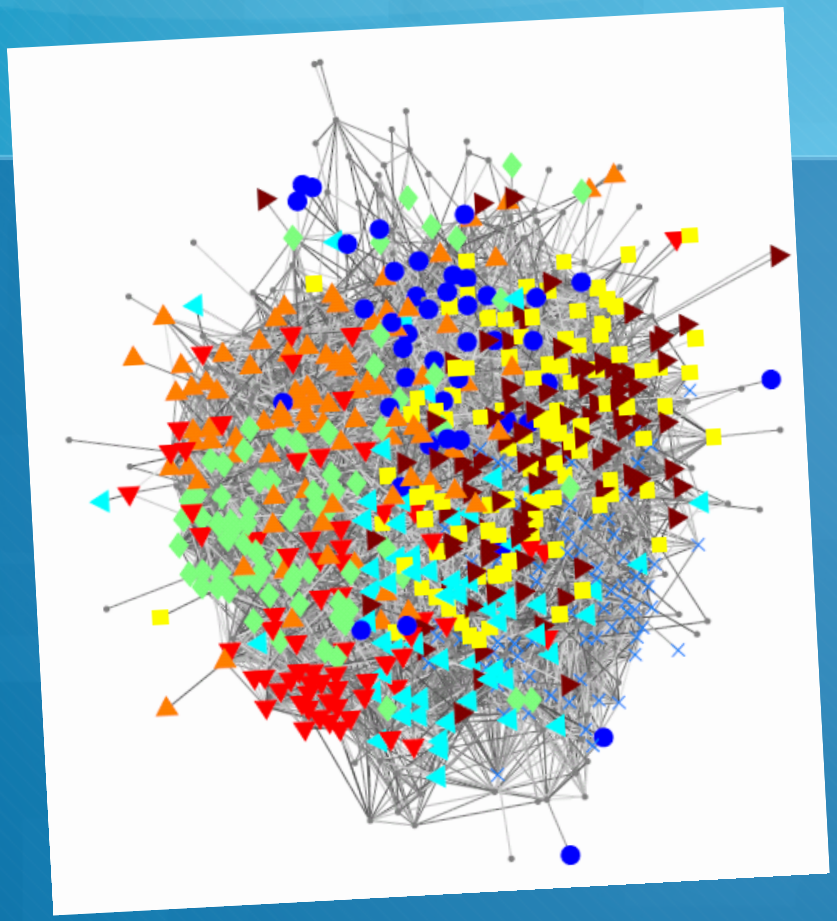


Facebook Friendship Networks

One way to try to understand the large-scale structure of networks is to group nodes into “communities” in which the nodes have a lot of ties among themselves (e.g. students in Somerville College)

Finding such groupings require the development of computer algorithms, which is one of the things that my students and I study.





Facebook Networks

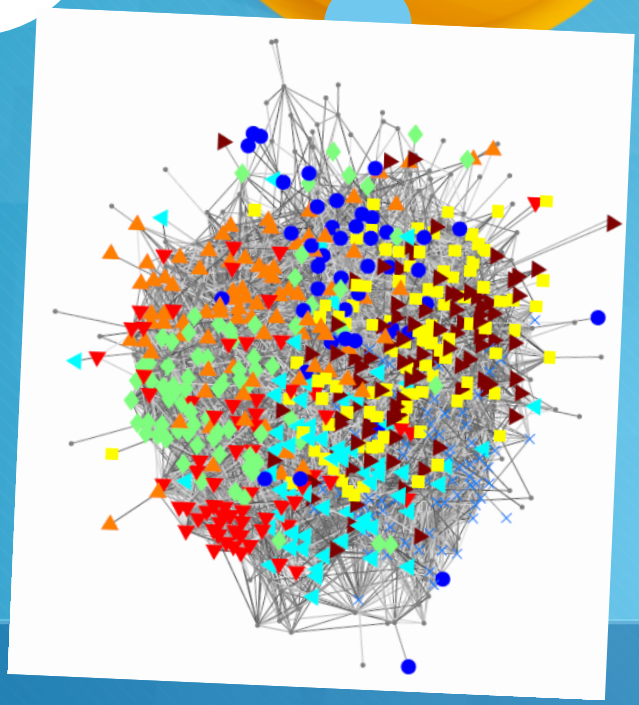
Nodes = individuals

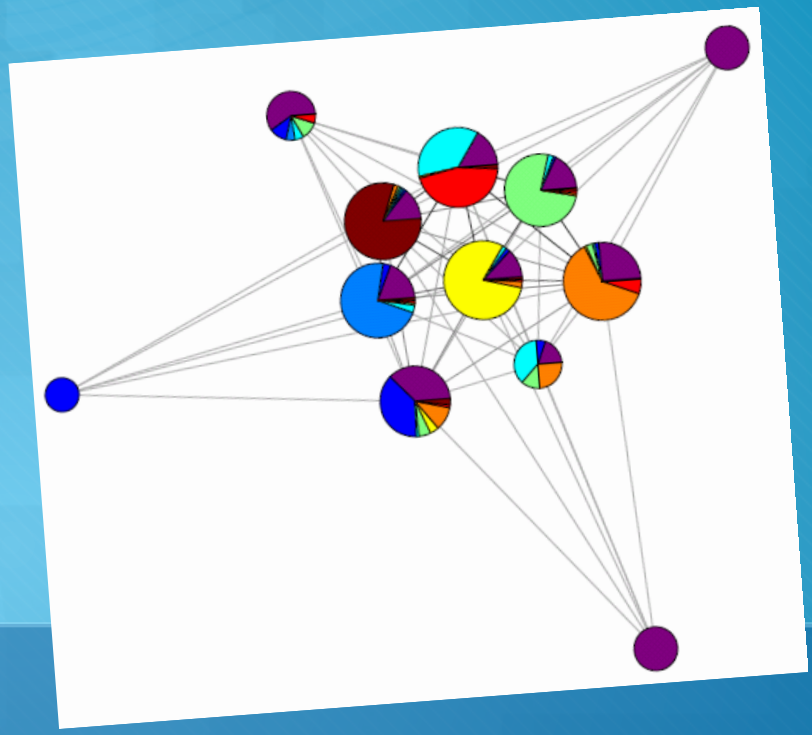
Links = self-identified friendships (1 or 0)

Data

- 100 different universities (full networks)
- Single-time snapshot: September 2005
  - Facebook was university-only
- Self-reported demographics
  - Gender, class year, high school, major, dormitory/"House"

Provided by Adam D'Angelo & Facebook





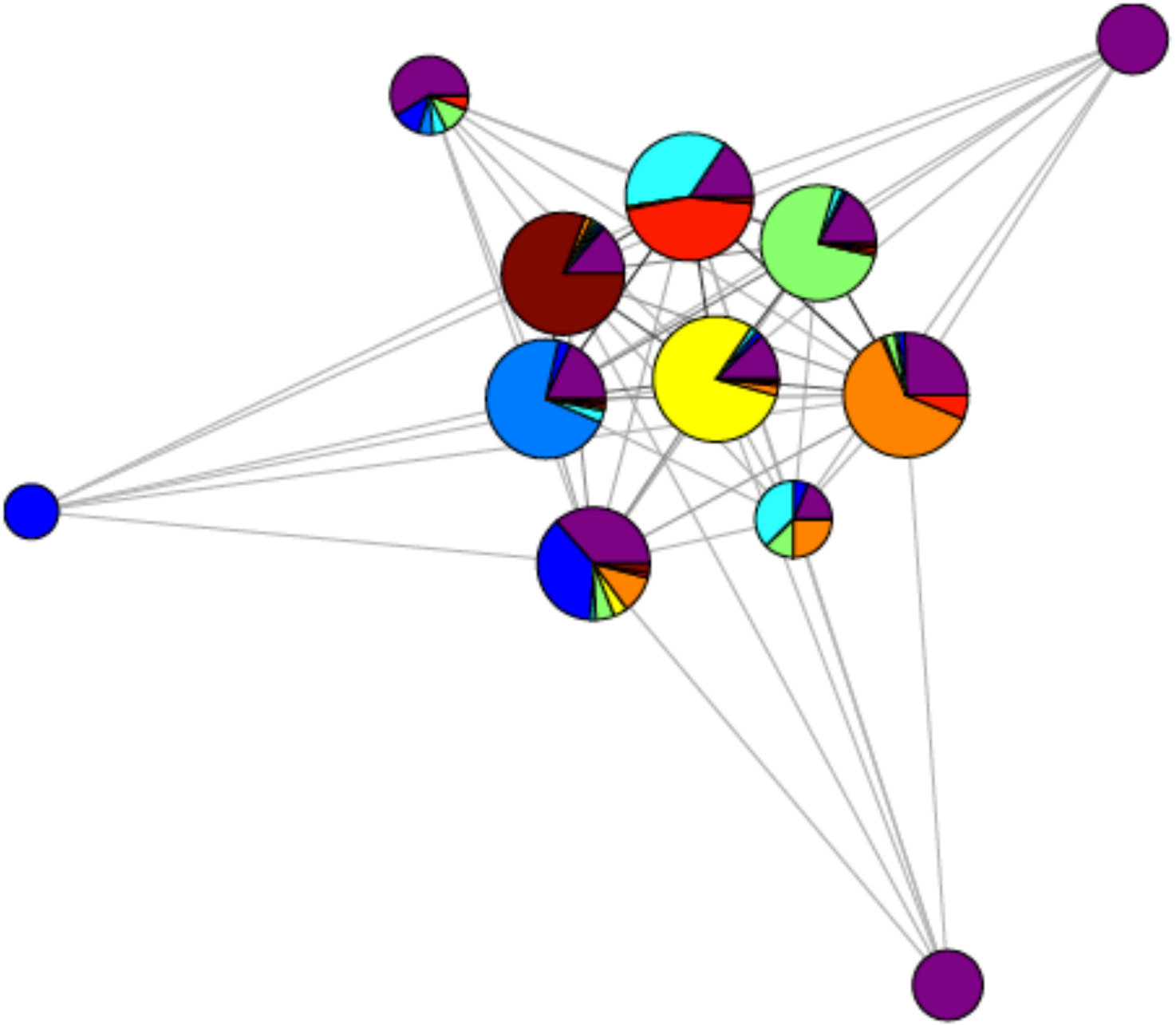
Detecting Communities

Develop and use computer algorithms to group circles of friends in an automated fashion.

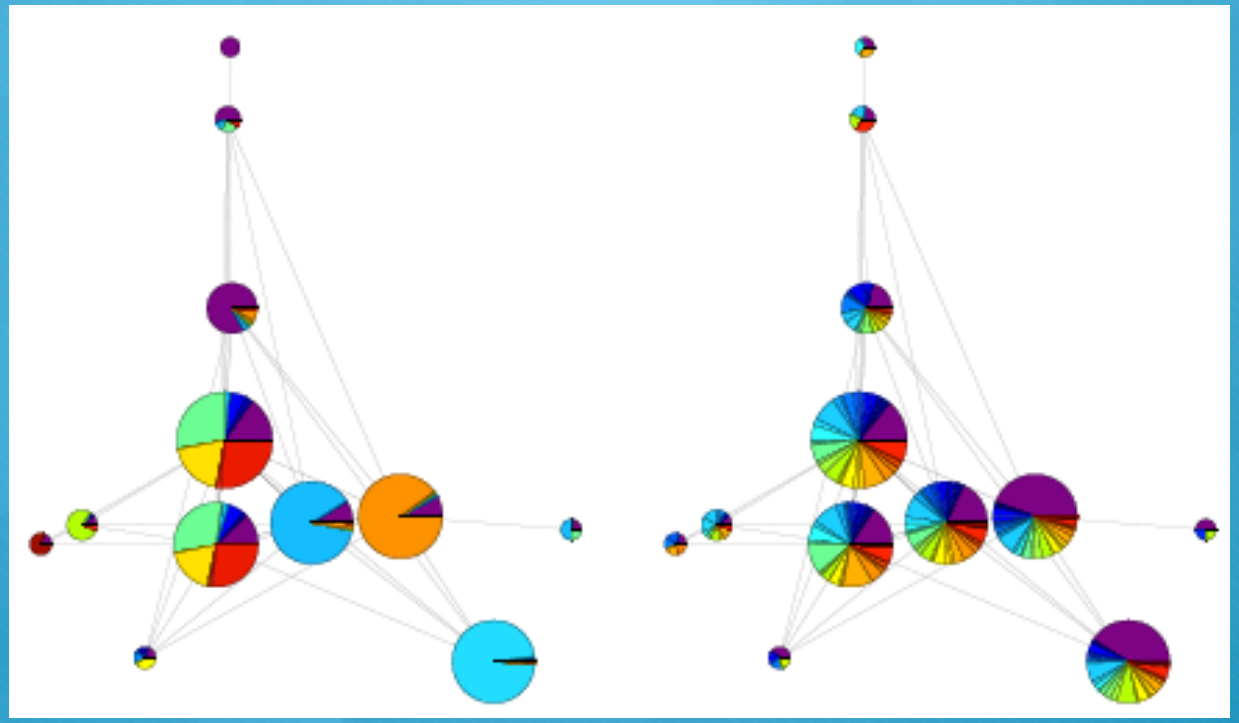
The problem is both very difficult and very interesting.

There are equations.

- (But I'm not going to show them to you.)
- What types of things might we try to do with these algorithms?
- How do we want to quantify tight connections among people?



Princeton: Class Year & Major

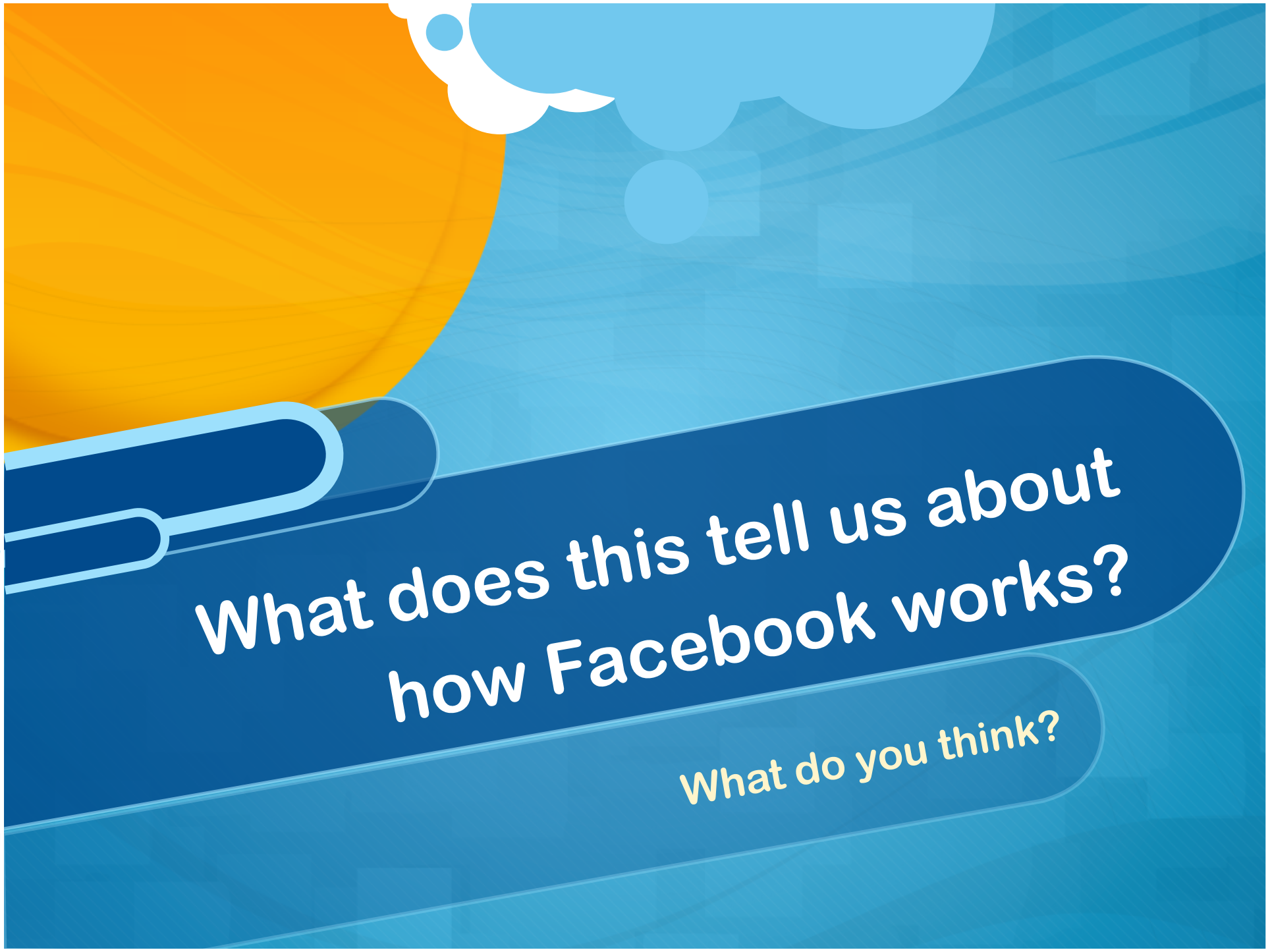


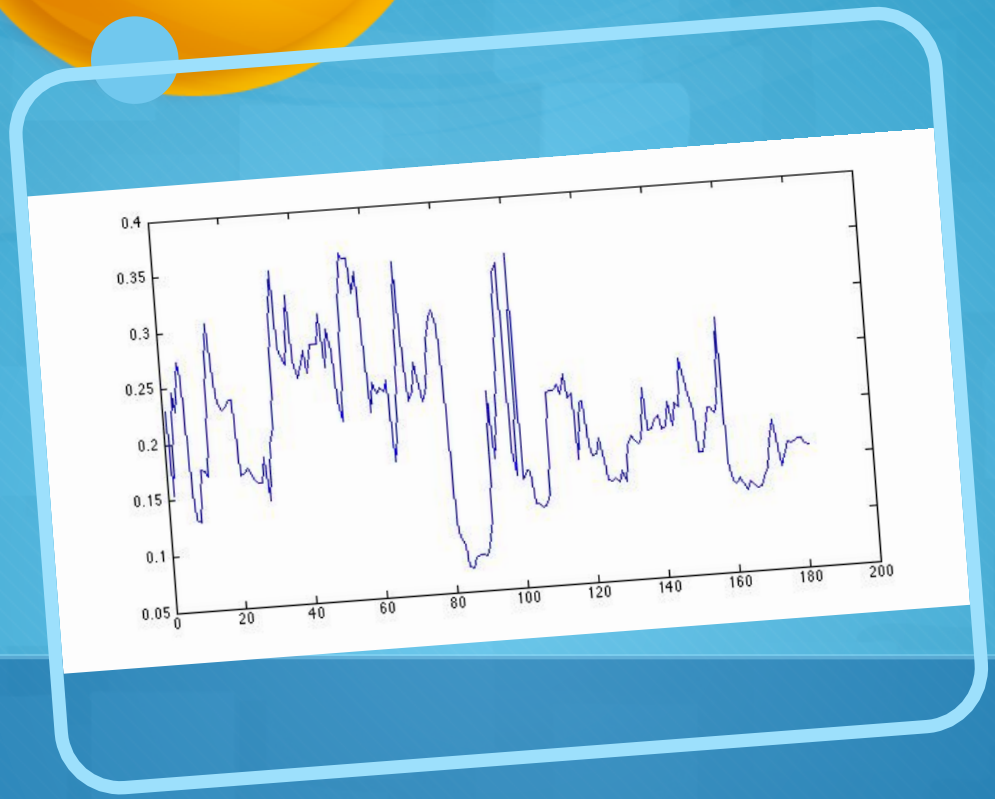
Quantitative Comparisons

- Princeton example: Is this random? Is is correlated? Visually, it's not clear!
- My collaborators and I have used quantitative methods (from statistics, network science, and information theory) to study the community structure of the Facebook networks from the 100 universities.

How do universities organize?

- Houses are important at Caltech (reality check for methodology)
  - We'd expect this to be the case at Oxford.
- High school is more important at large state universities
- Class year is the most important factor at most universities and dorm is often a very strong secondary factor
- Major has varying importance at different universities
- Our work suggests significant future research by social scientists


What does this tell us about how Facebook works?
What do you think?

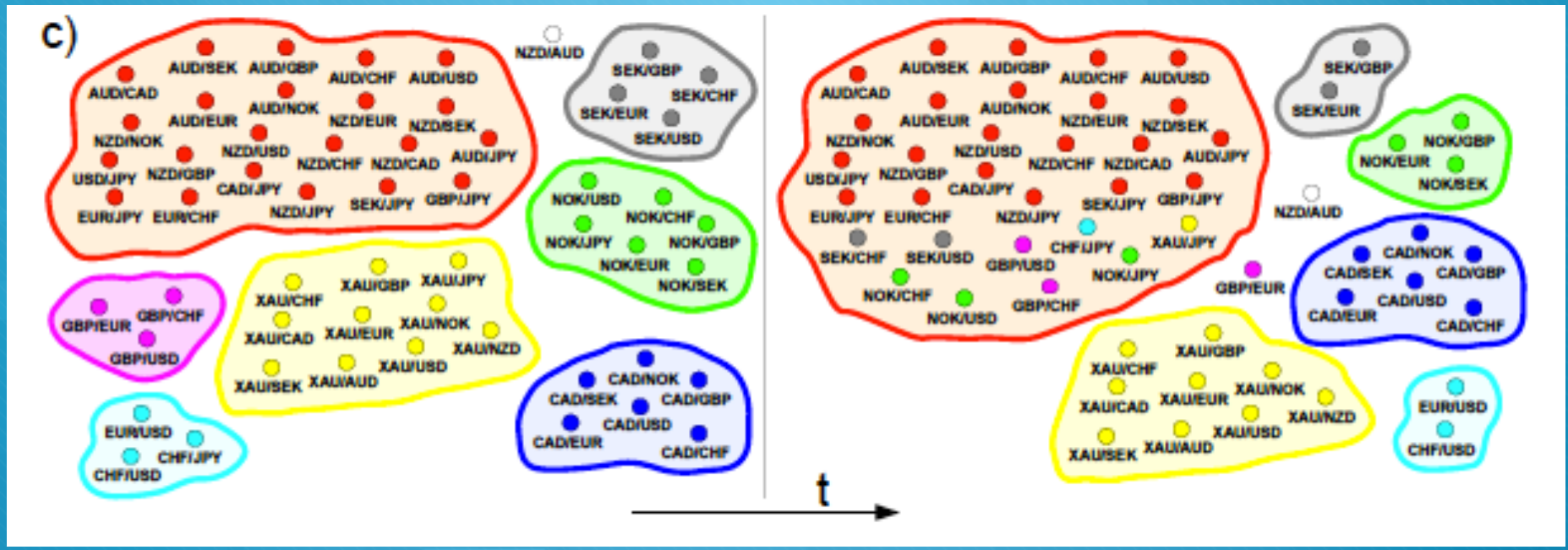


Application to Epidemics

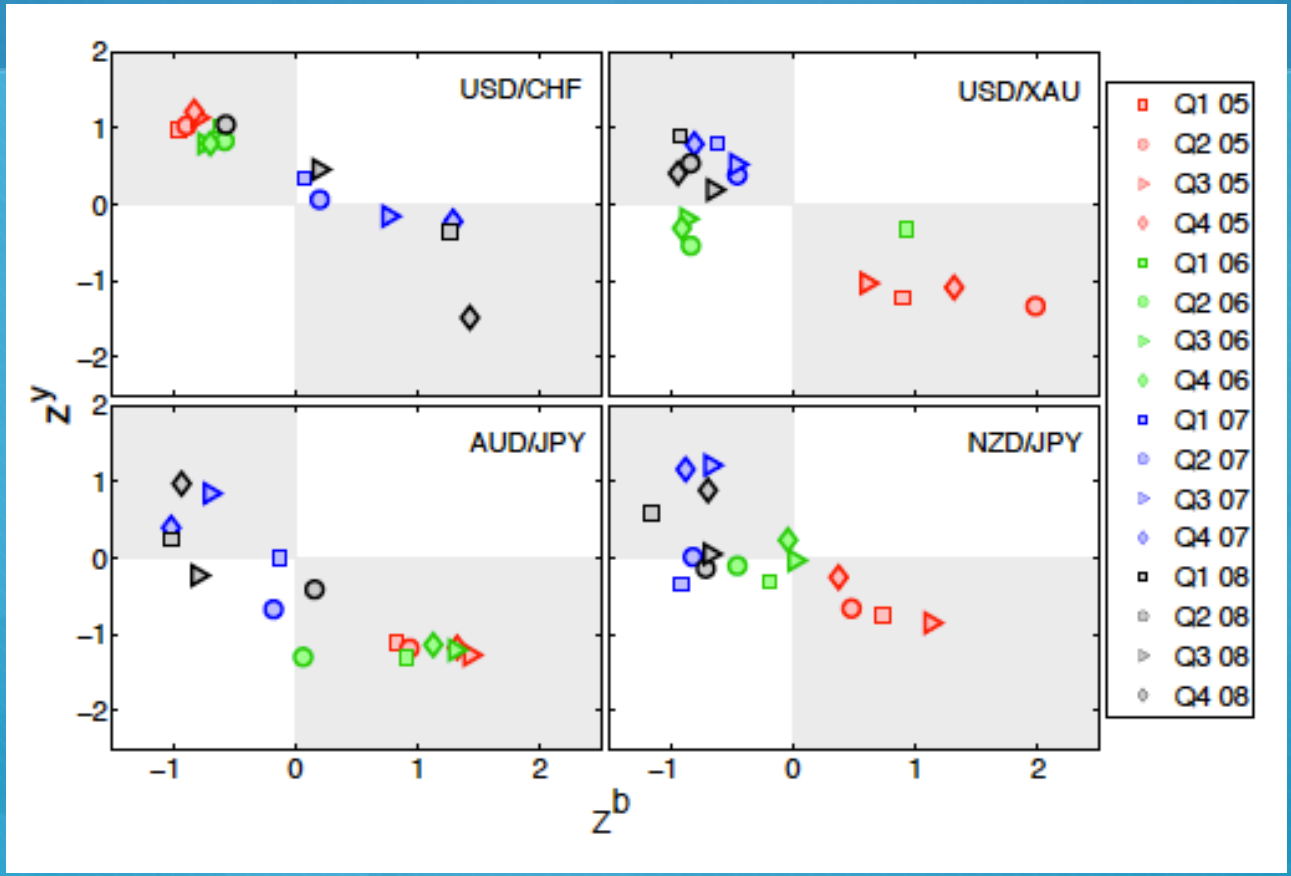
Student project by Yulian Ng (Somerville, 4th year, Mathematics)

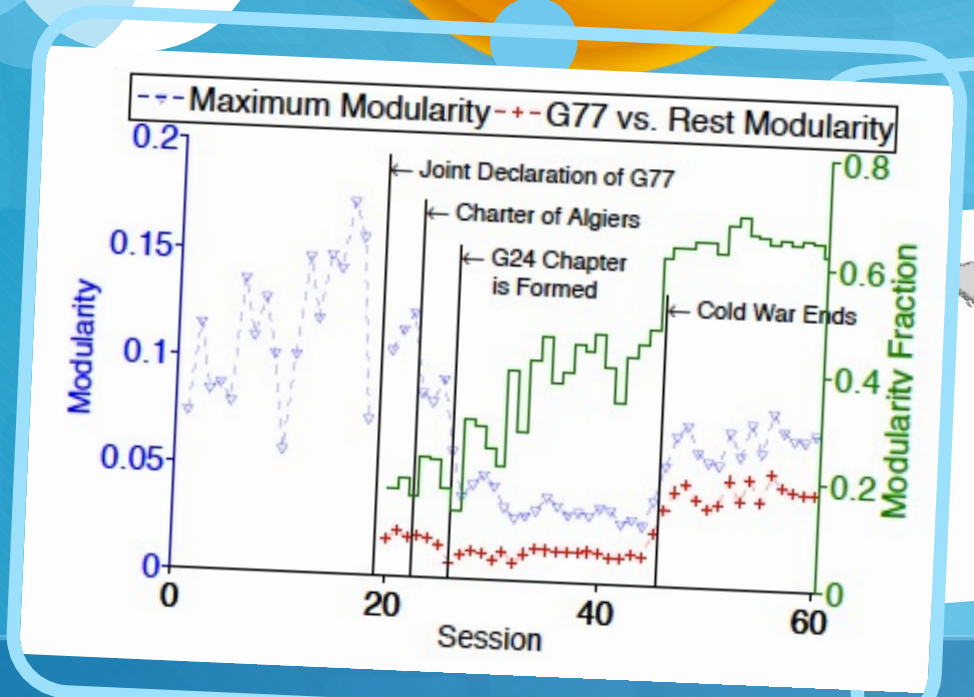
I have advised many undergraduate research projects in networks, and there will be many more in the future. So this opportunity awaits you if you come here.

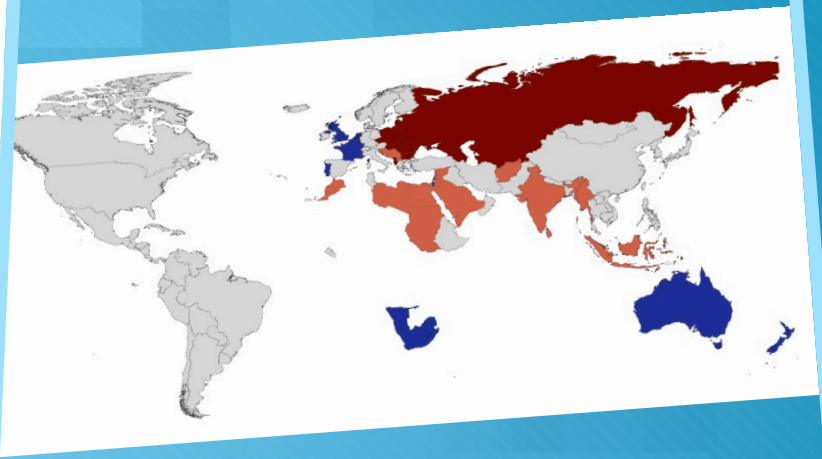
Application to Finance



Some exchange rates changed roles in the network right when the Credit Crunch began!

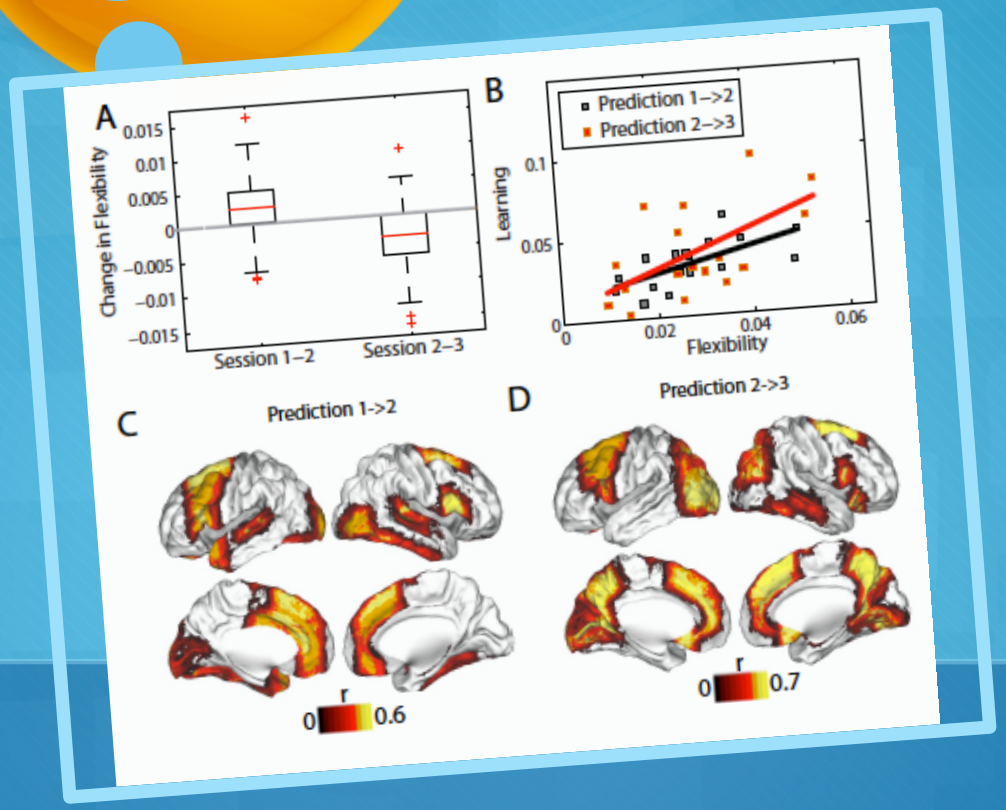




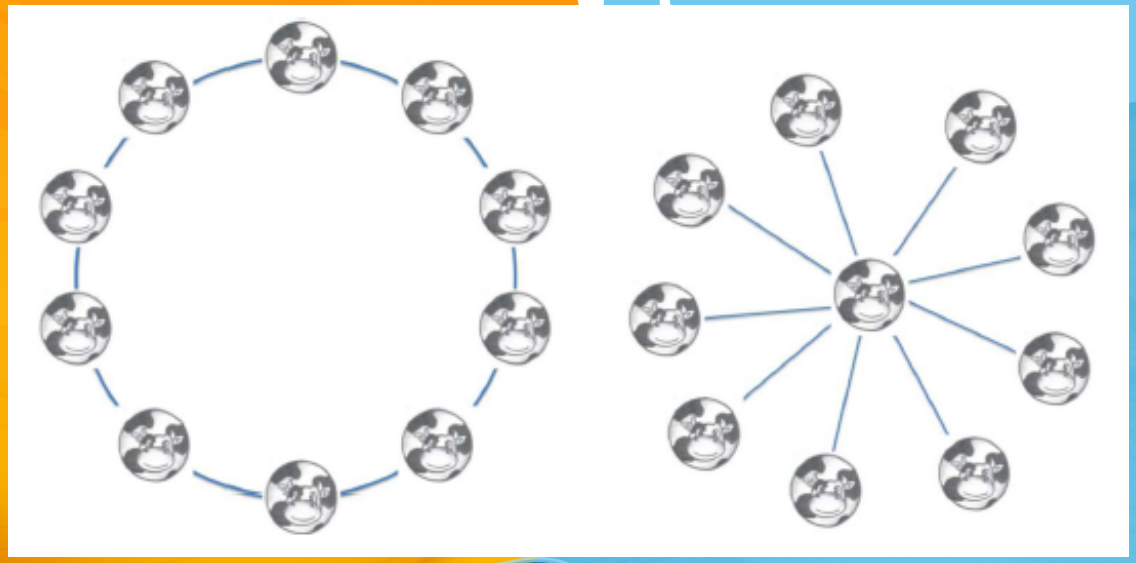


Voting Networks

One can also study things like voting networks, such as voting on resolutions in the United Nations General Assembly.



Configuration of Human Brain Networks During Learning



Questions?

porterm@maths.ox.ac.uk