Vacuum energy in cyclic universe (Draft)

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1 Cyclic universe

1.1 Cyclic universe. The assumption of huge-finite (or pseudo-finite) universe of the form $\mathbb{U}^4$ (instead of Minkowski $\mathbb{R}^4$), with $\mathbb{U}$ of the form $\mathbb{Z}_{\mathcal{N}} = \mathbb{Z}/\mathcal{N}\mathbb{Z}$ (the ring of residues of integer numbers modulo a highly divisible number $\mathcal{N}$) was considered in [Z25-1] and [Z25-2]. In particular, it was argued in [Z25-1] that because of cyclicity the appropriate numerical coordinate system should be a huge size finite field $\mathrm{F}_{\mathfrak{p}}$ ($\mathfrak{p}$ a prime number) or the residue ring $\mathrm{K} = \mathbb{Z}_{\mathcal{N}}$. More specifically, see also [Z25-2], $\mathbb{U}$ should be treated as a 1-dimensional K-module. And there is a useful abstractly defined map

$$\exp_{\mathfrak{p}} : \mathbb{U} \to \mathrm{F}_{\mathfrak{p}}$$

a homomorphism of the additive group of K onto the multiplicative group of $\mathrm{F}_{\mathfrak{p}}$ which is a good analogue of the classical exponentiation map.

In section 2 we work out some mathematics over the ring K. In section 3 we apply it to make a statement about the cosmological constant.

2 Physics over the ring K

2.1 A crucial point in the analysis of physics over such a cyclic universe is that it can be interpreted by the physicist in two ways - either by a **local** or by a **global** approximation. Conceptually, in local approximation the observer sees only the ininital interval of K, and in global approximation one sees K with its algebraic structure as a whole. The definition and a detailed mathematical analysis of these notions are in [Z26] but in order to understand the claims below it will suffice to quote the following:

Fact ([Z26]). Let K be a huge-finite field or ring. Then:

- a global approximation of K is $\bar{\mathbb{C}}$, the field of complex numbers compactified by adding ∞, written

$$\mathsf{Im}^{\mathrm{glob}}(\mathrm{K}) = \bar{\mathbb{C}}$$

- a local approximation of K is $\mathbb{R}$, the field of real numbers, written

$$\mathsf{Im}^{\mathrm{loc}}(\mathrm{K}) = \mathbb{R}.$$

2.2 Numerical system. Once our universe is assumed huge-finite and cyclic, with the underlying 1-dim cycle K of length $\mathcal{N}$, there is some absolute maximum of any *linear* units (length, time, energy,...) in any system. We assume that this maximum, counted in some absolute units, to be $\mathcal{N}$.

Example. Suppose at each point p of a set $\Pi(\mathrm{K})$ underlying a physical system Π there is a particle of energy E_p. How do we estimate the energy of the whole system? The natural count is

$$E = \sum_{p \in \Pi(K)} E_p \mod \mathcal{N}$$

because any additions along $\{0, 1, 2, \ldots \mathcal{N} - 1\}$ if it reaches the end value has to cyclically start from the beginning.

But note that if the system is relatively small, then we would expect $\sum_{p \in \Pi(K)} E_p << \mathcal{N}$, that is the calculations can be actually carried out in usual arithmetic, that is locally.

The following is going to be useful.

2.3 Lemma. Let $m \in \mathrm{K}$ is such that $\mathsf{Im}^{\mathrm{loc}}(m) = \mu \in \mathbb{R}_{>0}$, and

$$\Theta(\mathrm{K}) = \{(\omega, \mathbf{k}) \in \mathrm{K}^4 : \ \omega^2 = \mathbf{k}^2 + m^2\},$$

$$\Sigma(\mathrm{K}) := \{\mathbf{k} = (k_1, k_2, k_3) \in \mathrm{K}^3 : \ \exists \omega \in \mathrm{K} \ \omega^2 = \mathbf{k}^2 + m^2\}$$

where $\mathbf{k}^2 = k_1^2 + k_2^2 + k_3^2 + m^2$. And let

$$\Omega(\mathrm{K}) = \{\omega \in \mathrm{K} : \exists \mathbf{k} \in \mathrm{K}^3 \ \omega^2 = \mathbf{k}^2 + m^2\}$$

Then

$$\mathsf{Im}^{\mathrm{loc}}\Sigma(\mathrm{K}) = \mathbb{R}^3 \text{ and } \mathsf{Im}^{\mathrm{loc}}\Omega(\mathrm{K}) = \mathbb{R}_{\geq\mu} \cup -\mathbb{R}_{\geq\mu}. \tag{1}$$

We may assume that in local setting only positive ω are admissible then

$$\mathsf{Im}^{\mathrm{loc}}\Omega(\mathrm{K}) = \mathbb{R}_{\geq\mu}.$$

Proof. Clearly, Σ and Ω are projections of Θ,

$$\Omega(\mathrm{K}) = \mathrm{pr}_1\Theta(\mathrm{K}), \quad \Sigma(\mathrm{K}) = \mathrm{pr}_2\Theta(\mathrm{K}).$$

The approximation map

$$\mathsf{Im}^{\mathrm{loc}}: \ \Theta(\mathrm{K}) \to \Theta(\mathbb{R})$$

is by definition a map with domain $\Theta(\mathrm{K}_{\mathrm{loc}}) \subset \Theta(\mathrm{K})$ which preserves the algebraic relations (Zariki topology) and satisfies $\mathsf{Im}^{\mathrm{loc}}(\mathrm{K}_{\mathrm{loc}}) = \mathbb{R}$ as in 2.1. Such a map commutes with projection maps and hence (1). □

Commentary. $\Sigma(\mathrm{K})$ is the set of 3-momenta $\mathbf{k}$ over which there can be installed a particle of frequency ω such that $\omega^2 = \mathbf{k}^2 + m^2$. The lemma asserts that in local approximation this set looks like $\mathbb{R}^3$ and the set $\Omega(\mathrm{K})$ of possible ω looks like $\mathbb{R}_{\geq\mu}$ (in union with its negative mirror).

2.4 Now we want to introduce a map

$$\mathbf{k} \mapsto \omega_{\mathbf{k}}$$

such that

$$\{(\omega_{\mathbf{k}}, \mathbf{k}): \ \mathbf{k} \in \Sigma(\mathrm{K})\} \subseteq \Theta(\mathrm{K})$$

which locally behaves like $\mathbf{k} \mapsto \sqrt{\mathbf{k}^2 + m^2}$. We will require that when moving along axis of K^3 the triple $\mathbf{k} = (k_1, k_2, k_3)$ reaches $(\mathcal{N} - k_1, \mathcal{N} - k_2, \mathcal{N} - k_3) = -\mathbf{k}$ we get

$$\omega_{-\mathbf{k}} = -\omega_{\mathbf{k}} \tag{2}$$

This is clearly achievable when $\mathbf{k} \neq -\mathbf{k}$. In case this happen the components k_i of $\mathbf{k} = \mathbf{k}_0$ are either 0 or $-\frac{\mathcal{N}}{2}$ and $k_i^2 = 0$ ($\frac{\mathcal{N}}{4}$ is an integer because $\mathcal{N}$ is highly divisible). In each of the 8 cases

$$\mathbf{k}_0^2 + m^2 = m^2$$

and we assume

$$\omega_{\mathbf{k}_0} = m \tag{3}$$

2.5 As a corollary we have a well-defined notion of the sum

$$\mathrm{E}_0^{\mathrm{tot}}(\mathrm{K}) = \sum_{\mathbf{k}\in\Sigma(\mathrm{K})} \omega_{\mathbf{k}} = 8m \tag{4}$$

the value of which $8m << \mathcal{N}$.

Commentary. It is essential that the summation formula (4) is interpreted as global and K, respectively, is seen as approximating $\bar{\mathbb{C}}$. In this interpretation $\sqrt{\mathbf{k}^2 + m^2}$ is a square root of a complex number and it is natural to assign it both $\pm\sqrt{\mathbf{k}^2 + m^2}$ values.

2.6 Let $\sigma(\mathrm{K})$ be the number such that

$$0 \leq \sigma(\mathrm{K}) < \mathcal{N} \ \ \& \ \ \sigma(\mathrm{K}) = |\Sigma(\mathrm{K})| \,\mathrm{mod}\, \mathcal{N}$$

Working Conjectures (TODO)

$$|\Sigma(\mathrm{K})| = \mathrm{O}(\mathcal{N}^3) \tag{5}$$

$$\sigma(\mathrm{K}) = \mathrm{O}(\mathcal{N}). \tag{6}$$

3 Vacuum energy

3.1 Vacuum energy for a free global field. For a free field, classically, in convenient units, vacuum energy density

$$\rho_0(\mathbb{R}) = \int_{\mathbf{k}\in\mathbb{R}^3} d\mathbf{k}\, \sqrt{\mathbf{k}^2 + m^2} \tag{7}$$

We replace the definition, in accordance with 2.1 and 2.3 by the density

$$\rho_0(\mathrm{K}) := \frac{\mathrm{E}_0^{\mathrm{tot}}}{|\Sigma(\mathrm{K})|}$$

distributed over the 3-space $\Sigma(\mathrm{K})$

By 2.5

$$\rho_0(\mathrm{K}) = \mathrm{O}(\frac{1}{\mathcal{N}^3}). \tag{8}$$

Let us now look at the possible contribution of Higgs field interaction term ϕ^4. Its groundstate density is a constant $= v$ and we assume that $v \in \mathrm{K}$

and it corresponds to the contribution to energy at a given point $\mathbf{k}$, the whole contribution is

$$|\Sigma(\mathrm{K})| \cdot v = \sigma(\mathrm{K}) \cdot v$$

which is treated as an element of K, that is modulo $\mathcal{N}$.

Thus the contribution of interaction to the density is

$$\rho_{\mathrm{int}} = \frac{\sigma(\mathrm{K})}{|\Sigma(\mathrm{K})|} \cdot v = \mathrm{O}(\frac{1}{\mathcal{N}^2}) \cdot v$$

and the final estimate is

$$\rho_{\mathrm{vac}} = \rho_0(\mathrm{K}) + \rho_{\mathrm{int}}(\mathrm{K}) = \mathrm{O}(\frac{1}{\mathcal{N}^2}).$$

3.2 Now we address the question why it is consistent with the direct calculation, for example for the phonon field, supproted by experimental data, which returns much bigger values.

Phonon field vacuum energy Following [LancBl2014] and others the groundstate energy of periodic 1-dim field is

$$C \cdot \sum_{k=-N}^{N-1} |\sin k\frac{\pi}{N}| = 2C \cdot \sum_{k=0}^{N-1} \sin k\frac{\pi}{N} = 2C \cot\frac{\pi}{2N} \sim 4CN. \quad (9)$$

The answer is in the difference between **local** and **global** versions of models of the free field. The general theory is addressed in [Z26]. The phonon field by its physical nature is local, of some limited size in spacetime. Then the parameter N for this field is much much smaller than $\mathcal{N}$, the parameter for global fields. One can say that operations with numbers $k < N$ never reaches $\mathcal{N}$ and so these numbers should be treated like usual numbers (i.e. feasible numbers of [Z26] and [Z25-1]). This explains the different method of summation.

References

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