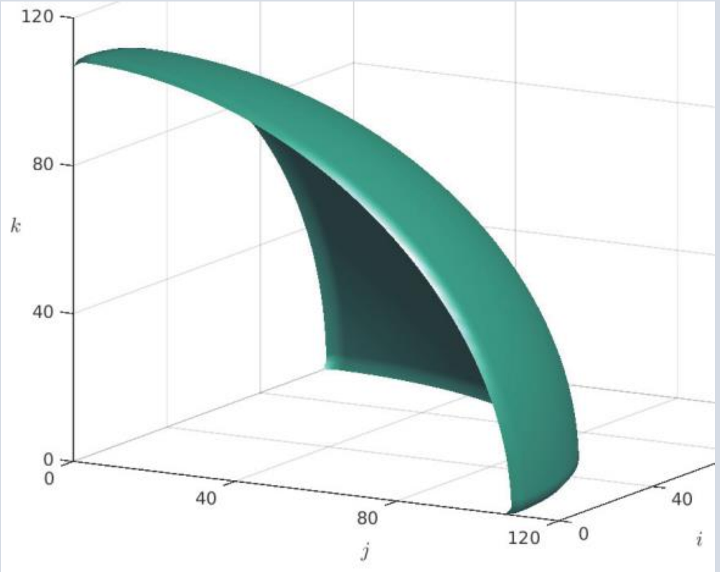
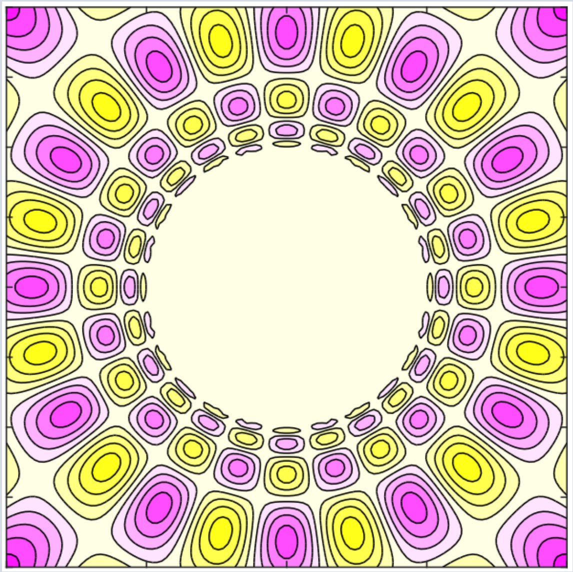
Memos





1976-77

Harvard

I. 4 November 1976

1) THESIS TOPIC: numerical rational Chebyshev approximation of an analytic function of one complex variable. To begin with at least, I am considering the case of a domain which is the closed region bounded by a closed Jordan curve: a Jordan region.

2) COMPARISON BETWEEN THE COMPLEX AND REAL PROBLEMS:

The prototypical problem is approximation on a real interval by polynomials of degree at most m. Here one has existence and uniquess of best approximations, which are characterized by having at least m+2 points of alternating extreme values of the error function.

In approximation on a real interval by rational functions of the form $r_{mn}(x) = p_m(x)/q_n(x)$, existence and uniqueness still hold, and best approximations are characterized by having at least m+n+2-min{m-degp,n-degq} points of alaternating extreme errors.

In complex polynomial approximation, existence and uniqueness still hold. A best approximation has at least m+2 points of extreme error, and a somewhat more complicated characterization in terms of extreme errors is available. [Rivlin and Shapiro — see p. 2.]

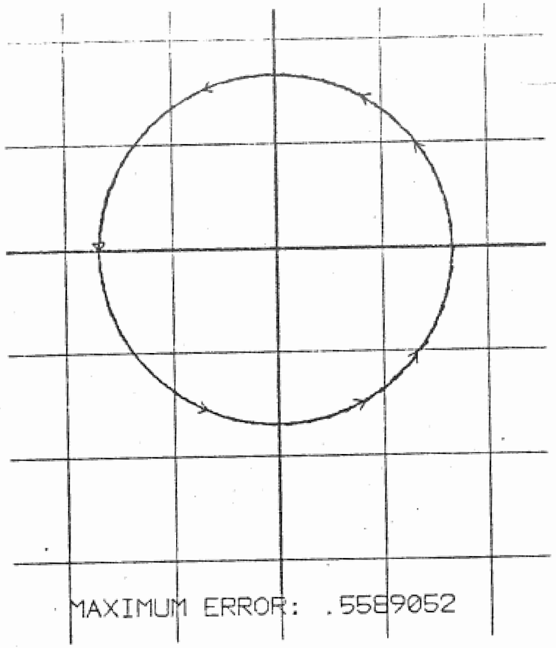
Finally, in complex rational approximation existence holds but uniqueness does not, and apparently no useful

1976-77
Harvard

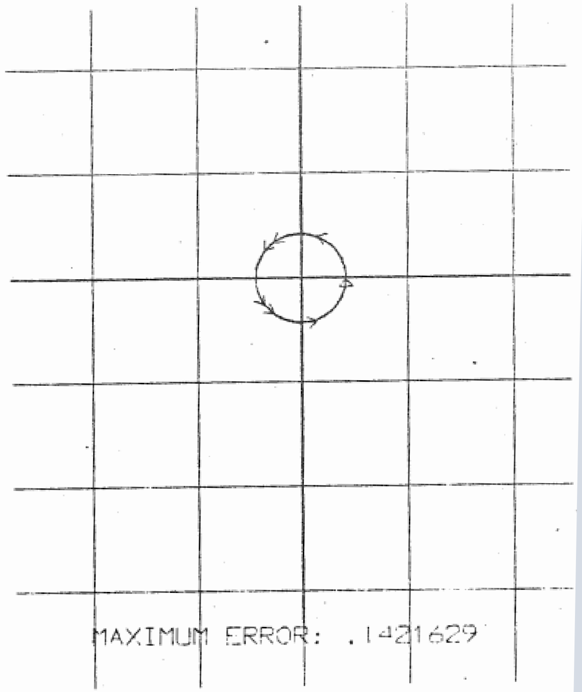
	f CONTINUOUS ON A REAL INTERVAL		f ANALYTIC ON A JORDAN REGION	
	$p_m(x)$	$r_{mn}(x) = \frac{p_m(x)}{q_n(x)}$	$p_m(z)$	$r_{mn}(z) = \frac{p_m(z)}{q_n(z)}$
WEIERSTRASS APPROXIMATION THEOREM	YES (Weierstrass 1885) [Meinardus, p. 7; Davis, chap. 6]	YES – generalization [Meinardus, p. 158]	YES (~~Walsh?~~ Runge?) [Meinardus, p. 10; Walsh, p. 36]	YES – same generalization ?
EXISTENCE OF BEST APPROX.	YES (Chebyshev 1899?) [Meinardus, p. 1; Lorentz, p. 17]	YES (Walsh) [Rice, vol. 1; Ralston, p. 297]	YES [Meinardus, p. 1; Lorentz, p. 17]	YES (Walsh?) [Walsh?]
UNIQUENESS OF BEST APPROX.	YES (Chebyshev 1899?) [Meinardus, p. 16; Lorentz, p. 26]	YES (Chebyshev 1899) [Ralston, p. 297]	YES [Meinardus, p. 1; Lorentz, p. 26]	NO [Walsh, p. 356]
NO. OF POINTS OF EXTREME ERROR	⩾ m+2, ALTERNATING (Chebyshev 1899) [Meinardus, p. 20; Davis, p. 143]	⩾m+n+2–min{m–degp, n–degq}, ALTERNATING [Meinardus, p. 161; Ralston, p. 297]	⩾ m+2 [Davis, p. 143; Lorentz, p. 26]	?
NO. OF POINTS OF ZERO ERROR	⩾ m+1	⩾ m+1	⩾ 0 [Saff]	⩾ 0 [Saff]
CHARACTERIZATION OF BEST APPROX.	⩾ m+2 ALTERNATING ERROR EXTREMA (Chebyshev 1899) [Davis, p. 151; Cheney, p. 77]	⩾m+n+2–min{m–degp, n–degq} ALTERNATING ERROR EXTREMA [Ralston, p. 297; Meinardus, p. 161]	more complicated (Rivlin & Shapiro) [Lorentz, p. 22; Rivlin & Shapiro]	none known

NOTES I-3

1976-77
Harvard



$f(z) = e^z$



$f(z) = \log(z-2)$

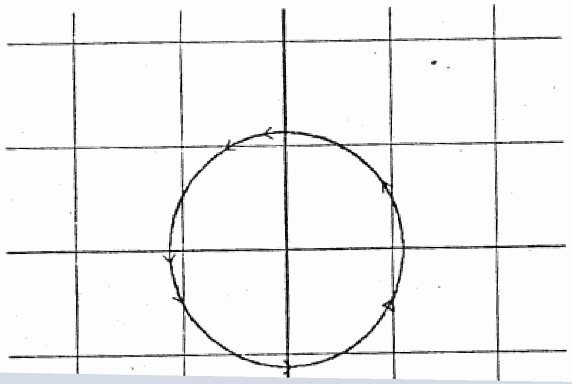


Figure 1. Images of the unit circle under the error function $p_1^*(f) - f$, for three different functions f.

1978
Stanford

SC Memo 1 10/8/78

FORMULATION OF THE SCHWARZ-CHRISTOFFEL MAPPING PROBLEM;
PROGRAM SC1:

Suppose we wish to map the upper half plane conformally onto the interior of a polygon $w_1, \ldots, w_n$ in the plane. The Schwarz-Christoffel mapping theorem tells us that every such map can be written in the form

$$w = C_2 + C_1 \int_0^z \prod_{k=1}^{n} (z-x_k)^{-\beta_k} \, dz \qquad (1)$$

where β_k is the exterior angle at w_k and each x_k lies on the x-axis. Moreover, we may set any three of the points x_k arbitrarily; the rest are then uniquely determined.

The computational problem is this: how do we find the remaining points x_k, given some arbitrary choice for three of them? We may formulate this for the computer by setting x_1, x_2, and x_3 arbitrarily and then solving the following nonlinear system in the n-3 unknowns $x_4, \ldots x_n$:

$$\frac{\int_{x_k}^{x_{k+1}} \prod_{j=1}^{n} |(z-x_j)|^{-\beta_j} \, dz}{\int_{x_1}^{x_2} \prod_{j=1}^{n} |(z-x_j)|^{-\beta_j} \, dz} = \frac{|w_{k+1} - w_k|}{|w_2 - w_1|}, \quad (3 \le k \le N-1) \qquad (2)$$

My first program followed this procedure, using $x_1=1$, $x_2=\infty$, $x_3=1$. (One term drops from the integral (1) when

Reason #1 for writing memos: they make a helpful record for the future.

Lab notebook

Article Talk

From Wikipedia, the free encyclopedia

A **laboratory notebook** (*colloq.* **lab notebook** or **lab book**) is a primary record of research. Researchers use a lab notebook to document their hypotheses, experiments and initial analysis or interpretation of these experiments. The notebook serves as an organizational tool, a memory aid, and can also have a role in protecting any intellectual property that comes from the research.[2]

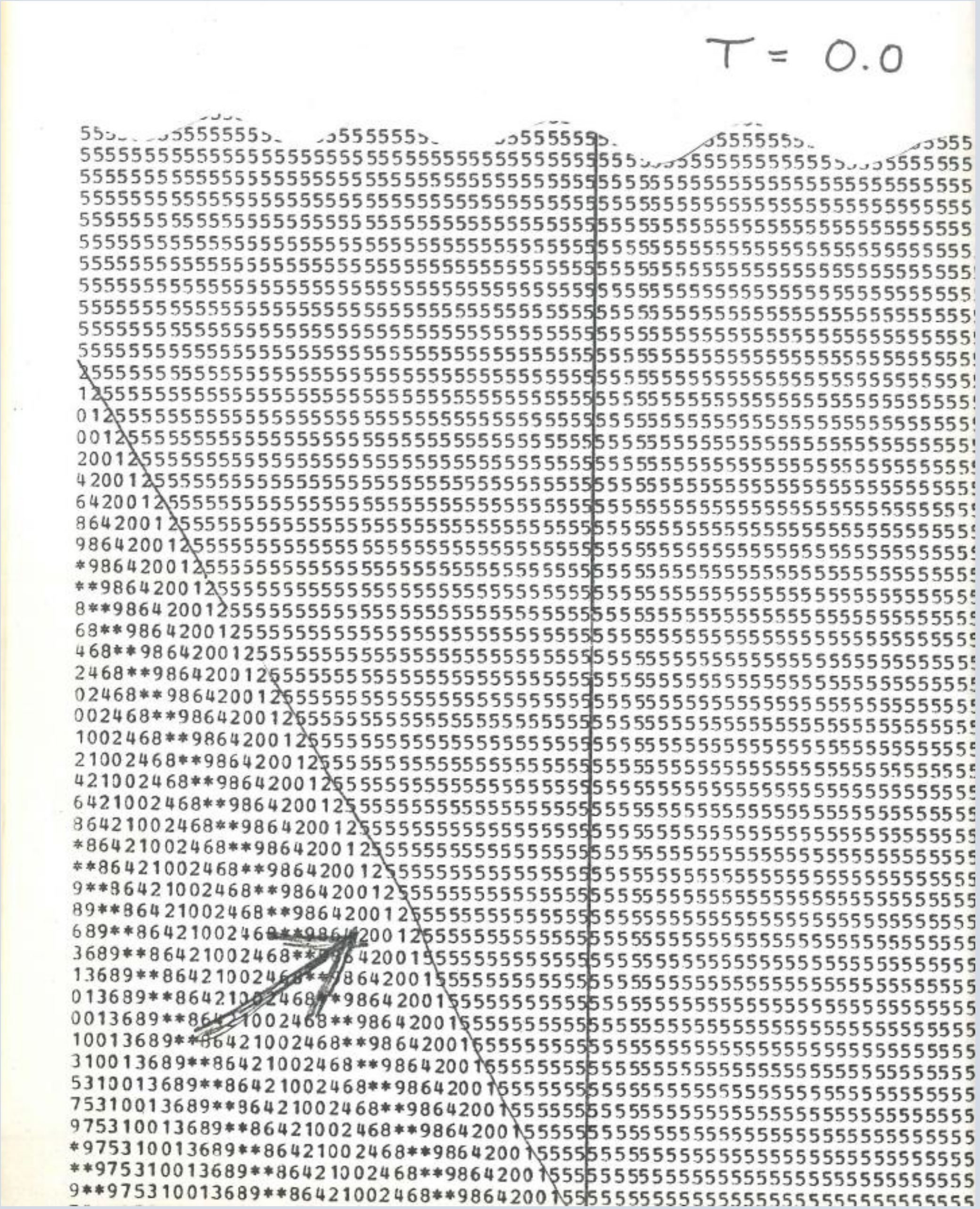
1981-82
Stanford

1981-82
Stanford

CONTENTS: LNT MEMOS ON WAVES

1981-82

Stanford



1981-82
Stanford

LNT
28 Sept. 1980

3. GROUP VELOCITY IN FINITE DIFFERENCE SCHEMES

I. THEORY

Memos 1 and 2 considered phase speeds, but since finite-difference schemes are dispersive these are not the whole story. Any linear scheme will satisfy a dispersion relation

$$\omega = \omega(k) \qquad (3.1)$$

relating frequency ω and wave number k. The phase speed is then

$$C = \frac{\omega(k)}{k}, \qquad (3.2)$$

while the group velocity for a smoothly varying wall packet will be

$$C_g = \frac{d\omega(k)}{dk}. \qquad (3.3)$$

Let LF and LW denote the leapfrog and Lax-Wendroff schemes for the first-order wave equation

1981-82
Stanford

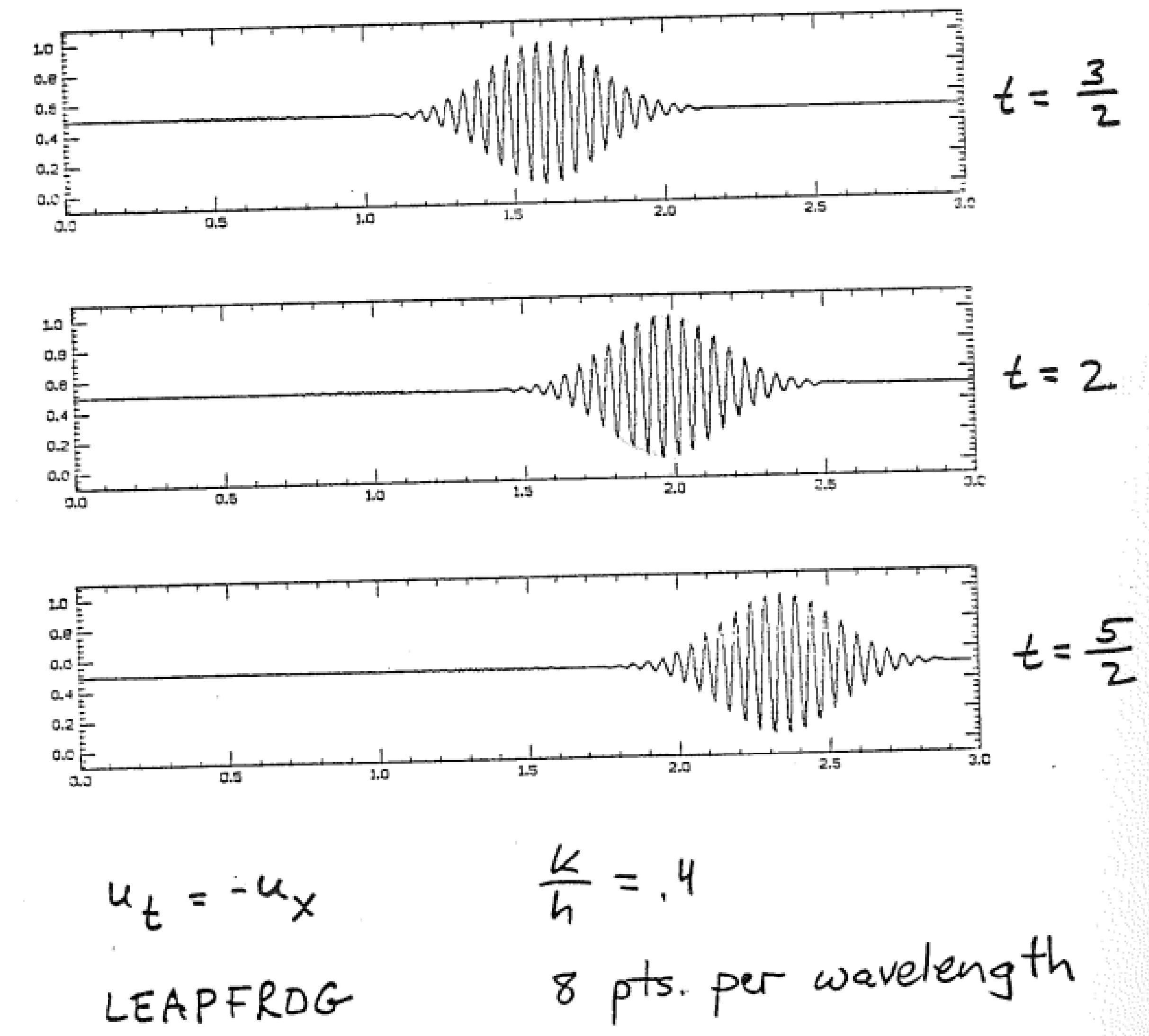
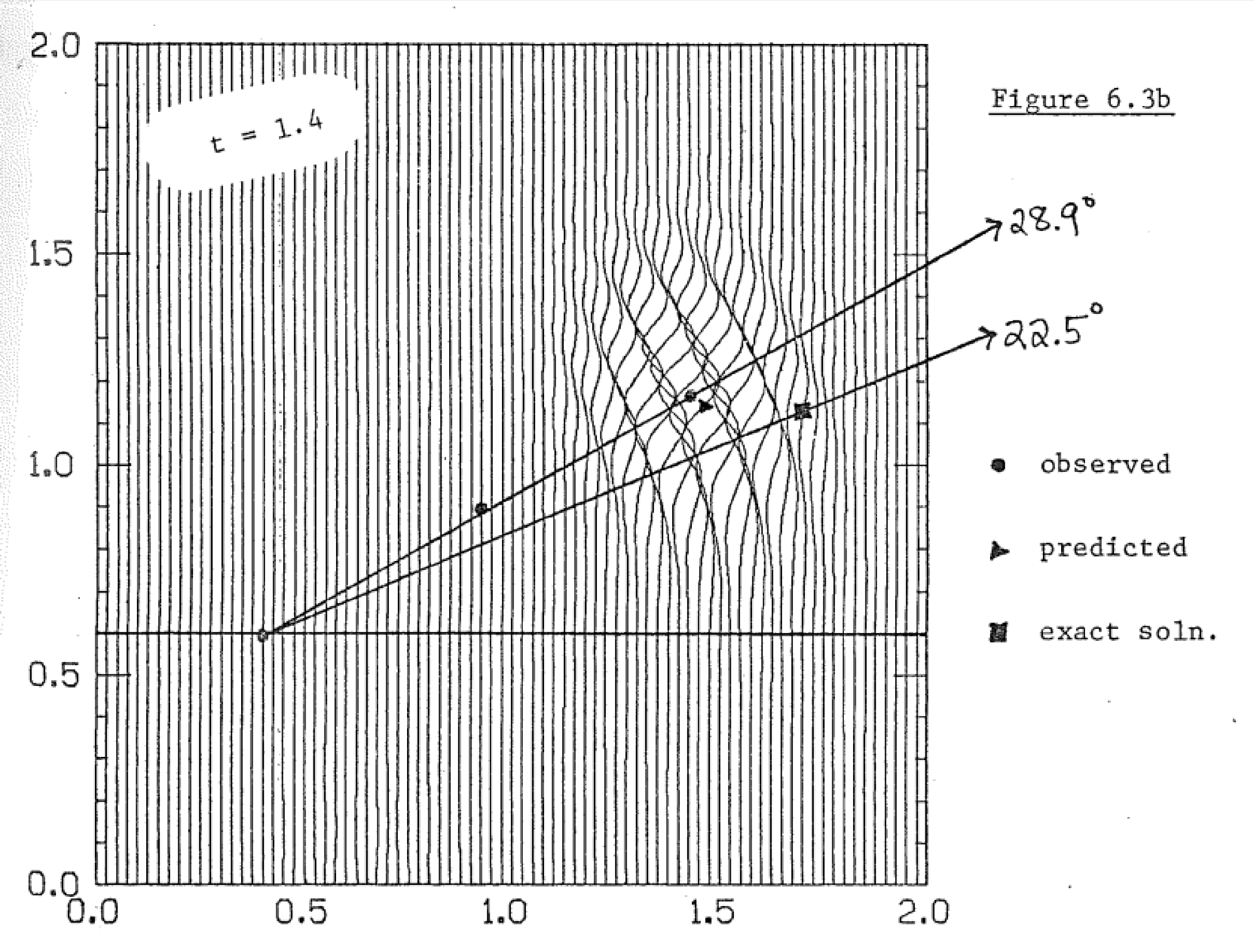


Figure 3.1

1981-82
Stanford



1981-82
Stanford

ω	λ	ξ obs/pred	c obs/pred	c_g obs?/pred
25	1	25.0/25.1	1.00/1.00	.98/.99
25	10	27.2/27.4	.91/.91	.78/.77
50	1	50.4/50.5	.99/.99	.96/.97
50	5	54.6/55.1	.91/.91	.72/.75
50	10	79.3/79.2	.66/.63	.25/.28

Table 11.2. Propagation of a forced wave under CN for $h = 1/250$, various ω and λ. See Figures 11.4 - 11.6.

1981-82
Stanford

LNT
5 February 1981

17. FOUR EXPLANATIONS OF GROUP VELOCITY

If a linear p.d.e. has sinusoidal solutions $e^{i(\omega t - \xi x)}$, where ω depends on ξ according to a dispersion relation $\omega = \omega(\xi)$, then energy associated with wave number ξ travels at the group speed

$$C_g = \omega'(\xi) . \qquad \text{GROUP SPEED} \qquad (*)$$

This seems magical — after all, how does energy propagating blithely with wave number ξ figure out what the derivative $\omega'(\xi)$ is equal to? Here I sketch four simple arguments to motivate (*) .

1. Beating between two sine waves (Lighthill, p. 247)

1981-82
Stanford

49. SLIDES FROM ORAL DEFENSE

WAVE PROPAGATION AND STABILITY FOR FINITE DIFFERENCE SCHEMES*

Lloyd N. Trefethen

* main concern: stability for difference models of hyperbolic initial boundary value problems

Reason #2 for writing memos: day-to-day motivation.

A book takes years.

A paper takes months.

A memo takes a day or two.

I’ve written around a thousand.

1983-84

NYU

4. FREE-STREAMLINE FLOW PAST WALLS AND HOUSES 26 July 1983

A. Introduction. In his fluid mechanics book Prandtl has a figure depicting the flow of air over a two-dimensional house:

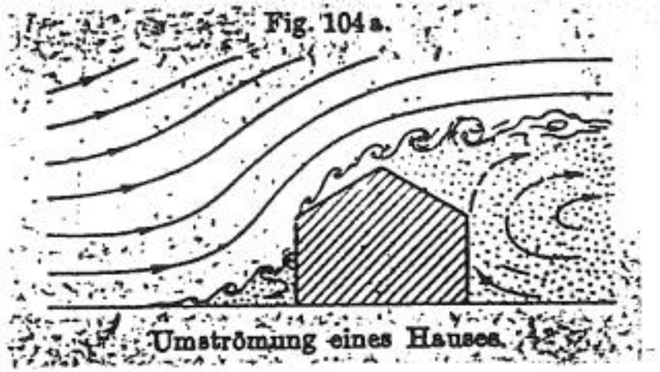


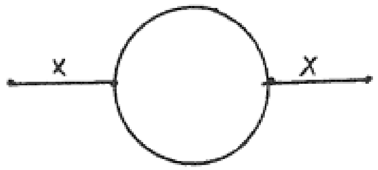
FIG. 4.1

If you make enough simplifying assumptions, this kind of problem can be treated by Schwarz-Christoffel maps after a hodograph transformation and then a log transformation. In particular, assume a motionless wake at ambient pressure extending to infinity and bounded by a sharp streamline, along which one then has has a constant speed $|v| = |v_\infty|$; let the flow everywhere else be ideal irrotational, incompressible 2D flow; and let there be no stagnant region in front of the house (despite Prandtl's picture). Let the hodograph variable be $\zeta = v_x - iv_y$, i.e. the conjugate of velocity.

1983-84

NYU

First, here is what one finds for a circle with two opposing slits of length x :

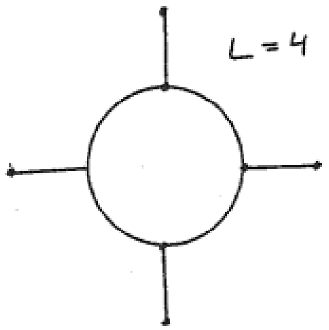


x	ρ
0	1.0
1/2	1.08333
1	1.25000
2	1.66667
3	2.12500
8	4.55556

Table 7.1

Obviously the general formula is $\rho = \frac{1}{2}[(x+1) + (x+1)^{-1}]$; this is probably easy to prove by exhibiting the map analytically.

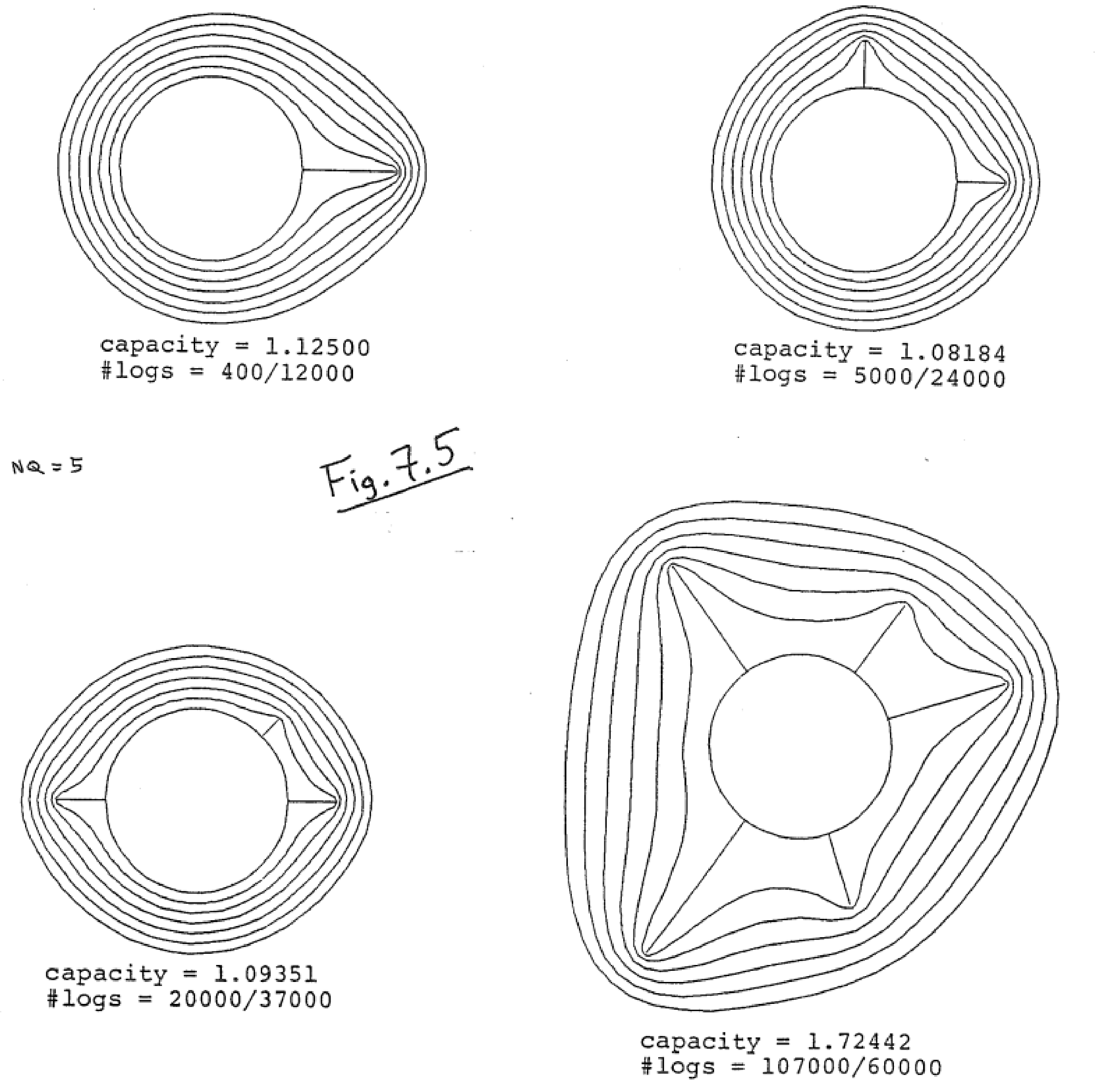
Second, here are results for a circle with L slits of length 1 equally spaced around it:



L	ρ
0	1.0
1	1.12500
2	1.25000
3	1.36284
4	1.45773
5	1.53448
6	1.59729
∞	2.0

Table 7.2

1983-84
NYU


capacity = 1.12500
#logs = 400/12000
capacity = 1.08184
#logs = 5000/24000
NQ = 5
Fig. 7.5
capacity = 1.09351
#logs = 20000/37000
capacity = 1.72442
#logs = 107000/60000

1983-84
NYU

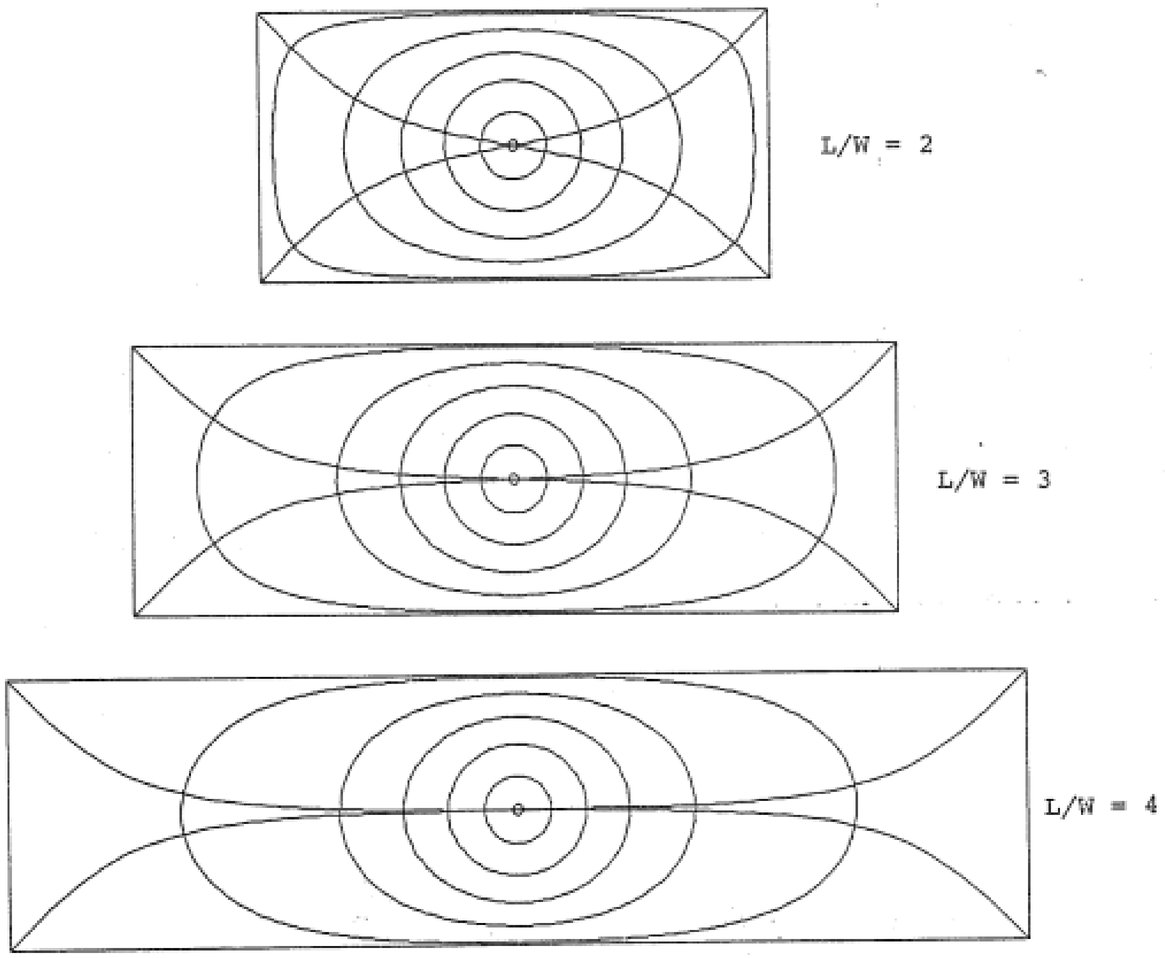


Figure 9.3. Conformal maps onto four rectangles. Images are plotted of radial segments and of concentric circles at radii .03, .2, .4, .6, .8, .97 in the disk. The crowding of prevertices as L/W increases can be seen in the exponentially decreasing angle formed between the radial segments at the origin.

1983-84

NYU

This is very beautiful, but a grim candidate for numerical integration. Perhaps the corresponding x domain looks something like this:

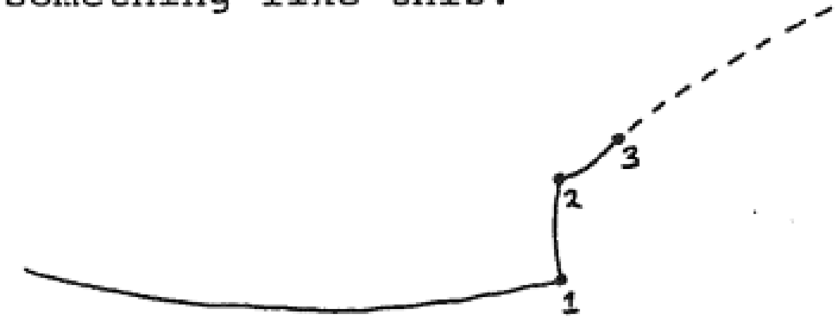


Fig. 13.3c

At any rate its boundary arcs are all curved.

1983-84

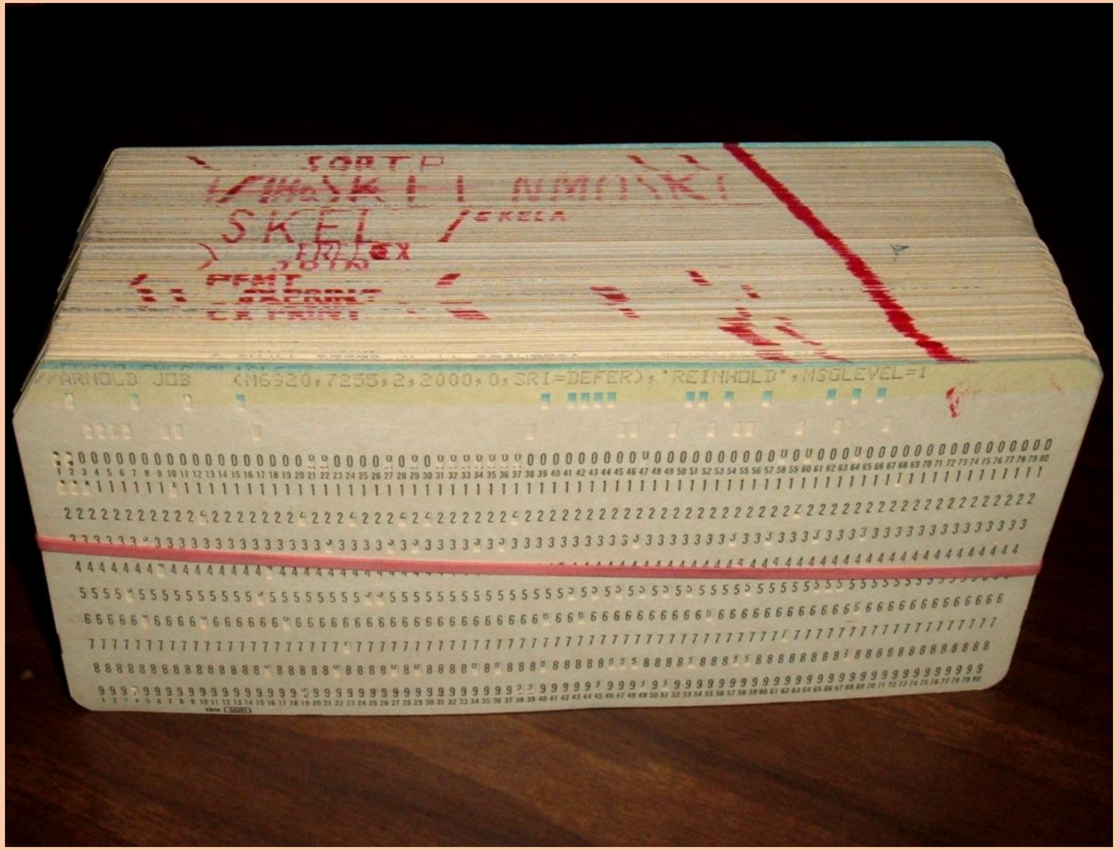
NYU

3. Program listing

The next few pages list the driver program I have used for these computations, together with the revised sections of SCPACK. The coding here is a bit ugly, due to the one-shot nature of the project.

```
      program h1
*
* FLOW OVER HOUSE
*
* Computes the ideal free-streamline flow over a "house"
*
* Variables:
*        x  physical plane
*        w  velocity potential
*        z  unit disk
*     zeta  hodograph;  zeta = dw/dz
*    omega  = -log(zeta)
*
* L. N. Trefethen, January 1984
*
      implicit double precision (a-b,d-h,p-v,y)
      implicit complex*16 (c,o,w,x,z)
      dimension z(6),betam(6),omega(6),qwork(300)
      common /xwc/ isave,nqq,theta,t(20,3),tw(20,3)
      common /nearwc/ x(13),w(13)
      common /logcnt/ nlog
      data n,x(2),w(2),x(4),w(4) /6, (0.d0,0.d0),(0.d0,0.d0),
     &  (0.d0,1.d0),(1.d0,0.d0)/
      data betam /-.5d0,-1.d0,0.d0,-1.d0,1.d0,-.5d0/
      pi = acos(-1.d0)
      pi2 = pi/2.d0
*
* input control data:
      print '(''nq,nqq,nstr ?'')'
      read *, nq,nqq,nstr
      tol = 10.d0**(-min(13,nq+3))
      print '(''angle of roof from vertical (fraction of pi) ?'')'
```

Reason #3 for writing memos: it gives you a repository of your old codes.

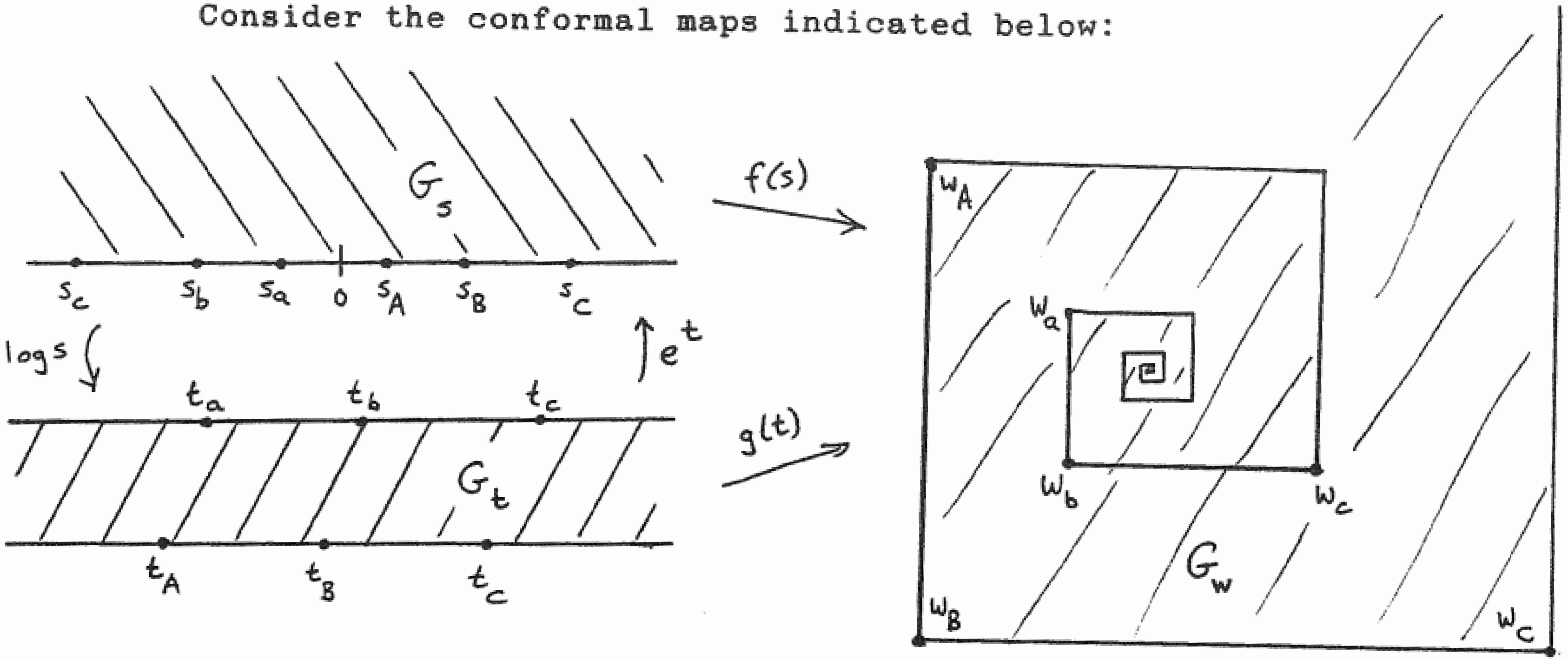


1987
Oxford visit

L. N. Trefethen
Oxford Computing Laboratory
June 9, 1987

SC B1. SCHWARZ-CHRISTOFFEL MAP OF A RECTILINEAR SPIRAL

Consider the conformal maps indicated below:



Reason #4 for writing memos: nice mementos for the future (e.g. at your retirement conference).

- Always put your name and the date on any document.
- I usually include the place, too.

2010
Oxford

RBF1. RATIONAL INTERPOLANTS BY PGV ALGORITHM

Nick Trefethen, 9 September 2010

Back at Oxford, with Bengt Fornberg in town for the autumn, I am getting involved with the project that we hope will lead to a Fornberg-Wright-Trefethen paper. The subject is rational interpolation for evaluating RBFs. Specifically, we want to evaluate an RBF interpolant s(e) depending on a smoothness parameter e in the small-e regime, where the direct method is impossibly ill-conditioned. The idea, due originally to Bengt, is to evaluate instead r(e), where r(e) is a rational function obtained by interpolating s (e) over a contour in a region of the complex e-plane where s(e) can be properly evaluated.

In my usual fashion I need to creep up on this problem to properly understand it, and I will do so via memos PUBLISHed in Matlab. I will try to keep the formatting so simple that it's enough to publish the M-files in html mode. To publish this one, for example, you just need to type open(publish('rbf1')). That's why it doesn't have proper LaTeX symbols. If this drives me crazy later on I may switch to LaTeX mode.

This first memo explores the simplest possible version of rational interpolation of a function with poles. I use the PGV algorithm, the beautiful new method developed here in the NA Group last year by Ricardo Pachon, Pedro Gonnet, and Joris Van Deun: see their report "Fast and stable rational interpolation in roots of unity and Chebyshev points" available at http://eprints.maths.ox.ac.uk/view/groups/nag/.

Reason #5 for writing memos: communicating progress to others.

2012

Oxford

CP1: First-order linear differential equations

Nick Trefethen, 7 August 2012

Contents

Ordinary Differential Equations by Carrier and Pearson (CP) is a differential equations text, a classic from 1968 that would probably be regarded as pretty advanced these days. I'll make a start of looking through it, following the CP chapter headings, with explorations in Chebfun.

The book has a striking feature: every other section, from beginning to end, is labeled "Problems". This gives an extraordinary emphasis to the importance

Reason #6 for writing memos: it's great practice in writing and LaTeXing.

2012
Oxford

EM 1. Memos on Euler-Maclaurin, contour integrals, and hyperfunctions

Nick Trefethen, 19 July 2012

Contents

1. A new series of memos

I am going to write a few memos to try to develop some ideas related to contour integrals, the Euler-Maclaurin formula, delta functions, hyperfunctions, rational barycentric formulas, etc. The audience I have in mind initially is myself, Mohsin Javed (now in his first year as a DPhil student), and André Weideman (planning several visits to Oxford in the upcoming year). Some other people who might be interested in this kind of thing include Stefan Güttel, George Klein, and Jean-Paul Berrut.

About News

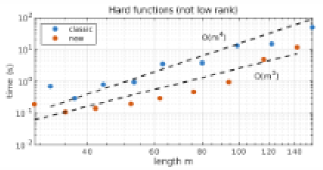
22 April 2016 / **200th Chebfun example posted!**

The Chebfun examples collection now has 200 examples.…

2012-2023
Oxford

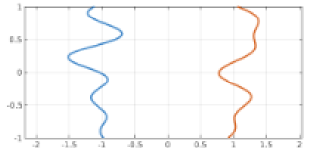
Chebfun3 speedups

Behnam Hashemi, Christoph Strössner, and Nick Trefethen, March 2023



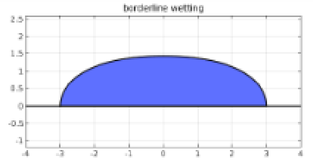
Distance between two curves

Nick Trefethen, November 2022



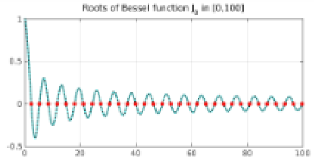
A droplet sitting on a surface

Ray Treinen and Nick Trefethen, October 2022



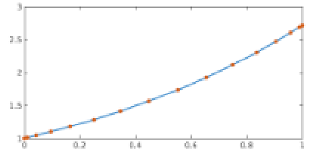
Rootfinding with the AAA algorithm

Stefano Costa, June 2022



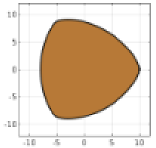
Delay differential equations in Chebfun

Nick Hale, June 2022



Polynomial level curve of constant width

Nick Trefethen, May 2022



Rat1. Rational approximation to data of Silantyev

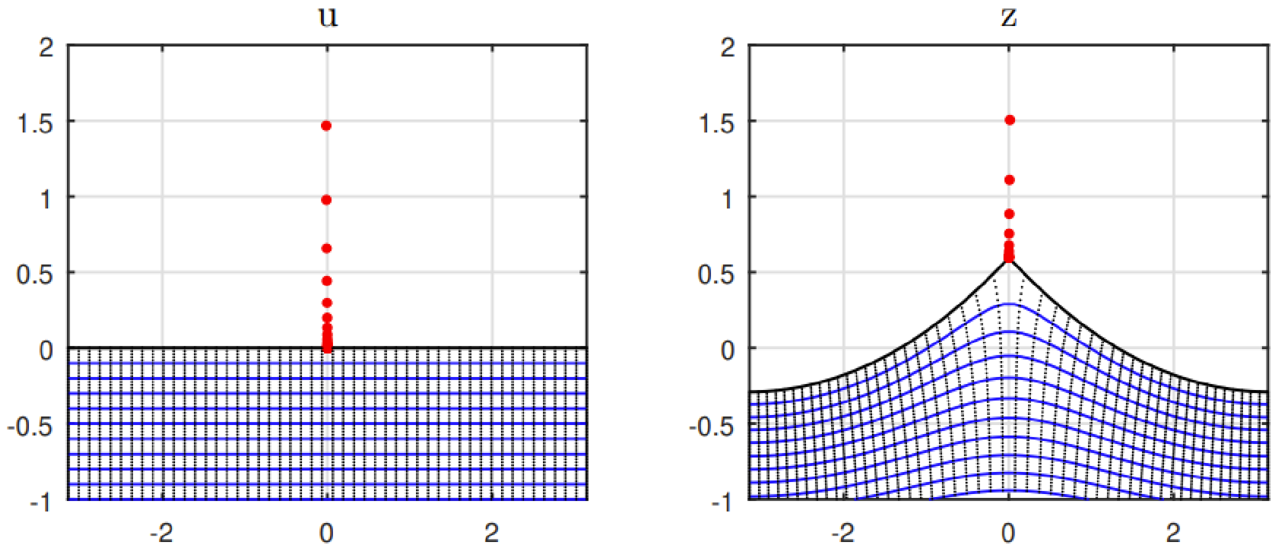
Nick Trefethen, New York, 21 September 2017

This is the first of a series of memos on various aspects of computing with rational functions.

Denis Silantiev has provided some data:

2017-
Oxford

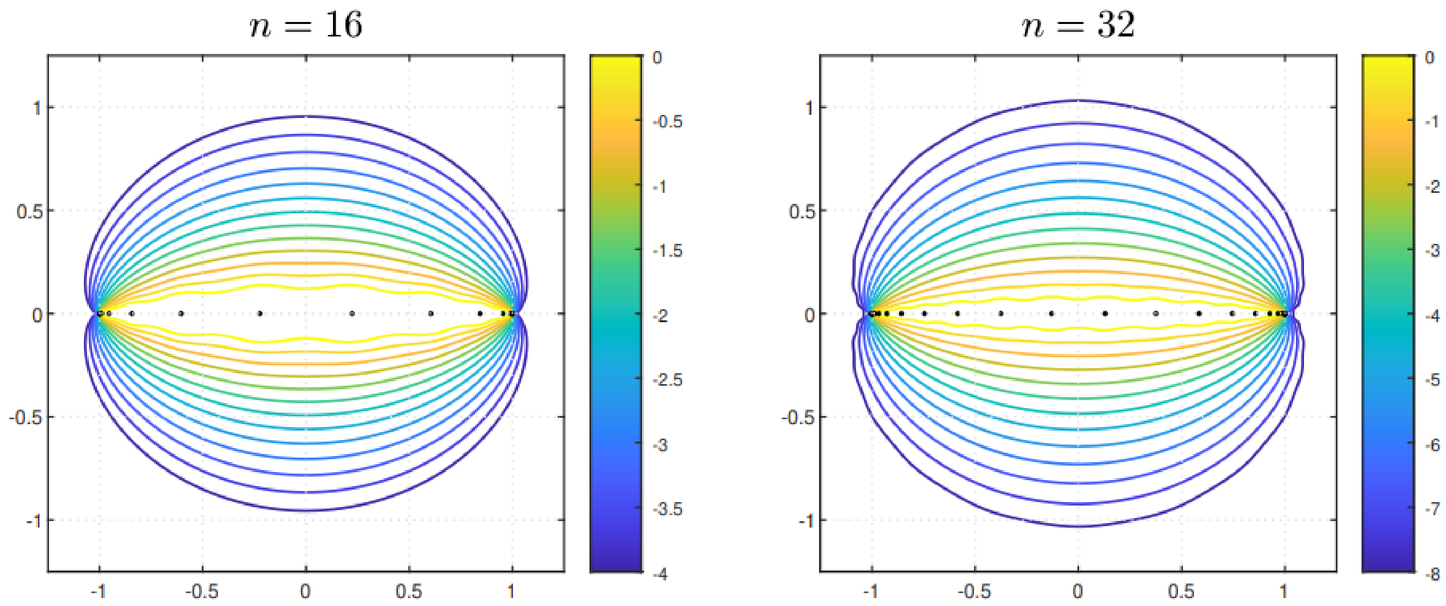
```
a = linspace(-pi,pi,50); b = linspace(-1,0,50);
[aa,bb] = meshgrid(a,b); uu = aa + 1i*bb;
subplot(1,2,1), plot(uu,'.k',MS,2), hold off
subplot(1,2,2), plot(r(uu),'.k',MS,2), hold off
```



2017-
Oxford

Rat100. DE quadrature and rational approximation

Nick Trefethen, Iffley, 27 March 2020

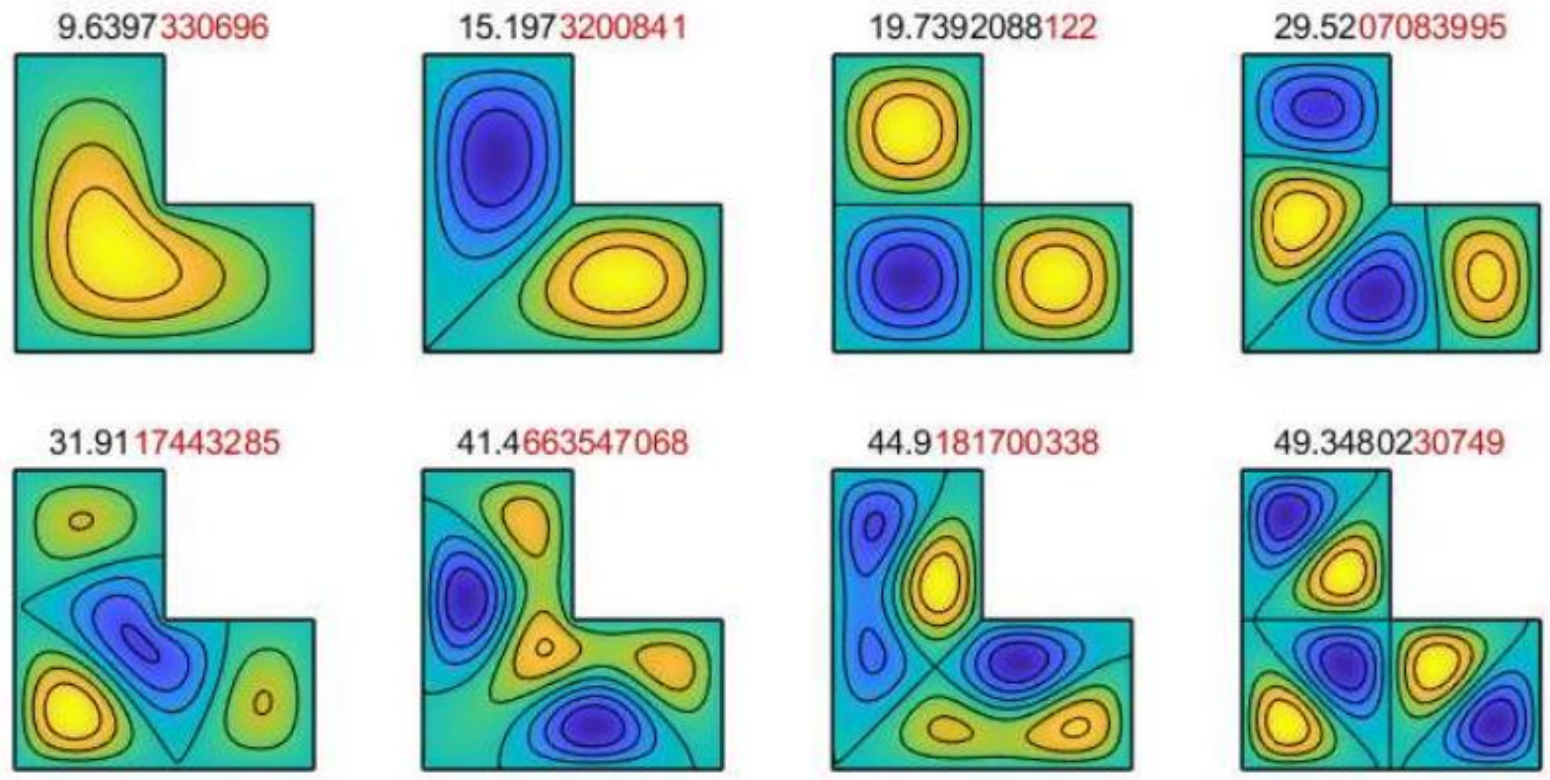


This pair of figures tells a very interesting story. The crucial fact is that the levels have been doubled from the left image to the right one, along with the value of n, yet the two figures look much the same. We are seeing an approximation whose accuracy is *exponential, not root-exponential.* (More precisely it is almost-exponential — adjusted by a log term.)

2017-
Oxford

Rat200. L-shaped eigenmodes with singular columns

Nick Trefethen, Iffley, 19 December 2021

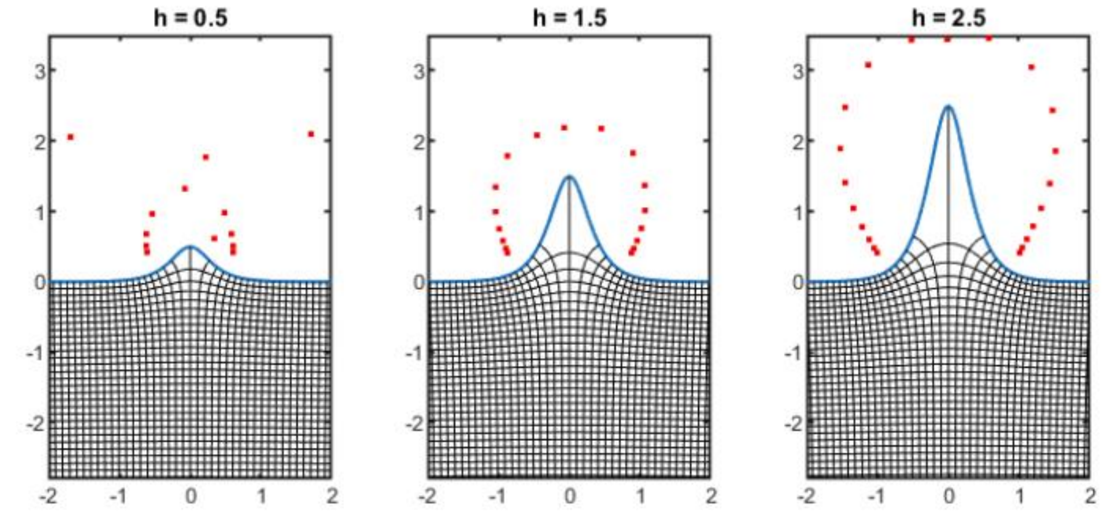


Rat287. “Crowding” and a half-plane

Nick Trefethen, Oxford, 4 August 2023

2017-
Oxford

Oct. 2024: Rat354



Yidan Xue has suggested a related image for comparison:


```
total_time_for_this_memo = toc
total_time_for_this_memo = 1.561
```

Reason #7 for writing memos: (wait a few years, you'll see).

"The horror of that moment," the King went on, "I shall never, never forget!" "You will, though," the Queen said, "if you don't make a memorandum of it."

My plan for the future:

From ~~287~~ 354 to ∞.

Thanks everybody for a wonderful week.

//