Annotated Selection of Publications

Lloyd N. Trefethen, December 2020
Titles and citations are abbreviated. Ordering of categories is chronological.
∗ = 25 most cited on Google Scholar.

SC Schwarz-Christoffel maps and computational complex analysis

∗ Numerical computation of the Schwarz-Christoffel transformation, *SISC* 1980
Introduction of what became the standard numerical algorithm for computing SC conformal maps, featuring compound Gauss-Jacobi quadrature.

SCPACK User's Guide, ICASE internal report 1983
A Fortran software package for computing SC maps. This led to Driscoll's SC Toolbox in Matlab.

Solution of Laplace's equation on a polygon with oblique derivative b.c.'s, with Williams, *JCAM* 1986
Shows how SC maps can be used to solve Laplace problems with oblique derivative boundary conditions such as arise in queuing theory.

Ideal jet flow in two dimensions, with Dias and Elcrat, *JFM* 1987
Together with an earlier paper on free-streamline flows, shows that one should use a modified SC formula for such computations rather than the hodograph.

A modified Schwarz-Christoffel transformation for elongated regions, with Howell, *SISC* 1990
Develops a modified SC formula for mapping from rectangles or strips to circumvent the "crowding" problem for highly elongated polygons.

Green's functions for multiply connected domains via conformal mapping, with Embree, *SIREV* 1999
Uses SC maps to compute Green's functions for multiply connected geometries with a line of symmetry, including a union of real intervals.

Numerical solution of the omitted area problem, with Banjai, *Comp Meth Func Th* 2001
Shows numerically that the largest area of the unit disk that the image of a standard univalent function can omit is 0.2385813248π.

∗ *Schwarz-Christoffel Mapping*, with Driscoll, Cambridge U Press 2002
Monograph on history, theory, algorithms, and applications for Schwarz-Christoffel mapping.

A multipole method for SC mapping of polygons with thousands of sides, with Banjai, *SISC* 2003
Solves the SC parameter problem by a fast multipole method, enabling conformal mapping of polygons with thousands of sides.

Quantifying the ill-conditioning of analytic continuation, *BIT* 2020
It is a well-posed problem with an infinite condition number. For the Weierstrass chain-of-disks method, digits are lost at the rate exp(-*ex*).

CF Carathéodory-Fejér approximation

Near-circularity of the error curve in complex Chebyshev approximation, *J Approx Th* 1981
Shows that error curves in complex Chebyshev approximation are often spectacularly close to circular, and invents CF approximation to explain this.

Rational Chebyshev approximation on the unit disk, *Numer Math* 1981
Extension of the above results from polynomial to rational approximation: closely related to the "AAK" theory of Adamjan, Arov & Krein.

Real polynomial Chebyshev approximation by the CF method, with Gutknecht, *SINUM* 1981
Development of an analogous method for real approximation, with powers of *z* replaced by Chebyshev polynomials.

The CF method for real rational approximation, with Gutknecht, *SINUM* 1983
Extension of above to rational case, and as an application, the discovery of the number 9.28903… in the "1/9 conjecture" for approximation of exp(*x*) on (−∞,0].

MATLAB programs for CF approximation, *Approximation Theory V*, 1986
This appears to be the first research paper ever published with a MATLAB program.

The CF table, with Hayashi and Gutknecht, *Constr Approx* 1990
Extension of the theory of rational CF approximation to functions that are just continuous, not analytic.

A robust implementation of the CF method for rational approximation, with Van Deun, *BIT* 2011
Twenty years later, CF becomes a practical tool for polynomial and rational approximation and is included in Chebfun.

GV Group velocity, wave propagation, and stability in finite difference schemes

∗ Group velocity in finite difference schemes, *SIREV* 1982
Survey of dispersion relations and group velocity effects for finite difference models of PDE. Discreteness makes finite difference schemes behave like crystals.

Group velocity interpretation of the stability theory of Gustafsson, Kreiss, and Sundström, *JCP* 1983
Shows that the GKS stability criterion for hyperbolic IBVPs is equivalent to a condition involving group velocities.

Instability of finite difference models for hyperbolic IBVPs, *CPAM* 1984
Theorems about GKS/group velocity effects in various contexts. This article won the first Fox Prize in Numerical Analysis.

∗ Well-posedness of absorbing boundary conditions and one-way wave eqs, with Halpern, *Math Comp* 1986
Shows that only the two diagonals of the Padé table recommended by Engquist and Majda generate well-posed one-way wave equations.

Wide-angle one-way wave equations, with Halpern, *J Acoust Soc Amer* 1988
Investigation of six families of rational approximations of $\sqrt{1-x^2}$ on [−1,1] and the one-way wave equations they generate.

∗ *Finite Difference and Spectral Methods for Ordinary and Partial Differential Equations*, unpublished 1996
Graduate textbook, freely available at `people.maths.ox.ac.uk/trefethen/pdetext.html`.

RAT Rational functions and approximations

Real vs. complex rational Chebyshev approximation on an interval, with Gutknecht, *Trans AMS* 1983
Shows that type (m,n) complex best rational approximations on an interval can be arbitrarily more accurate than real ones if $n \geq m+3$.

Nonuniqueness of best rational Chebyshev approximations on the unit disk, with Gutknecht, *J Approx Th* 1983
Shows that best rational approximations on the unit disk may be nonunique. Previously, nonuniqueness had been established only for certain non-circular domains.

Square blocks and equioscillation in the Padé, Walsh, and CF tables, Springer *LNM* 1105, 1984
Explains the structural similarity between these three approximation problems and their differences, with consequences for continuity of approximation operators.

Padé, stable Padé, and Chebyshev-Padé approximation, with Gutknecht, *Algorithms for Approximation*, 1987
Extensive and systematic treatment of Chebyshev-Padé and associated methods of rational approximation.

∗ Talbot quadratures and rational approximations, with Weideman and Schmelzer, *BIT* 2006
Shows that every quadrature formula corresponds to a rational approximation and explores the approximations associated with quadrature on Hankel contours.

Robust rational interpolation and least-squares, with Gonnet and Pachón, *ETNA* 2011
Proposes an algorithm based on the SVD for computation of rational interpolants in roots of unity without spurious poles, i.e., Froissart doublets.

Robust Padé approximation via SVD, with Gonnet and Güttel, *SIREV* 2013
Analogous algorithm for robust Padé approximation, based on hopping across square blocks identified via SVD; proposal of theoretical implications.

The AAA algorithm for rational approximation, with Nakatsukasa and Sète, *SISC* 2018
"Adaptive Antoulas-Anderson" algorithm for rational approximation appears to be the first to fully exploit rational barycentric representations.

Rational minimax approximation, with Filip and Nakatsukasa, *SISC* 2018
Remez algorithm based on barycentric representations computes approximations in 16-digit arithmetic for which Varga, et al. required 200 digits.

Rational approximation of x^n, with Nakatsukasa, *Proc AMS* 2018
Shows that degree n rational approximations to x^n on $[-1,1]$, as with $\exp(x)$ on $(-\infty,0]$, converge at the rate $C/(9.28903\ldots)^n$.

Exponential node clustering at singularities…, with Nakatsukasa and Weideman, Numer Math, to appear
Shows that rational approximations optimize accuracy via linearly-tapered exponential clustering of nodes at singularities.

An algorithm for real and complex rational minimax approximation, with Nakatsukasa, *SISC*, to appear
Introduces the AAA-Lawson algorithm, implemented in Chebfun, for rational minimax approximation in the complex plane.

KMT Kreiss matrix theorem and stability theory for ODEs and PDEs

On the resolvent condition in the Kreiss Matrix Theorem, with LeVeque, *BIT* 1984
Conjectures the sharp form of the Kreiss matrix theorem and proves the conjecture up to a factor of 2. The conjecture was later proved by Spijker.

Stability of the method of lines, with Reddy, *Numer Math* 1992
Development of theorems that show that stability of the methods of lines is equivalent to pseudospectra lying inside a stability region.

Stiffness of ODEs, with D. Higham, *BIT* 1993
Shows that stiffness cannot be characterized by eigenvalues and proposes an alternative characterization in terms of pseudospectra.

From the Buffon needle problem to the Kreiss Matrix Theorem, with Wegert, *Amer Math Monthly* 1994
Elementary proof of Spijker's sharp Kreiss matrix theorem using the "Buffon noodle", here independently rediscovered as the "Buffon paper clip".

The Kreiss matrix theorem on a general complex domain, with Toh, *SIMAX* 1999
Generalization of the Kreiss matrix theorem from the unit disk to a complex domain.

GE Gaussian elimination and classical numerical linear algebra

Fourier analysis of the SOR iteration, with LeVeque, *IMAJNA* 1988
Shows that SOR optimal ω can be derived by Fourier analysis if the grid is tilted, and that the same tilting is the basis of Garabedian's PDE argument.

∗ Average-case stability of Gaussian elimination, with Schreiber, *SIMAX* 1990
Investigates the mystery that GE with partial pivoting is unstable in the worst case but "always" stable in practice. See also Chap. 26 of Trefethen & Bau.

∗ *Numerical Linear Algebra*, with Bau, SIAM 1997
SIAM's all-time bestselling textbook. Also appeared in Chinese translation.

Householder triangularization of a quasimatrix, *IMAJNA* 2010
Generalizes Householder factorization to matrices whose columns are functions of a continuous variable, requiring first a generalization of the QR idea.

Gaussian elimination as an iterative algorithm, with Townsend, *SIAM News* 2013
Shows that like CG, GE has both a direct and an iterative side, the latter being the basic algorithm for low-rank approximation of matrices.

Continuous analogues of matrix factorizations, with Townsend, *Proc Roy Soc* 2015
Develops a theory of LU, QR, SVD, and Cholesky factorizations of 2D functions. A key notion is that of a *triangular quasimatrix*.

Vandermonde with Arnoldi, with Brubeck and Nakatsukasa, *SIREV, to appear*
Shows that for a wide variety of problems in data-fitting and solution of PDEs, Stieltjes orthogonalization eliminates the usual stability barrier.

KRY Krylov iterations

∗ How fast are nonsymmetric matrix iterations?, with Nachtigal and Reddy, *SIMAX* 1992
Identifies classes of matrices for which methods of types (1) CG, (2) BiCG, and (3) CGN maximally outperform the others.

A hybrid GMRES algorithm, with Nachtigal and Reichel, *SIMAX* 1992
Argues that two-step Krylov iterations should be based on polynomial coefficients, not eigenvalue estimates.

GMRES/CR and Arnoldi/Lanczos as matrix approximation problems, with Greenbaum, *SISC* 1994
Defines and proves existence and uniqueness for the matrix optimization problems underlying Krylov iterations.

The Chebyshev polynomials of a matrix, with Toh, *SIMAX* 1998
The degree n Chebyshev polynomial of a matrix A is the monic polynomial that minimizes $\|p(A)\|$. Existence and uniqueness are proved and examples explored.

From potential theory to matrix iterations in six steps, with Driscoll and Toh, *SIREV* 1998
The six approximations are (1) finite n, (2) estimated spectrum, (3) nonnormality, (4) particular RHS, (5) quasi-minimization, (6) floating point.

PSA Pseudospectra

∗ Pseudospectra of matrices, in *Numerical Analysis 1991*, 1992
Introduction of the idea of pseudospectra, illustrated by thirteen highly nonnormal matrices.

Pseudozeros of polynomials and pseudospectra of companion matrices, with Toh, *Numer Math* 1994
Shows that pseudozero sets of polynomials are closely approximated by pseudospectra of balanced companion matrices, implying practical stability of "ROOTS".

∗ Pseudospectra of linear operators, *SIREV* 1997
Investigation of pseudospectra of ten highly nonnormal linear operators: Zabczyk, Hille-Phillips, convection-diffusion, Papkovich-Fadle, Poiseuille flow,….

∗ Computation of pseudospectra, *Acta Numer* 1999
Great speedups are achievable by preliminary triangularization, projection to an invariant subspace, and Arnoldi iteration. This paper led to EigTool.

Large-scale computation of pseudospectra using ARPACK and eigs, with Wright, *SISC* 2001
Shows that for matrices of large dimension, pseudospectra can be computed at negligible additional cost once the spectrum is computed.

Pseudospectra of rectangular matrices, with Wright, *IMAJNA* 2002
Observes that whereas spectra of rectangular matrices rarely exist, pseudospectra always exist and are meaningful.

∗ *Spectra and Pseudospectra*, with Embree, Princeton U Press 2005
Monograph with sixty short chapters reviewing all aspects of theory and applications of pseudospectra.

TOEP Toeplitz/twisted Toeplitz matrices and analogous differential operators

∗ Eigenvalues and pseudo-eigenvalues of Toeplitz matrices, with Reichel, *Lin Alg Appl* 1992
Illustrations of the exponential nonnormality of nonsymmetric Toeplitz matrices, with theorems relating pseudospectra to the symbol.

Piecewise continuous Toeplitz matrices and operators, with Böttcher and Embree, *SIMAX* 2002
Demonstrates that if the symbol is only piecewise continuous, the resolvent norms grow only very slowly with the dimension.

Wave packet pseudomodes of twisted Toeplitz matrices, with Chapman, *CPAM* 2004
Proves general theorems asserting that properties of the symbol imply existence of exponentially localized pseudomodes.

Wave packet pseudomodes of variable coefficient differential operators, *Proc Roy Soc* 2005
Continuous analogue of above. The theorems are structurally stable, different from what one gets by microlocal analysis.

TT Transition to turbulence

∗ Hydrodynamic stability without eigenvalues, with A. Trefethen, Reddy, and Driscoll, *Science* 1993
Computes pseudospectra for Couette and Poiseuille flow; explains why eigenvalues do not determine stability; proposes attention to narrow basins of attraction.

A mostly linear model of transition to turbulence, with Baggett and Driscoll, *Phys Fluids* 1995
Shows how 3-variable ODE models capture the key fluid flow features of (a) nonnormal linearly stable state and (b) chaotic global solutions.

Low-dimensional models of subcritical transition to turbulence, with Baggett, *Phys Fluids* 1997
Compares six published models of transition, finding that they share the features of a nonnormal linearly stable state with a narrow basin of attraction.

Spectra and pseudospectra for pipe Poiseuille flow, with A. Trefethen and Schmid, *Comm Meth Appl Mech* 1999
Computes spectrum of linearized pipe flow — a countable collection of curves in the left half-plane — and associated pseudospectra involved in transition.

∗ Linearized pipe flow to Reynolds number 10^7, with Meseguer, *JCP* 2003
New discretization enables computation of eigenvalues and eigenmodes for pipe flow to an unprecedentedly high Reynolds number.

RAND Probability and stochastic processes

Condition numbers of random triangular matrices, with Viswanath, *SIMAX* 1998
The inverses of random triangular matrices grow exponentially (almost surely) with the dimension. Here exact Lyapunov constants are derived for various cases.

Growth and decay of random Fibonacci sequences, with Embree, *Proc Roy Soc* 1999
Computes Lyapunov constants for randomized variants of the Fibonacci recurrence, implying almost sure growth or decay depending on a parameter.

Computing Lyapunov constants for random recurrences with smooth coefficients, with Wright, *JCAM* 2001
Analogous results for random recurrences with coefficients from smooth distributions.

Spectra, pseudospectra and localization for random bidiagonal matrices, with Contedini and Embree, *CPAM* 2001
Shows by analysis of an analogous bidiagonal problem that the essence of the Hatano-Nelson eigenvalue model is nonnormality.

Smooth random functions, random ODEs, and Gaussian processes, with Filip and Javeed, *SIREV* 2019
Shows how technicalities of stochastic analysis can be avoided by working with random Fourier series rather than Wiener processes: Chebfun `randnfun`.

SPEC Spectral methods for ODE and PDE

* *Spectral Methods in MATLAB*, SIAM, 2000
Textbook of Chebyshev spectral collocation methods based on MATLAB programs p1,…,p40.

* Fourth-order time-stepping for stiff PDEs, with Kassam, *SISC* 2003
Shows that the Cox-Matthews ETDRK4 formula gives robust 4th-order convergence for KdV, Kuramoto-Sivashinsky, Burgers and Allen-Cahn equations.

A rational spectral collocation method with adaptively transformed Chebyshev grid points, with Tee, *SISC* 2006
Combines Padé localization of singularities, conformal mapping, and rational barycentric interpolation to achieve spectral accuracy on highly distorted grids.

Block operators and spectral discretizations, with Aurentz, *SIREV* 2017
Presents a continuous analogue of block matrices as a methodology for linear and nonlinear spectral discretization, featuring rectangular differentiation matrices.

DRUMS Polygonal drums and other eigenvalue problems

Eigenvalues and musical instruments, with Howle, *JCAM* 2001
Proposes that frequencies and decay rates of eigenmodes of musical instruments can be analysed in tandem by plotting complex eigenvalues in the complex plane.

Reviving the method of particular solutions, with Betcke, *SIREV* 2005
Explains why the Fox-Henrici-Moler "MATLAB logo" method often fails, and fixes the difficulty by sampling inside the polygon as well as on the boundary.

Computations of eigenvalue avoidance in planar domains, with Betcke, *Proc Appl Math Mech Mech* 2004
Rectangles have degenerate Laplace eigenmodes. Here it is shown how perturbing the shape makes them nondegenerate.

Computed eigenmodes of planar regions, with Betcke, *Contemp Math* 2006
Surveys a wide range of physical effects through examination of highly accurate solutions of eigenproblems in various geometries.

TDA Challenge problems and Ten Digit Algorithms

A hundred-dollar, hundred-digit challenge, two articles in *SIAM News* 2002
A computational challenge springing from the "Problem Solving Squad" at Oxford. This led to English and German books published by Bornemann, et al.

Ten digit algorithms, Oxford technical report, 2004
Argues that most computational scientists should spend most of their time computing in the mode of "ten digits, five seconds, and just one page".

Ten digit problems, in *An Invitation to Mathematics*, Springer 2011
Discussion of the Oxford Problem Solving Squad with four example problems.

INTERP Interpolation and approximation in 1D and nD

* Barycentric Lagrange interpolation, with Berrut, *SIREV* 2004
Survey of the barycentric formula: in principle old, but known and used by few. This article led to widespread recognition of barycentric formulas.

Six myths of polynomial interpolation and quadrature, *Maths Today* 2011
Discussion of common misconceptions, with themes that (a) polynomial interpolation is better and (b) Gauss quadrature is not as optimal as is widely believed.

* *Approximation Theory and Approximation Practice*, SIAM 2013 and extended edition 2019
Chebfun-based textbook of "approximation theory for the 21st century" emphasizng Chebyshev polynomials and historical origins, with an annotated bibliography.

Impossibility of fast stable approximation from equispaced samples, with Kuijlaars and Platte, *SIREV* 2011
Proves that no stable approximation scheme based on equispaced data, linear or nonlinear, can be geometrically convergent.

Cubature, approximation, and isotropy in the hypercube, *SIREV* 2017
Reviews grid-alignment and related issues for low-rank approximation, sparse grids, quasi-Monte Carlo, and multivariate polynomial approximation.

Multivariate polynomial approximation in the hypercube, *Proc. AMS* 2017
Proves that "Euclidean degree", rather than total or maximal degree, determines accuracy of multivariate polynomial approximation in the hypercube.

CHEBFUN Chebfun and numerical computing with functions

* An extension of MATLAB to continuous functions and operators, with Battles, *SISC* 2004
Introduction of Chebfun and the associated idea of overloading discrete methods by the their continuous analogues.

Computing numerically with functions instead of numbers, *Math Comput Sci* 2007 and *Comm ACM* 2015
Proposal of a "floating point arithmetic for functions", the basis of Chebfun-style computing.

Piecewise smooth chebfuns, with Pachón and Platte, *IMAJNA* 2010
Describes extension of Chebfun from global to piecewise polynomial representations: a chebfun is composed of one or more "funs".

The chebop system for automatic solution of differential equations, with Driscoll and Bornemann, *BIT* 2008
Extension of Chebfun to solve ODEs via automatic Chebyshev spectral methods.

A sinc function analogue of Chebfun, with Richardson, *SISC* 2011
Reimplementation of core Chebfun via transplanted sinc functions instead of Chebyshev polynomials. This resolves general endpoint singularities, but at great cost.

An extension of Chebfun to two dimensions, with Townsend, *SISC* 2014
Extension of Chebfun to computation with functions on a rectangle, based on low-rank approximations.

∗ *The Chebfun Guide,* edited with Driscoll and Hale, 2014
Online and privately published 20-chapter user's guide to Chebfun.

An extension of Chebfun to periodic functions, with Wright, Javed, and Montanelli, *SISC* 2015
Chebfun is extended from Chebyshev to Fourier representations, leading to observations about differences between the two settings.

Chopping a Chebyshev series, with Aurentz, *TOMS* 2017
Presents the new (as of 2015) Chebfun algorithm for the surprisingly challenging problem of robustly chopping series to prescribed precision.

Chebfun in three dimensions, with Hashemi, *SISC* 2017
Extension of Chebfun to computations with functions in a box, again based on low-rank approximations.

CONTOUR Numerical methods based on contour integrals

∗ Parabolic and hyperbolic contours for computing the Bromwich integral, with Weideman, *Math Comp* 2007
Optimal parameters are derived for geometric convergence in trapezoidal discretizations of Hankel contours.

Computing A^α, log(A), and related matrix functions by contour integrals, with Hale and N. Higham, *SINUM* 2008
After a conformal map of a doubly slit plane to an annulus, the trapezoidal rule gives a very efficient algorithm for computing $f(A)$.

Numerical algorithms based on analytic function values at roots of unity, with Austin and Kravanja, *SINUM* 2014
Investigates relationships between polynomial interpolation and discretized contour integrals, and shows that rational analogues may do even better.

Computing eigenvalues with rational filters, with Austin, *SISC* 2015
In "FEAST"-related algorithms of Polizzi and Sakurai, contour integrals can be replaced by more flexible rational interpolants and filters.

QUAD Quadrature

∗ Is Gauss quadrature better than Clenshaw-Curtis?, *SIREV* 2008
Shows that C-C quadrature converges at the same rate as Gauss for non-analytic functions, and even in a practical sense for many analytic ones.

New quadrature formulas from conformal maps, with Hale, *SINUM* 2008
Shows that Gauss quadrature is suboptimal by a factor of $\pi/2$, and develops near-optimal quadrature formulas via a conformal map of an ellipse to a strip.

A trapezoidal rule error bound unifying the Euler-Maclaurin formula…, with Javed, *Proc Roy Soc* 2013
Derives a bound that gives the Euler-Maclaurin formula in one limit and exponential convergence for periodic integrands in another.

∗ The exponentially convergent trapezoidal rule, with Weideman, *SIREV* 2014
Comprehensive review of the exponentially convergent trapezoidal rule and its use in scientific computing.

Euler-Maclaurin and Gregory interpolants, with Javed, *Numer Math* 2015
Generalizes to E-M and Gregory quadrature the fact that the equispaced trapezoidal rule is equivalent to integration of a trigonometric interpolant.

Trigonometric interpolation and quadrature in perturbed points, with Austin, *SINUM* 2017
Shows stability and accuracy of interpolants and quadrature formulas in perturbed points, contradicting expectations based on Kadec's ¼ theorem.

ODE Ordinary differential equations

Exploring ODEs, with Birkisson and Driscoll, SIAM 2018
Chebfun-based textbook on ordinary differential equations (*not* on numerical algorithms) freely available online.

Eight perspectives on an exponentially ill-conditioned equations, *SIREV* 2020
Uses the equation εy"-xy'+y=0 to link boundary layer theory, dynamical systems, adjoint analysis, Lewy nonexistence, and other topics.

RATPDE Rational functions for solution of PDE and related problems

Series solution of Laplace problems, *ANZIAM J* 2018
Demonstration of the surprising power of elementary series for solving PDEs in smooth 2D regions.

New Laplace and Helmholtz solvers, with Gopal, *PNAS* 2019
Demonstration that for regions with corners, root-exponential convergence can be achieved with rational functions with exponentially clustered fixed poles.

Solving Laplace problems with corner singularities via rational functions, with Gopal, *SINUM* 2019
Introduction of the "lightning Laplace solver" with theorems establishing its root-exponential convergence.

Representation of conformal maps by rational functions, with Gopal, *Numer Math* 2019
Root-exponentially convergent representations of conformal maps, both forward and inverse, via rational functions from AAA approximation.

Numerical conformal mapping with rational functions, *Comp Meth Func Th* 2020
Lightning Laplace solution of conformal mapping problems; the results can then be further compressed by the method of the paper above.

Reciprocal-log approximation and planar PDE solvers, with Yakatsukasa, arXiv 2020
A new kind of approximation goes beyond rational functions, giving exponential rather than root-exponential convergence for Laplace problems.

OTHER Other technical contributions

The (Unfinished) PDE Coffee Table Book, with K. Embree, 2000, freely available online
34 two-page spreads giving key facts about different PDEs, with computed illustrations. If completed, there would have been 100 PDEs.

How many shuffles to randomize a deck of cards?, with L. M. Trefethen, *Proc Roy Soc* 2000
Shows that Diaconis's "cutoff effect" is absent if randomization is measured by bits of information. This article was covered widely in newspapers and radio.

Four bugs on a rectangle, with Chapman and Lottes, *Proc Roy Soc* 2011
The generalization of Martin Gardner's famous problem from a square to a rectangle brings a phase transition to 1D motion and gigantic numbers like $10^{427,907,250}$.

Mathematics of the Faraday cage, *SIREV* 2015
Though Faraday shielding dates to 1836 and is widely exploited by engineers, this is the first mathematical analysis and shows the effect is weaker than thought.

ESSAYS Opinion pieces and index cards

The definition of numerical analysis, SIAM News 1992, reprinted in Trefethen & Bau textbook 1997
Defines NA as the study of algorithms for problems of continuous mathematics, *not* the study of rounding errors.

Maxims about numerical mathematics, computers, science, and life, *SIAM News* 1998
40 aphorisms touching subjects from the fractal structure of scientific revolutions to the probability of extraterrestrial life.

Predictions for scientific computing fifty years from now, *Maths Today*, 2000
This essay won the *Maths Today* Catherine Richards Prize for 2000.

An American at Oxford, *Oxford Magazine*, May 2003
This essay, discussed in the *THES* and elsewhere, criticizes Oxford for relying on 600 admissions committees to select each year's incoming class.

Obituary: Gene H. Golub (1932-2007), *Nature* 2007
Obituary of a leading numerical analyst of the 20th century.

G. H. Hardy, applied mathematician, *SIAM News* 2008
Reflections on the prize book won by Hardy for being the best physics student in year 1891 at Winchester School.

Numerical analysis, in *Princeton Companion to Mathematics* 2008
Survey of history and current state of the field, including a table of 29 important numerical algorithms and their originators.

Trefethen's Index Cards, World Scientific Publishing 2011
4"×6" book with a selection of short notes on philosophical and scientific subjects accumulated from the age of 14. Newer cards on blog listed below.

President's columns in *SIAM News*, 2011-2012
Twenty columns on various mathematical subjects published as President of SIAM. Available online.

BMI (Body Mass Index), people.maths.ox.ac.uk/trefethen/bmi.html and bmi_calc.html, 2013
Proposal of a modified BMI formula with exponent 2.5, covered by newspapers and radio stations around the world, with 1.3 million hits at the web pages.

Surprises of the Faraday cage, *SIAM News* 2016
The story of the Faraday cage problem and its incorrect treatment in the *Feynman Lectures on Physics.*

Inverse Yogiisms, *Notices of the AMS* 2016, reprinted by Princeton in *The Best Writing on Mathematics 2017*
Yogi Berra said things that were literally nonsensical, yet conveyed a truth. Mathematicians often prove theorems that are literally true, yet convey a falsehood.

Index Card Notes, blog at `https://trefethen.net`, beginning September 2013
Notes on various subjects, 1-2 per month.

Rational functions and beyond, *SIAM News* December 2020
Essay based on SIAM John von Neumann lecture displaying AAA, lightning, and log-lightning approximations.

Notes of a Numerical Analyst in *LMS Newsletter,* beginning September 2020
Regular column on various topics at the interface of theoretical and computational mathematics.