A special corner in the landscape

Balázs Szendrői, University of Oxford

String Phenomenology, UPenn, 31 May 2008

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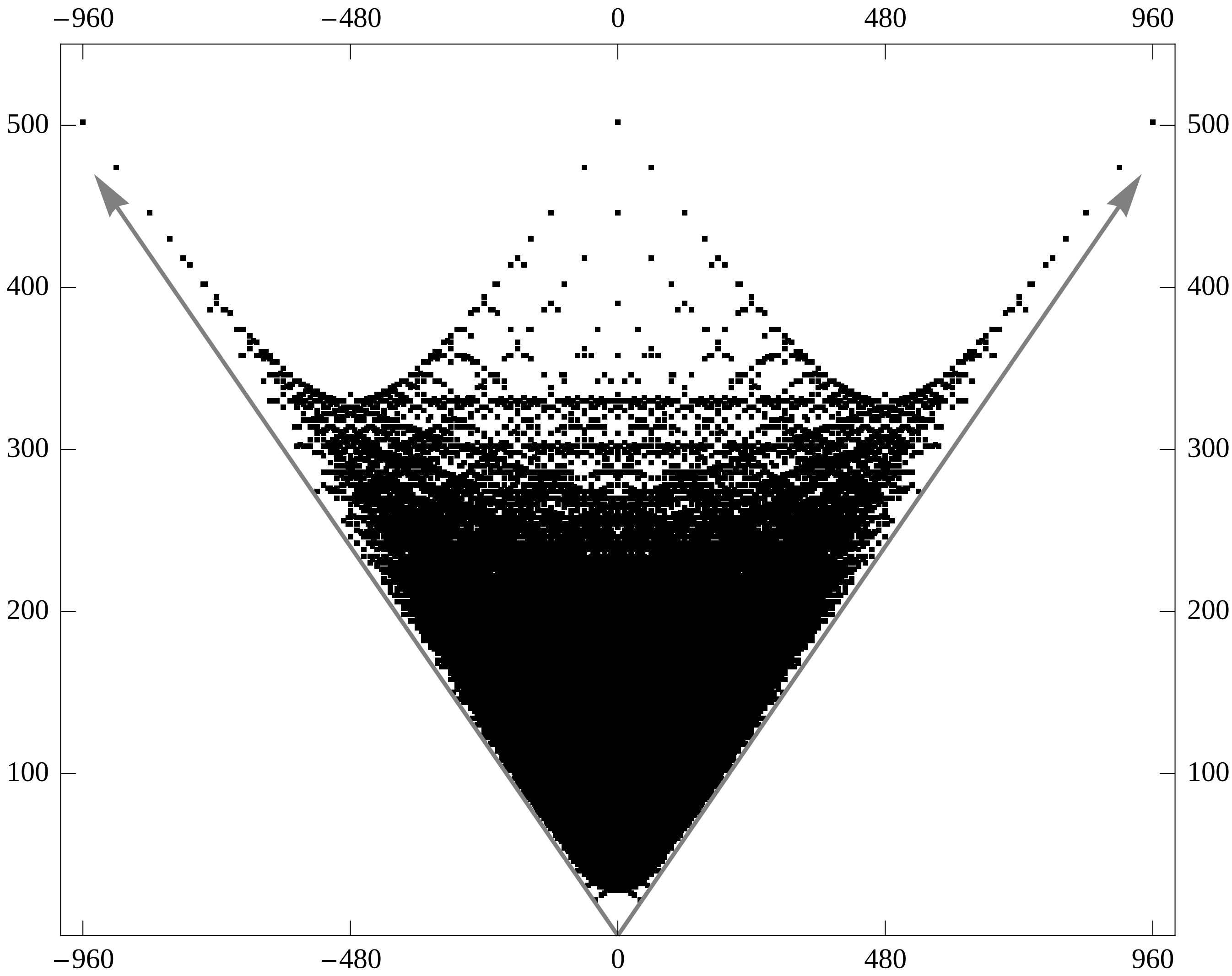
Also work by Philip Candelas and Ryan Davies



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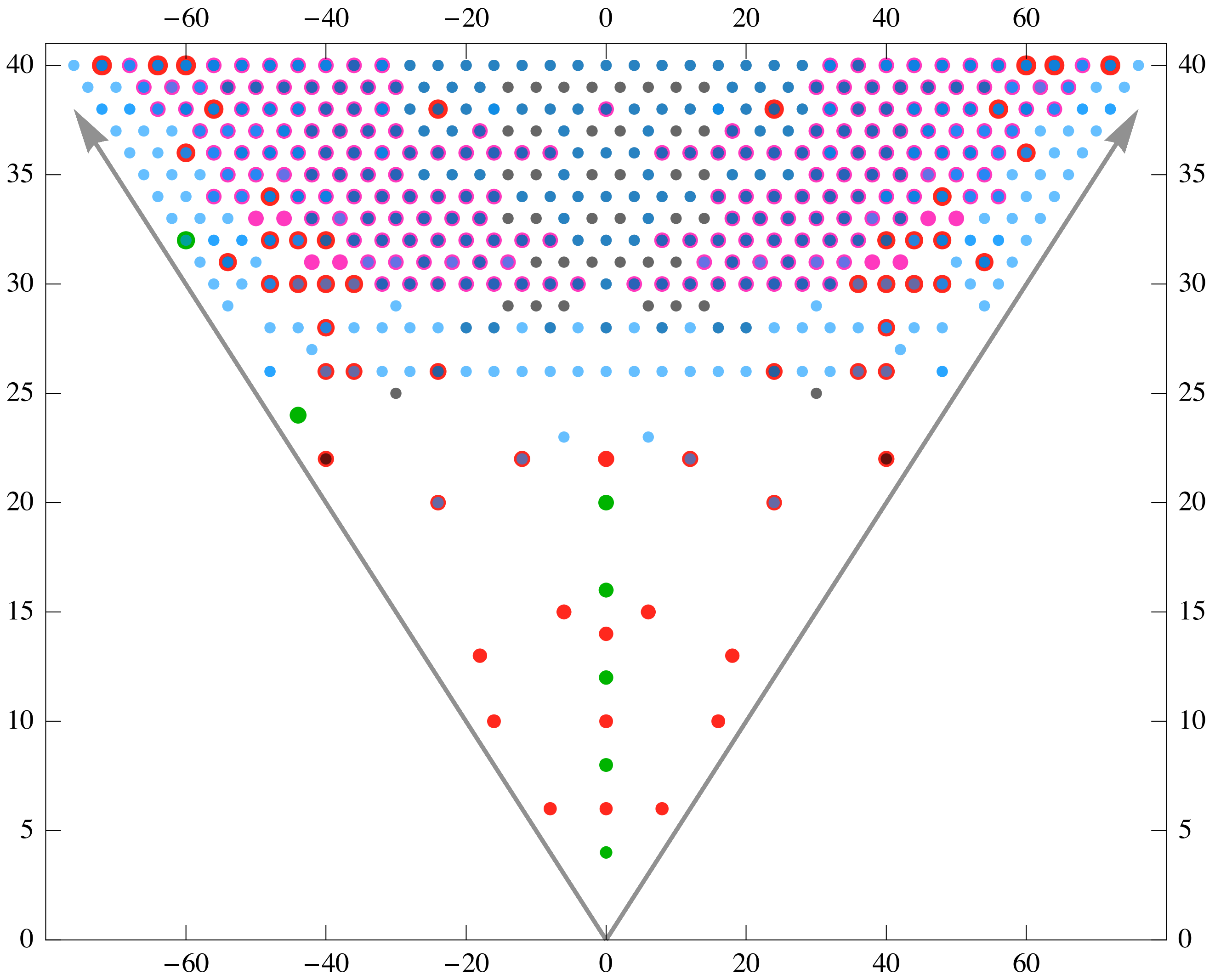

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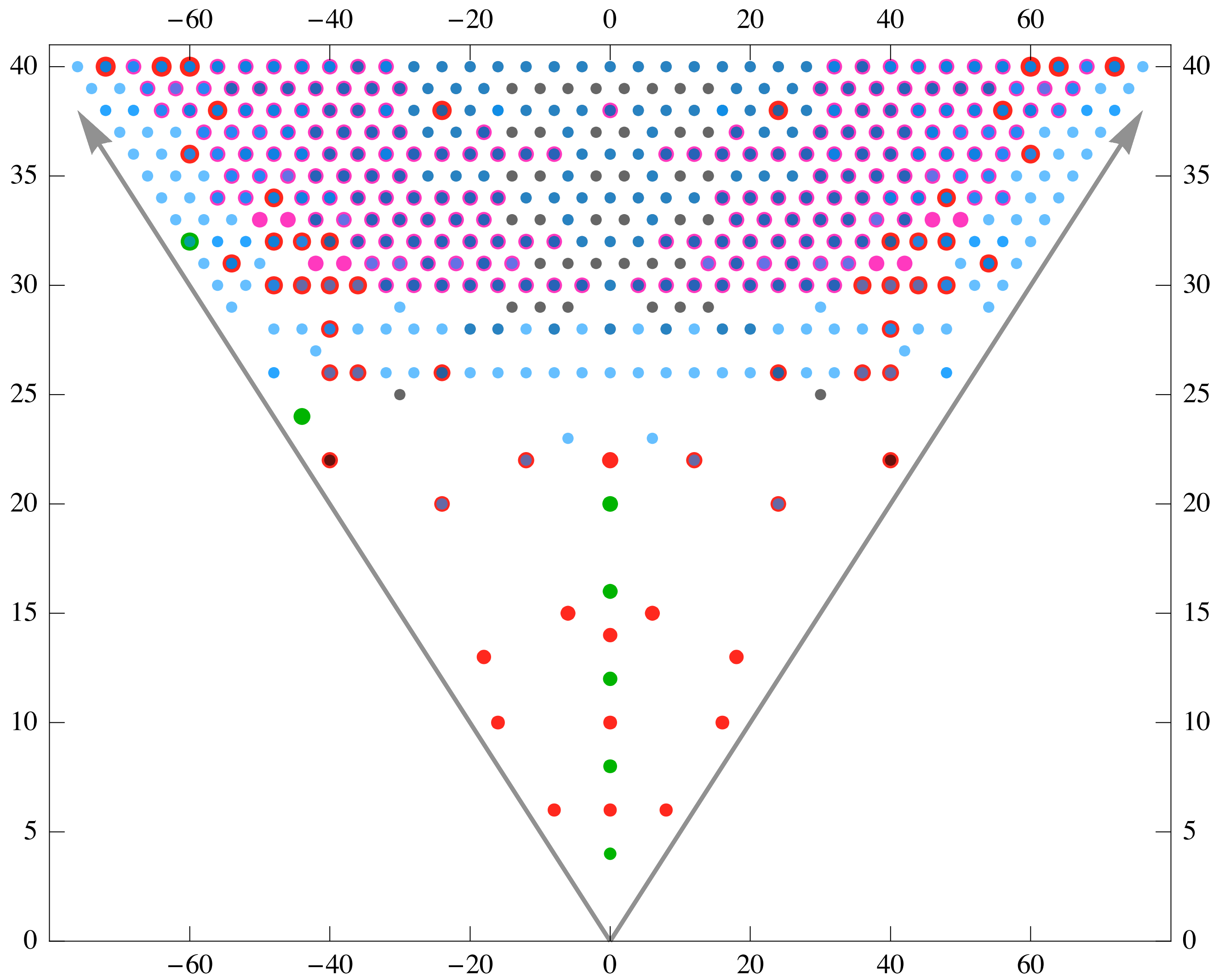

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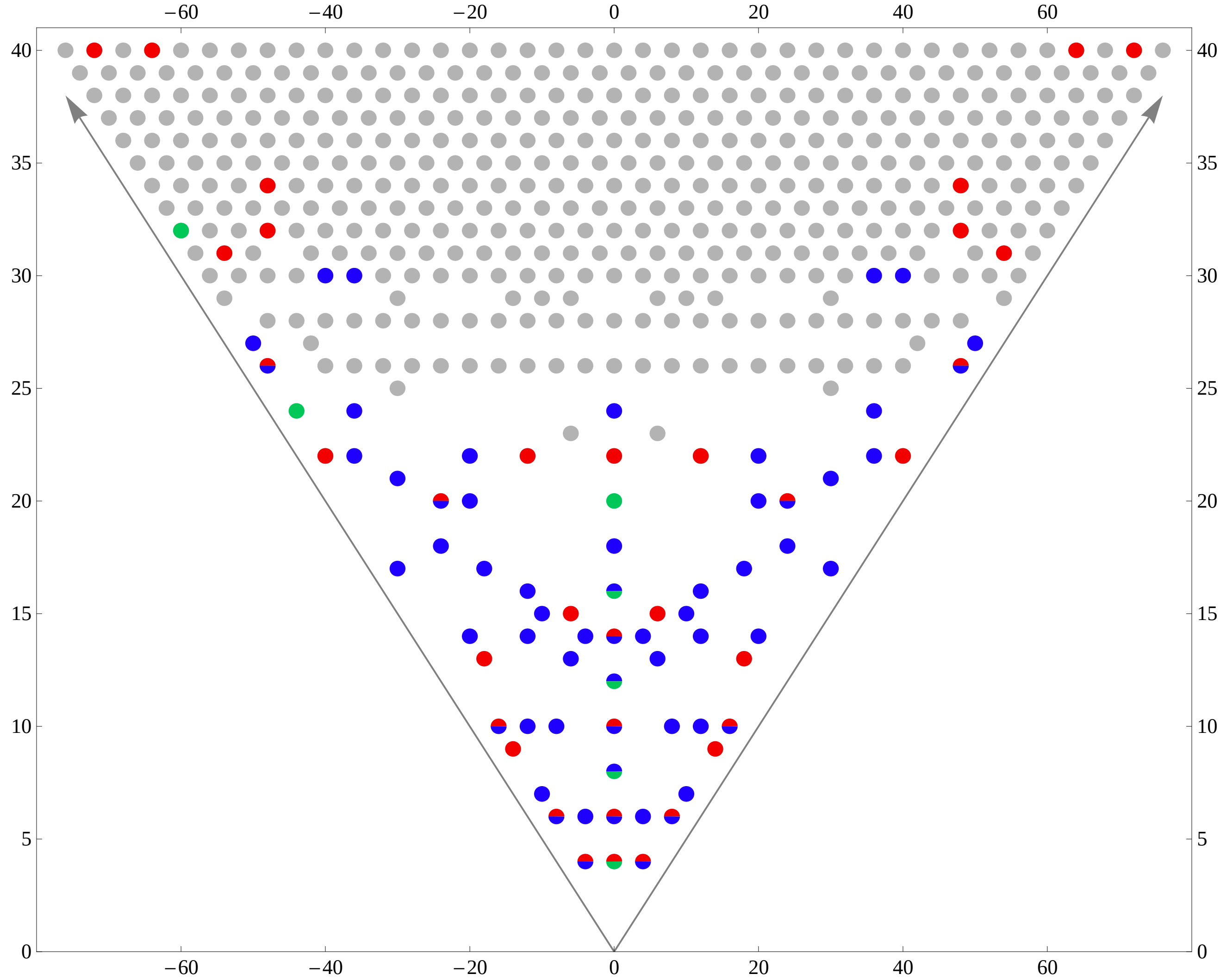

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Outline

1. Some classical manifolds with small Hodge numbers
2. A conifold transition
3. Transgression of bundles
4. An example
5. Outlook

The Tian–Yau example

- Start with complete intersection in $\mathbb{P}^3 \times \mathbb{P}^3$

$$\tilde{M}^{14,23} = \left[\begin{array}{ccc} 1 & 3 & 0 \\ 1 & 0 & 3 \end{array} \right] \subset \mathbb{P}^3 \times \mathbb{P}^3.$$

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- Tian–Yau manifold:

$$M^{6,9} = \left[\begin{array}{ccc} 1 & 3 & 0 \\ 1 & 0 & 3 \end{array} \right]_{/A}$$

where $A \cong \mathbb{Z}/3$ acts on $\mathbb{P}^3 \times \mathbb{P}^3$ by

$$A : \left(x_0 : x_j \,;\, y_0 : y_k \right) \mapsto \left(x_0 : x_{j+1} \,;\, y_0 : y_{k+1} \right).$$

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- Free quotient, $\pi_1(M) = \mathbb{Z}/3$, Euler number $e(M) = -6$.
- Standard embedding gives a three-generation model, with Wilson lines breaking gauge group.

A bielliptic threefold

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- Take quotient

$$N^{7,7} = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 0 & 3 \end{bmatrix}_{/A},$$

where $A = \mathbb{Z}/3$ acts by

$$A : \left(t_l\,;\, x_j\,;\, y_k\right) \mapsto \left(t_l\,;\, x_{j+1}\,;\, y_{k+1}\right).$$

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- There is a construction of an MSSM-ish model, using an SU(4)-bundle, on one of these quotients [Braun–He–Ovrut–Pantev].

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giving singular threefold M_0 with nodal singularities.

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- Can we use the tangent bundle of the Tian–Yau manifold M to construct a bundle on the bielliptic threefold N?

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- Form diagram:

$$\begin{array}{ccc} & \check{M} & \hookleftarrow \mathbb{P}^1 \\ & \downarrow & \\ M_t & \rightsquigarrow & M_0 \end{array}$$

Local transgression

- Tangent bundle $\mathcal{T}_t$ of M_t: sections are vector fields V^j on $\mathbb{C}^4$ satisfying

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- Instead, first deform $\mathcal{T}_t$ to a bundle $\mathcal{E}_{t,h}$ whose sections are defined by

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- In the limit $t \to 0$, the space of allowed sections remains 3-dimensional even at the conifold point, so we obtain a bundle $\mathcal{E}_{0,h}$ on M_0.

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- Diagram:

$$\begin{array}{ccccccc} \mathcal{T}_t & \leftsquigarrow & \mathcal{E}_{0,h} & \rightsquigarrow & \mathcal{E}_{0,h} & & \check{\mathcal{E}}_h \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ M_t & = & M_t & \rightsquigarrow & M_0 & \leftarrow & \check{M}. \end{array}$$

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for some μ, subject for all ρ to the identification
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- Conditions are consistent by the Euler relation $x^j \frac{\partial p}{\partial x^j} = 5p$. The two conditions reduce the dimension of the space of allowed vectors from 5 to the correct value of 3.

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- Estimate the dimension of the space of first order deformations of $\mathcal{T}_Q$. For the r_j, a total of $5\binom{4+5-1}{4} = 350$ parameters. The constraint (1) imposes $\binom{5+5-1}{5} = 126$ conditions. We thus constructed a $350-126 = 224$-dimensional family of deformations of $\mathcal{T}_Q$.

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- If we discard the constraint (1), then we are really deforming the (semistable) SU(4)-bundle $\mathcal{T}_Q \oplus \mathcal{O}_Q$ on Q.

The tangent bundle of the Tian–Yau manifold

- Tian–Yau manifold: defining equations p^1, p^2, p^3:

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- Identification:

$$(X^a + \rho x^a, \; Y^b + \sigma y^b) \simeq (X^a, \; Y^b) \; .$$

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- In order to maintain the identification, we require the constraints

$$x^a\left(\frac{\partial p^\alpha}{\partial x^a} + r^\alpha{}_a\right) = n^\alpha{}_\beta\, p^\beta\ , \qquad y^b\left(\frac{\partial p^\alpha}{\partial y^b} + s^\alpha{}_b\right) = \tilde{n}^\alpha{}_\beta\, p^\beta$$

for some constant matrices $n^\alpha{}_\beta$, $\tilde{n}^\alpha{}_\beta$.

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- Transgression naturally leads us to SU(4) and SU(5)-bundles on the resolution, the bielliptic Calabi–Yau!

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- Physical processes behind transgression?