Lecture 1. Gaussian elimination and low-rank approximation Nick Trefethen, 5 Sept. 2017

Two faces of GE: elimination; low-rank approximation.

We will describe and compare the two, then explore one topic related to elimination and three related to low-rank approximation.

3 x 3 matrix example: A = [1 2 3; 2 8 11; 1 14 24]

ELIMINATION	LOW-RANK APPROXIMATION
A = square matrix	A = square or rectangular matrix or continuous analogue (quasimatrix, cmatrix)
at each step: subtract multiples of one row from others	at each step: subtract a rank-1 matrix
aim: LU factorization $A = LU$	aim: sum of rank 1 matrices $A = E_1 + E_2 + \dots$
finite dimensional	large finite, infinite, or continuous dimensional
direct	iterative
$\mathrm{cond}(A)$ finite	$\mathrm{cond}(A)$ infinite
orthogonal alternative: QR	orthogonal alternative: SVD
pivoting: row/column interchange (usually just rows – "partial") …	pivoting: row/column selection
applications: $Ax = b$, $\det(A)$	applications: compression, completion
history: back to Gauss and the ancient Chinese	history: booming in recent years

A curious parallel in the histories of GE and conjugate gradients
[Townsend + T, "Gaussian elimination as an iterative algorithm," *SIAM News*, 2013]

1. The unsolved problem of stability of GE
   [T & Schreiber, "Average-case stability of Gaussian elimination," *SIMAX* 1990; T + Bau, *Numerical Linear Algebra*, 1997, chap 22; Driscoll + Maki, "Searching for rare growth factors using muticanonical Monte Carlo methods," *SIREV* 2007]

2. Computing with multivariate functions
   [Townsend + T, Chebfun2 software and "An extension of Chebfun to two dimensions," *SISC 2013;* Hasehmi + T, Chebfun3 software and "Chebfun in three dimensions," *SISC*, to appear; Oseledets, "Tensor train decomposition," *SISC* 2011; Grasedyck, Kressner + Tobler, "A literature survey of low-rank tensor approximation techniques," *GAMM-Mitteilungen* 2013]

3. Low-rank approximation and alignment with axes
   [T, "Cubature, approximation, and isotropy in the hypercube," *SIREV* 2017]

4. What's the continuous analogue of LU factorization?
   [Townsend + T, "Continuous analogues of matrix factorizations," *Proc. Roy Soc. A* 2015]