Physics on a circle and geometry

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The 1st Strathmore University
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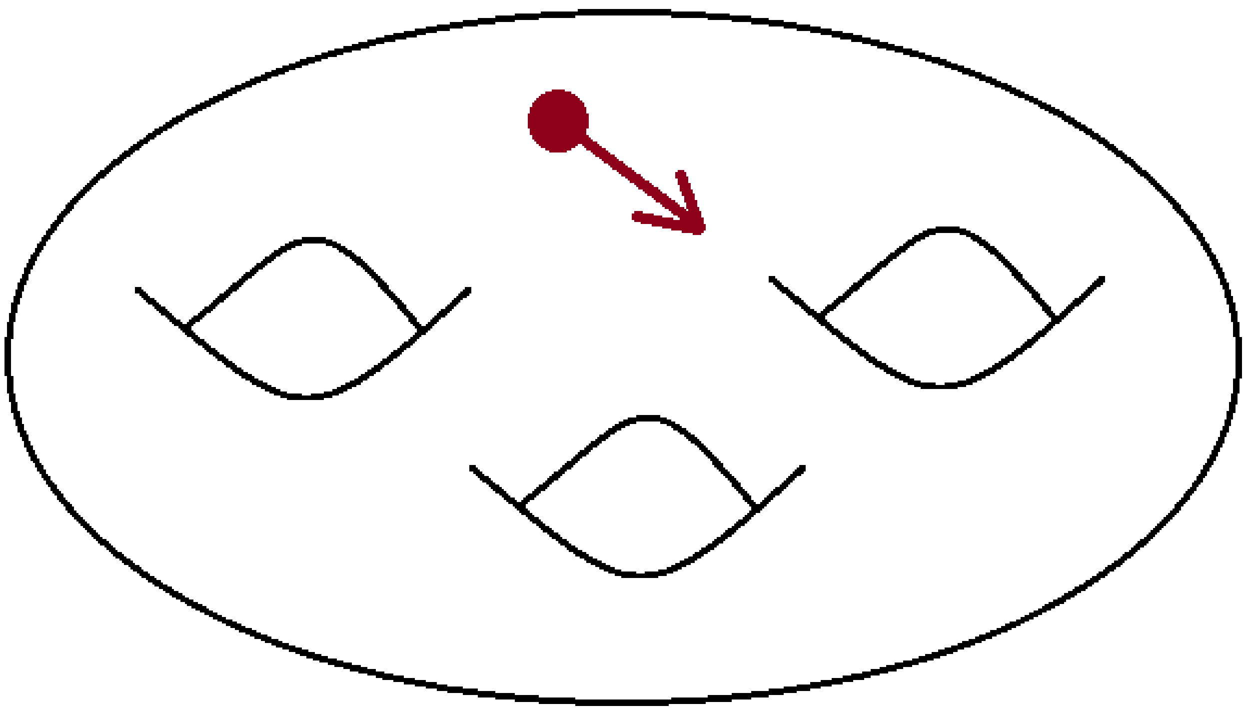
18-20 August 2011

Classical physics

Classical physics deals with *particles* moving in *physical space* under the influence of *forces*



Classical physics

Classical physics deals with *particles* moving in *physical space* under the influence of *forces*

Choose a simple physical space:

the one-dimensional *circle*

Classical physics on a circle

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Classical physics on a circle

- Equation of motion: Newton's equation

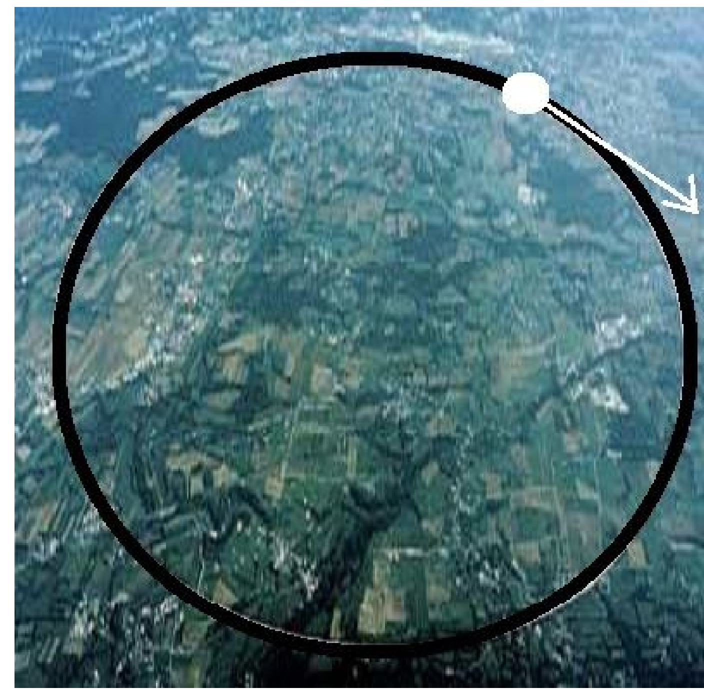
$$d^2x/dt^2 = 0$$

- Energy of particle

$$E = ½ \cdot v^2$$

v a real constant

- Continuous spectrum!



Quantum physics on a circle

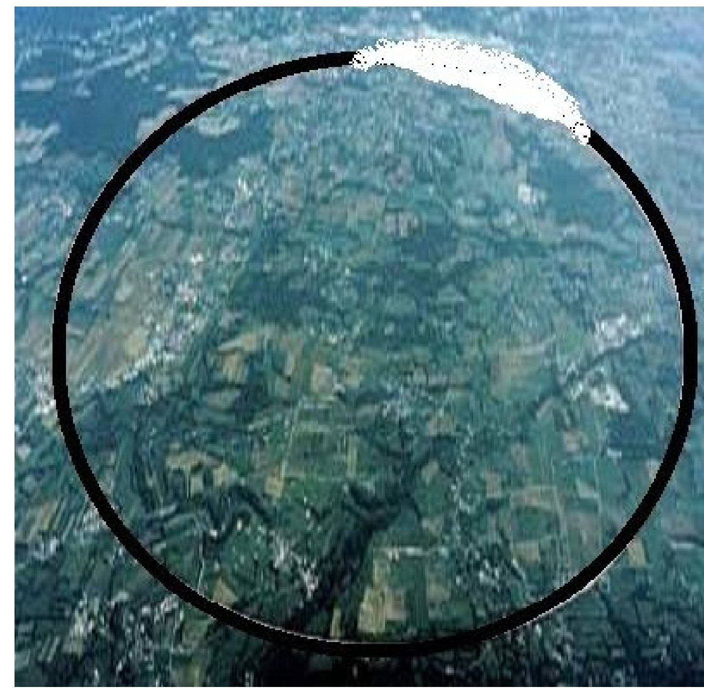
- Equation of motion: Schrödinger's equation

- Energy of particle

$$E = n^2/R^2$$

n: an integer number

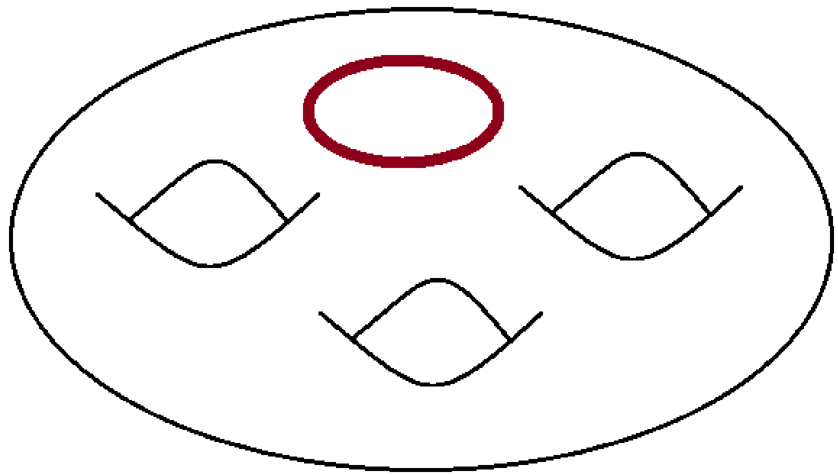
R: radius of circle



- Discrete spectrum!

String theory

- Idea: replace *particles* by *strings.*



- Strings move in space just like particles
- Strings have internal motion patterns, giving rise to physically particles

String theory on a circle

- Geometric input: string can wind round the circle!

- Energy of states:

$$E = n^2/R^2 + m^2R^2$$

n: quantum number
m: winding number
(integers)

R: radius of circle

String theory on a circle

- Energy levels:

$$E = n^2/R^2 + m^2R^2$$

n,m integers; R: radius of circle

- Energy spectrum *invariant* under the transformation

$$(n, m, R) <=======> (m, n, 1/R)$$

Duality in string theory on a circle

(n, m, R) <=======> *(m, n, 1/R)*

small circle <=======> *large circle*

winding state <=======> *particle state*

<=======>

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Duality in string theory on a circle

(n, m, R) <=======> $(m, n, 1/R)$

small circle <=======> *large circle*

winding state <=======> *particle state*

Physics on a small circle is indistinguishable from physics on a large circle!

Duality in string theory

- This example can be generalized: there are many examples, where *geometrically very different spaces* give rise to *identical physics in string theory.*
- One much studied example: *mirror symmetry*
- A famous example because of a historially important maths/physics debate

Rational curves on a quintic: maths

- Problem in classical enumerative geometry:

  "find the number n_d of rational curves of degree d on a quintic threefold"

- Mathematics results:

$$n_1 = 2875 \text{ (Kleiman, 1979)}$$

$$n_2 = 609{,}250 \text{ (Katz, 1986)}$$

$$n_3 = 2{,}682{,}549{,}425 \text{ (Ellingsrud et al 1990)}$$

Rational curves on a quintic: phys

$$
\begin{aligned}
Y_1^1 = 5 &+ 2875\,\frac{1^3 q}{1-q} + 609250\,\frac{2^3 q^2}{1-q^2} + 317206375\,\frac{3^3 q^3}{1-q^3} + 242467530000\,\frac{4^3 q^4}{1-q^4} \\
&+ 229305888887625\,\frac{5^3 q^5}{1-q^5} + 248249742118022000\,\frac{6^3 q^6}{1-q^6} \\
&+ 295091050570845659250\,\frac{7^3 q^7}{1-q^7} + 375632160937476603550000\,\frac{8^3 q^8}{1-q^8} \\
&+ 503840510416985243645106250\,\frac{9^3 q^9}{1-q^9} \\
&+ 704288164978454686113488249750\,\frac{10^3 q^{10}}{1-q^{10}} \\
&+ 1017913203569692432490203659468875\,\frac{11^3 q^{11}}{1-q^{11}} \\
&+ 1512323901934139334751675234074638000\,\frac{12^3 q^{12}}{1-q^{12}} \\
&+ 2299488568136266648325160104772265542625\,\frac{13^3 q^{13}}{1-q^{13}} \\
&+ 3565959228158001564810294084668822024070250\,\frac{14^3 q^{14}}{1-q^{14}} \\
&+ 5624656824668483274179483938371579753751395250\,\frac{15^3 q^{15}}{1-q^{15}} \\
&+ 9004003639871055462831535610291411200360685606000\,\frac{16^3 q^{16}}{1-q^{16}} + \dots
\end{aligned}
$$

Rational curves on a quintic: phys

- Candelas et al in 1991 claims:

$$n_1 = 2875$$

$$n_2 = 609{,}250$$

$$\sout{n_3 = 2{,}682{,}549{,}425}$$

$$n_3 = 317{,}206{,}375$$

$$n_4 = \ldots \text{ etc}$$

Rational curves on a quintic: verdict

Date: Wed, 31 Jul 91 11:06:34 MDT

From: Herb Clemens

To: candelas@YYY.edu

Subject: Physics wins!

String theory in geometry

- Such computations come from exploiting the power of duality in string theory
- These ideas had an enormous influence on the development of pure mathematics!
- Development of subjects such as Gromov-Witten theory, derived geometry, non-commutative algebraic geometry,...